

線形代数 1 演習 6 解答例

練習 6.1 次の行列の逆行列を求めよ．検算も書くこと！

$$(1) \begin{pmatrix} 2 & -1 & 0 \\ 2 & -1 & -1 \\ 1 & 0 & -1 \end{pmatrix} \quad (2) \begin{pmatrix} -3 & -6 & 2 \\ 3 & 5 & -2 \\ 1 & 3 & -1 \end{pmatrix} \quad (3) \begin{pmatrix} 1 & -1 & -3 \\ 1 & 1 & -1 \\ -1 & 1 & 5 \end{pmatrix} \quad (4) \begin{pmatrix} 2 & 0 & 1 & 0 \\ 0 & -1 & 1 & -2 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 3 \end{pmatrix}$$

解答 (1)

$$\begin{aligned} & \left(\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 2 & -1 & -1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{array} \right) \rightarrow \left(\begin{array}{ccc|ccc} 0 & -1 & 2 & 1 & 0 & -2 \\ 0 & -1 & 1 & 0 & 1 & -2 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{array} \right) \rightarrow \\ & \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & -1 & 2 & -1 & 0 & 2 \\ 0 & -1 & 1 & 0 & 1 & -2 \end{array} \right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & -1 & 2 & -1 & 0 & 2 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array} \right) \rightarrow \\ & \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & -2 & -1 & 0 & 2 \\ 0 & 0 & -1 & 1 & -1 & 0 \end{array} \right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & 1 & -1 & 0 \end{array} \right) \end{aligned}$$

【検算】

$$\begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & 2 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 2 & -1 & 0 \\ 2 & -1 & -1 \\ 1 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

よって求める逆行列は

$$\begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & 2 \\ 1 & -1 & 0 \end{pmatrix}.$$

(2)

$$\begin{aligned} & \left(\begin{array}{ccc|ccc} -3 & -6 & 2 & 1 & 0 & 0 \\ 3 & 5 & -2 & 0 & 1 & 0 \\ 1 & 3 & -1 & 0 & 0 & 1 \end{array} \right) \rightarrow \left(\begin{array}{ccc|ccc} 0 & 3 & -1 & 1 & 0 & 3 \\ 0 & -4 & 1 & 0 & 1 & -3 \\ 1 & 3 & -1 & 0 & 0 & 1 \end{array} \right) \rightarrow \\ & \left(\begin{array}{ccc|ccc} 1 & 3 & -1 & 0 & 0 & 1 \\ 0 & -4 & 1 & 0 & 1 & -3 \\ 0 & 3 & -1 & 1 & 0 & 3 \end{array} \right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & -2 \\ 0 & -1 & 0 & 1 & 1 & 0 \\ 0 & -3 & -1 & 1 & 0 & 3 \end{array} \right) \rightarrow \\ & \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & -2 \\ 0 & -1 & 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 4 & 3 & 3 \end{array} \right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -4 & -3 & -3 \end{array} \right) \end{aligned}$$

【検算】

$$\begin{pmatrix} -1 & 0 & -2 \\ -1 & -1 & 0 \\ -4 & -3 & -3 \end{pmatrix} \begin{pmatrix} -3 & -6 & 2 \\ 3 & 5 & -2 \\ 1 & 3 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

よって求める逆行列は

$$\begin{pmatrix} -1 & 0 & -2 \\ -1 & -1 & 0 \\ -4 & -3 & -3 \end{pmatrix}$$

(3)

$$\begin{aligned} & \left(\begin{array}{ccc|ccc} 1 & -1 & -3 & 1 & 0 & 0 \\ 1 & 1 & -1 & 0 & 1 & 0 \\ -1 & 1 & 5 & 0 & 0 & 1 \end{array} \right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & -1 & -3 & 1 & 0 & 0 \\ 0 & 2 & 2 & -1 & 1 & 0 \\ 0 & 0 & 2 & 1 & 0 & 1 \end{array} \right) \rightarrow \\ & \left(\begin{array}{ccc|ccc} 1 & -1 & -3 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1/2 & 1/2 & 0 \\ 0 & 0 & 1 & 1/2 & 0 & 1/2 \end{array} \right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1/2 & 1/2 & 0 \\ 0 & 1 & 1 & -1/2 & 1/2 & 0 \\ 0 & 0 & 1 & 1/2 & 0 & 1/2 \end{array} \right) \rightarrow \\ & \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/2 & 1/2 & 1 \\ 0 & 1 & 0 & -1 & 1/2 & -1/2 \\ 0 & 0 & 1 & 1/2 & 0 & 1/2 \end{array} \right) \end{aligned}$$

【検算】

$$\begin{pmatrix} 3/2 & 1/2 & 1 \\ -1 & 1/2 & -1/2 \\ 1/2 & 0 & 1/2 \end{pmatrix} \begin{pmatrix} 1 & -1 & -3 \\ 1 & 1 & -1 \\ -1 & 1 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

したがって求める逆行列は

$$\begin{pmatrix} 3/2 & 1/2 & 1 \\ -1 & 1/2 & -1/2 \\ 1/2 & 0 & 1/2 \end{pmatrix}$$

(4)

$$\begin{aligned} & \left(\begin{array}{cccc|cccc} 2 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & -2 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 3 & 0 & 0 & 0 & 1 \end{array} \right) \rightarrow \left(\begin{array}{cccc|cccc} 0 & 0 & -1 & 0 & 1 & 0 & -2 & 0 \\ 0 & -1 & 1 & -2 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 3 & 0 & 0 & 0 & 1 \end{array} \right) \rightarrow \\ & \left(\begin{array}{cccc|cccc} 0 & 0 & -1 & 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 3 & 0 & 0 & 0 & 1 \end{array} \right) \rightarrow \left(\begin{array}{cccc|cccc} 0 & 0 & -1 & 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 3 & -1 & -3 & 2 & -2 \end{array} \right) \rightarrow \\ & \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 & -3 & 2 & -2 \\ 0 & 0 & 1 & 0 & -1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{array} \right) \end{aligned}$$

【検算】

$$\begin{pmatrix} 1 & 0 & -1 & 0 \\ -1 & -3 & 2 & -2 \\ -1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 1 & 0 \\ 0 & -1 & 1 & -2 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

したがって求める逆行列は

$$\begin{pmatrix} 1 & 0 & -1 & 0 \\ -1 & -3 & 2 & -2 \\ -1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$