ERRATA
FOR HOMOTOPY EXACT SEQUENCES AND ORBIFOLDS (PRELIMINARY VERSION)

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In the paper, we gave a homotopy exact sequence for a fibration that is not necessarily proper and separable. The first term of this exact sequence is the étale fundamental group of a geometric fiber. In this article, we add an omitted condition on the choice of this geometric fiber in the case where the fibration is not proper, and correct the other errors. The author thanks Professors Hélène Esnault and Vasudevan Srinivas for pointing out this omitted condition and Professor Yuichiro Hoshi for his comments on this error.

Definition [2, 15.7.1]**.** Let $f\colon X \to S$ be a morphism between schemes, and s a point on S. Then f is said to be *proper at s* if there exists an open neighborhood U of s in S such that the base change of f via the open immersion $U \to S$ is proper.

Definition 4.25 and Theorems 1.1 and 4.22. Take a geometric fiber $i\colon X_0 \to X$ of f and a geometric point $\overline{x}_0$ on X_0. Put $\overline{x} := i(\overline{x}_0)$ and $\overline{s} := f(\overline{x})$. We denote the image of $\overline{s}$ on S by s. In Definition 4.25 and Theorem 1.1, we assume that s satisfies the following condition:

(B-0) s is a regular point on S and not contained in the closure of $\operatorname{Supp} B$.

In Theorems 1.1 and 4.22, we assume that s satisfies one of the following conditions:

(B-1) s is the generic point of S;
(B-2) f is proper at s.

Remark. (a) In the definition of $\pi_1(S, B, \overline{s})$ (Definition 3.19), Condition (B-0) is assumed to be satisfied. (b) If (B-1) or (B-2) is satisfied, then Condition (C) (Definition 4.17) implies that there exists an open neighborhood U of s in S such that any fiber of f over U is geometrically connected (see [2, 9.7.7] for (B-1)). (c) In Theorem 1.1, (B-1) implies (B-0). (d) Both Theorems 1.1 and 4.22 are incorrect without (B-1) or (B-2) [3, X.1.10] (see also [4, 1.10 (4)] for Theorem 4.22).

Proof of Theorem 4.22. We have only to prove that $\operatorname{Ker} f_* \subset \operatorname{Im} i_*$. We have to show the following [3, V.6.11]: let $\xi\colon Y \to X$ be a connected étale covering space; take the base change $\xi_0\colon Y_0 \to X_0$ of ξ via i; assume that there exists a connected component Y_1 of Y_0 such that the restriction $\xi_0|_{Y_1}\colon Y_1 \to X_0$ is an isomorphism; then there exists an étale covering space $S' \to S$ such that $Y \cong X \times_S S'$ over X.

Choose ξ and Y_1 as above. Take the normalization $Y \xrightarrow{h} S' \xrightarrow{u} S$ of S in the composite $Y \xrightarrow{\xi} X \xrightarrow{f} S$. Then S' is normal and integral (Propositions 4.5 (1)), $u\colon S' \to S$ is an étale covering space (Propositions 4.5 (2) and 4.8 (3)), and (Y, S', h) satisfies Condition (C) (Proposition 4.5 (3)). Take the factorizations $Y \xrightarrow{\xi'} X' \xrightarrow{u'} X$ of ξ and $Y \xrightarrow{\xi'} X' \xrightarrow{f'} S'$ of h given by Proposition 4.8. Note that $X' \cong X \times_S S'$ over

X since u is étale. Thus, we have only to prove that ξ' is an isomorphism. Since X' is connected and ξ' is finite and étale (Proposition 4.8 (1)), we have to show that $\deg \xi' = 1$.

Take the base change $Y_0 \overset{\xi'_0}{\to} X'_0 \overset{u'_0}{\to} X_0$ of the sequence $Y \overset{\xi'}{\to} X' \overset{u'}{\to} X$ via i. Then $\deg \xi' = \deg \xi'_0$. Put $X'_1 := \xi'_0(Y_1)$. Since ξ'_0 is finite and étale, the image X'_1 is a connected component of X'_0. We denote the restriction of $Y_0 \overset{\xi'_0}{\to} X'_0 \overset{u'_0}{\to} X_0$ to Y_1 and X'_1 by $Y_1 \overset{\xi'_1}{\to} X'_1 \overset{u'_1}{\to} X_0$. Since $\deg \xi'_1 \cdot \deg u'_1 = \deg \xi_0|_{Y_1} = 1$, the equality $\deg \xi'_1 = 1$ holds. Since any point on $u^{-1}(s)$ satisfies (B-1) or (B-2) with respect to both f' and h, any fiber of f' and h over $u^{-1}(s)$ is geometrically connected (Remark (b)). Thus, since X_0 is the geometric fiber of f over s, both X'_0 and Y_0 consist of d connected components, where $d := \deg u$. Thus, the equality $\deg \xi'_0 = \deg \xi'_1$ holds. As a result, we obtain the equalities $\deg \xi' = \deg \xi'_0 = \deg \xi'_1 = 1$.

Proof of Theorem 1.1. We have only to change the proof of Theorem 4.22 in the following way: replace "an étale covering space $S' \to S$" by "an orbifold étale covering space $(S', B') \to (S, B)$"; replace "$X \times_S S'$" by "the normalization of $X \times_S S'$"; use Theorem 4.24 instead of Proposition 4.8 (3).

Definition 3.6. Take the Henselization $\mathcal{O}^{\mathrm{h}}_{S,s}$ of $\mathcal{O}_{S,s}$: $\mathcal{O}_{S,s} \subset \mathcal{O}^{\mathrm{h}}_{S,s} \subset \mathcal{O}^{\mathrm{sh}}_{S,s}$. Put $J_s := \operatorname{Frac} \mathcal{O}^{\mathrm{h}}_{S,s}$. When the residue field of $\mathcal{O}_{S,s}$ is not separably closed (i.e., $J_s \neq K_s$), we have to add the following condition: B_s/J_s is Galois.

Remark. The extension theorem of valuations [5, II.8.1] implies the following: put $I_s := \operatorname{Frac} \mathcal{O}_{S,s}$; take a separable algebraic valuation field extension L of I_s; then an I_s-embedding $L \to \overline{K_s}$ between valuation fields is unique up to conjugate of $\overline{K_s}$ over J_s. In particular, in Definition 3.10, the assumption on u at s' is equivalent to the following: the image of any I_s-embedding $B'_{s'} \to \overline{K_s}$ between valuation fields contains B_s. Thus, this assumption does not depend on the choice of an I_s-embedding $K_s \to K'_{s'}$ in Lemma 3.9, which is unique up to conjugate of K_s over J_s.

Lemma 3.14. Replace the first statement by the following [1, Alg. comm., Chap. 8, §8, Prop. 3]: the L-algebra $M \otimes_L N$ is L-isomorphic to a finite product of field extensions of L in $\widetilde{Q}$ one of whose factors is L-isomorphic to Q.

Remark. In the proof of Proposition 5.4, the above fact implies that we may take Y'_K and Y' so that $Y'_K \cong Y_K$ over P_K and $Y' \cong Y$ over P, respectively.

Proof of Theorem 3.21. In (G1), the disjoint union of a finite number of objects in $\mathcal{C}_{(S,B)}$ is their direct sum. In (G3), the target of u is an arbitrary object in $\mathcal{C}_{(S,B)}$. **After Definition 3.22.** Replace "(étale)" by "(finite étale)". **Proof of Proposition 4.5.** Replace "faithfully" by "transitively" (i.e., S'/S is a Galois covering). **Proposition 4.8 (b).** Replace "$u' \circ h$" by "$u \circ h$". **Example 4.16.** Replace "three" by "one" (i.e., $E = E_{\mathrm{red}}$). **Proof of Proposition 5.3.** Replace "n_i/d" by "n_s/d". **Lemma 5.10.** Replace "$\{X_iY_j\}_{0\le i,j\le n}$" by "$\{X_iY_i\}_{0\le i<n}$". **Lemma 6.3.** Choose finite flat surjective morphisms $w\colon D \to C'$ and $u\colon C' \to C$ satisfying that $v = u \circ w$. Replace "$h = f' \circ \xi'$" by "$w \circ h = f' \circ \xi'$". **Proof of Lemma 6.14.** Put $C' := D$. By (1) and (2) of Lemma 6.12, we may assume that ξ' is an isomorphism. **Proof of Theorem 6.23, Paragraph 2, Line 10.** Replace "ξ' is étale" by "ξ is étale". **Remark 6.24.** Replace "Hopf surface" by "primary Hopf surface" (i.e., $n = 1$). **Lemmas A.1, A.5, and A.6.** Replace

"$q^2 - 1$" by "$q - 1$ or $q + 1$". **Proof of Lemma A.5.** Replace "$XYZ = 1$" by "$XYZ = I$". **Proposition A.7 (1).** Replace "$abc \mid \sharp G$" by "Each of a, b, and c divides $\sharp G/(q + 1)$ or $\sharp G/(q - 1)$". **Theorem A.9.** Replace "$p^2 - 1$" by "$p - 1$ or $p + 1$".

References

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