



RIMS conference “ Automorphic form, automorphic L -functions and related topics ”

Abstract

Jan. 24 (Mon.)

13:20 – 14:20 **Shingo Sugiyama** (Nihon University)

Title : On a weighted density of low-lying zeros of symmetric power L -functions

Abstract : As evidence of the Hilbert-Pólya conjecture, Montgomery observed a coincidence of the two distributions: the one is on zeros of the Riemann zeta function, and the other is on eigenvalues of random matrices (Gaussian Unitary Ensemble). Later, Katz and Sarnak suggested a philosophy, what is called the Density Conjecture, that low-lying zeros of general L -functions in a family has a density which is the same as that of eigenvalues of random matrices in a classical compact Lie group. A weighted version of the Density Conjecture has been studied by Kowalski, Saha, and Tsimerman (2012) for the spinor L -functions of Siegel modular forms, and by Knightly and Reno (2019) for the standard L -functions of elliptic modular forms. In this talk, we give an explicit form of a weighted density of low-lying zeros of symmetric power L -functions attached to Hilbert modular forms, and compare this with density arising from random matrix theory.

14:35 – 15:35 **Seiji Kuga** (Kyushu University)

Title : A resolvent trace formula of Jacquet-Zagier type for Hilbert Maass forms

Abstract : Zagier found a generalized Eichler-Selberg trace formula involving symmetric square L -functions by means of the Rankin-Selberg method in computing the trace formula of Hecke operators of elliptic cusp forms. Moreover, Sugiyama and Tsuzuki generalized Zagier’s formula for Hilbert modular forms with square-free levels in adelic setting and proved a non-vanishing property of symmetric square L -functions. In this talk, we give an analogy of Sugiyama-Tsuzuki’s trace formula for Hilbert Maass forms by using the resolvent kernel function of the Laplace operator as the test function at infinite places.

15:50 – 16:50 **Kohta Gejima** (Osaka City University Advanced Mathematical Institute)

Title : Inner product formula for Shintani lift

Abstract : Skoruppa-Zagier constructed a certain lifting map from the space of holomorphic Jacobi forms of integral weight to the space of elliptic cusp forms of integral weight. Gross-Kohnen-Zagier constructed the kernel function for this lifting map and obtained the inverse lifting map as the adjoint with respect to the Petersson scalar products.

In this talk, we reformulate the lifting map from the space of elliptic cusp forms of integral weight to the space of holomorphic Jacobi forms of integral weight in terms of theta lifts and investigate the Petersson norm for this lifting map. This is joint work with Atsushi Murase.

Jan. 25 (Tue.)

9:45 – 10:45 **Miki Hirano** (Ehime University), **Taku Ishii** (Seikei University),
Tadashi Miyazaki (Kitasato University)

Title : Whittaker functions on $GL(4, \mathbb{R})$ and archimedean zeta integrals

Abstract : We give explicit formulas of Whittaker functions for all irreducible generic representations of $GL(4, \mathbb{R})$. As an application we give test vectors for Bump-Friedberg archimedean zeta integrals which interpolate the standard and the exterior square L -functions simultaneously. If time allows, we mention Jacquet-Shalika integrals for the exterior square L -functions.

11:00 – 12:00 **Hiroaki Narita** (Waseda University)

Title: An explicit construction of CAP forms on $O(1,5)$ for the case of any definite quaternion algebras

Abstract: In the RIMS workshop of two years ago we presented an explicit lifting construction of CAP forms on the non-quasisplit group $O(1,5)$ for the case of definite quaternion algebras with prime discriminants. In this talk we generalize this result to the case of those with any discriminants. Non-holomorphic theta lifts formulated by Borcherds lead to these CAP forms. We have an explicit formula for the Fourier coefficients of the CAP forms and determine all local components of the cuspidal representations generated by them. The explicit structures of the CAP representations just mentioned are obtained by a detailed study on the Hecke theory of the (adelized) CAP forms. This is a joint work with Ameya Pitale and Siddhesh Wagh.

13:20 – 14:20 **Seidai Yasuda** (Hokkaido University)

Title : Local newforms for the general linear groups

Abstract : Through Casselman's work, Atkin-Lehner theory of newforms became a local theory. Jacquet, Piatetski-Shapiro, and Shalika generalized the local theory to generic representation of the general linear groups. Since then, the theory of local newforms for the generic representations of classical groups has been studied by many people. In this talk, I would like to propose how to construct the theory of local newforms for the irreducible smooth representations, which are not necessarily generic, of the general linear groups. The talk is based on the joint work with Hiraku Atobe and Satoshi Kondo.

14:35 – 15:35 **Nozomi Ito** (Kyoto University)

Title : On branching laws of Sp $_n$ representations and local zeta integrals

Abstract : In this talk, we discuss the branching laws of the Sp $_n$ representations associated to irreducible generic representations of GL_2 over p -adic fields with respect to block diagonal subgroups. We introduce an analogue of doubling zeta integrals to study the branching laws and give some results on it. These results are part of the local theory of the Miyawaki lifting for unitary groups.

15:50 – 16:50 (IST 8:50-9:50) **Maxim Gurevich** (Technion- Israel Institute of Technology)

Title: On finite-length smooth representations of p -adic GL_n

Abstract : While the local Langlands reciprocity is a known elegant approach to irreducible smooth representations of general linear groups, the nature of reducible finite-length representations remains elusive for many concrete needs. In particular, canonical models for realizing a given L -parameter, or its relative properties, often rely first on our understanding of some particulars of this structure

(e.g. Langlands quotient theorem.). Yet, a systematic study of these categories requires adopting new outlooks.

I will survey recent developments on this topic, with emphasis on tools from the study of type A quantum groups and the well developed representation theory attached to them. As a specific avenue of research, I would like to present how the combinatorial RSK transform may be harnessed (through a work initiated with Erez Lapid) to produce new effective models for irreducible representations. Such construction imports themes occurring in modular representation theory (such as that of cyclotomic Hecke algebras) into the p -adic setting.

Jan. 26 (Wed.)

9:45 – 10:45 **Hirotaka Kakuham** (Kyoto University)

Title : Local theta correspondences for quaternionic dual pairs and Langlands parameters

Abstract : Let F be a local field of characteristic zero and let (G, H) be a quaternionic dual pair over F . Then, there is a map from $\text{Irr}(G)$ to $\text{Irr}(H) \cup \{0\}$, which is called the local theta correspondence. We want to describe the correspondence in terms of the Langlands parameters which are completely formulated by Kaletha. In this talk, we discuss this problem for the Archimedean cases and some non-Archimedean cases.

11:00 – 12:00 (10:00 - 11:00 in Singapore time) **Lei Zhang** (National University of Singapore)

Title: The unramified computation of Bessel model cases

Abstract : In this talk, we will discuss the unramified calculation of the local Rankin-Selberg Integrals of quasi-split groups for the Bessel model cases. More precisely, we extend Soudry's results on the Bessel models for the split special orthogonal group cases to all the quasi-split $\text{SO}(n)$ and $\text{U}(n)$ cases. The goal is to obtain the tensor product L -functions over all pure inner forms of $\text{SO}(n)$ and $\text{U}(n)$ with the general linear groups. We follow Soudry's approach by using analytic continuation from Whittaker models to the Bessel models, by expressing the local integrals in terms of the Whittaker models of the representations. Then we may reduce to the known cases due to Soudry and Kaplan. This is a joint work with Dihua Jiang and David Soudry.

13:20 – 14:20 **Shuji Horinaga** (Tokyo University of Science)

Title : On the algebraicity of critical L values attached to vector valued Siegel cusp forms

Abstract : The algebraicity of critical values of automorphic L -functions is of great interest in connection with the Deligne conjecture. In the 1990s, Goro Shimura proved the algebraicity of L -functions associated with scalar-valued Siegel modular forms using the holomorphic projection. In this talk, I will give a generalization of the holomorphic projection and introduce its application to vector-valued Siegel modular forms. This work is joint work with Ameya Pitale (Oklahoma University), Ralf Schmidt (University of North Texas) and Abhishek Saha (Queen Mary University of London).

14:35 – 15:35 (13:35 - 14:35 in Taiwan time) **Shih-Yu Chen** (Institute of Mathematics, Academia Sinica)

Title: On Deligne's conjecture for symmetric fifth L -functions of modular forms

Abstract : Deligne proposed a conjecture on the algebraicity of values of motivic L -functions at critical points in terms of motivic periods. A special class of examples are the symmetric n -th power L -functions

associated to modular forms. The conjecture was considered by various authors when $n = 1, 2, 3, 4, 6$. In this talk, we will introduce a proof of the conjecture when $n = 5$.

15:35 – 16:00 **On the RIMS workshop next academic year. (Japanese Only)**

Jan. 27 (Thu.)

9:45 – 10:45 **Yuya Murakami** (Tohoku University)

Title: Quantum modularity of quantum invariants of non-Seifert manifolds

Abstract : Witten-Reshetikhin-Turaev (WRT) invariants are important quantum invariants of 3-manifolds. Gukov-Pei-Putrov-Vafa constructed q -series invariants called homological blocks in a physical way in order to categorify WRT invariants and conjectured that radial limits of homological blocks are WRT invariants. Their conjecture is proved only for Seifert manifolds. In this talk, I will prove their conjecture for some non-Seifert manifolds. As a consequence, it turns out that their WRT invariants yield quantum modular forms of depth two. This is a joint work with Akihito Mori in Tohoku University.

11:00 – 12:00 **Toshiki Matsusaka** (Nagoya University)

Title : Modular transformation formulas for homological blocks

Abstract : Recently, Gukov-Pei-Putrov-Vafa introduced important q -series called homological blocks for any plumbed 3-manifolds. In this talk, for any Seifert fibered homology 3-sphere, we give modular transformation formulas of homological blocks. Moreover, based on the modular transformation formulas, we have explicit asymptotic expansion formulas for the Witten-Reshetikhin-Turaev invariants, which give a new proof of a version by Andersen of the Witten asymptotic conjecture. This is joint work with Yuji Terashima (Tohoku University).

13:20 – 14:20 **Kazuma Ohara** (The University of Tokyo)

Title : Hecke algebras for tame supercuspidal types

Abstract : Let G be a connected reductive group over a non-archimedean local field F . When we study the category of smooth complex representations of $G(F)$, constructing types and determining the structure of Hecke algebras associated with the types are important. In 2001, Yu constructed many types which are called tame supercuspidal types and conjectured that Hecke algebras associated with these types are isomorphic to Hecke algebras associated with depth-zero types, which are well-understood. In this talk, we explain the proof of this conjecture.

14:35 – 15:35 **Masao Oi** (Kyoto University, Hakubi Center)

Title : On Iwahori-Hecke algebras and local L -factors of unramified representations

Abstract : Unramified representations are one of the most fundamental classes in representations of p -adic reductive groups. The unramified Hecke algebra acts on unramified representations and the local Langlands correspondence and the local L -factors of unramified representations are constructed based on this action. On the other hand, the Iwahori-Hecke algebra, which is larger than the unramified Hecke algebra, also acts on unramified representations. In Taylor's thesis (1988), it was observed that the spin L -functions of unramified representations of $\mathrm{GSp}(4)$ have a simple expression in terms of the Iwahori-Hecke algebra. In this talk, I will explain about a generalization of Taylor's result to split reductive groups. This is joint work with Ryotaro Sakamoto (RIKEN) and Hiroyoshi Tamori (Hokkaido University).

15:50 – 16:50 (UTC 6:50-7:50) **Abhishek Saha** (Queen Mary University of London)

Title : The Manin constant and p -adic bounds on denominators of the Fourier coefficients of newforms at cusps

Abstract : The Manin constant c of an elliptic curve E over \mathbb{Q} is the nonzero integer that scales the differential ω_f determined by the normalized newform f associated to E into the pullback of a Néron differential under a minimal modular parametrization $\phi: X_0(N)_{\mathbb{Q}} \rightarrow E$. Manin conjectured that $c = \pm 1$ for optimal parametrizations. I will talk about some recent joint work that makes progress towards this conjecture by establishing an integrality property of ω_f necessary for this conjecture to hold. Our result implies in particular that $c \mid \deg(\phi)$ under a minor assumption at 2 and 3 that is not needed for cube-free N or for parametrizations by $X_1(N)_{\mathbb{Q}}$.

We reduce the above results to p -adic bounds on denominators of the Fourier expansions of f at *all* the cusps of $X_0(N)_{\mathbb{C}}$. We succeed in proving stronger bounds in the more general setup of newforms of general weight and levels by approaching the problem representation-theoretically. These results follow from sharp lower bounds that we prove for the p -adic valuations of the values of the Whittaker newform of GL_2 over a nonarchimedean local field of characteristic 0, using techniques that were originally developed by me in the context of the analytic sup-norm problem. For local fields of odd residue characteristic, this allows us to ultimately reduce to the classical facts about p -adic valuations of Gauss sums. To overcome obstacles at 2, we analyze nondihedral supercuspidal representations of $\mathrm{GL}_2(\mathbb{Q}_2)$.

This is joint work with Kęstutis Česnavičius and Michael Neururer.

Jan. 28 (Fri.)

9:45 – 10:45 **Yuichi Sakai** (Multiple Zeta Research Center, Kyushu University)

Title : Modular linear differential equations and generalized Rankin-Cohen brackets

Abstract : The aim in this talk is to give expressions for modular linear differential equations (MLDEs) of any order. We also give more uniform descriptions of MLDEs in terms of canonically defined higher Serre derivatives and an extension of Rankin-Cohen brackets. This is a joint work with Kiyokazu Nagatomo and Don Zagier.

11:00 – 12:00 **Shin-ya Koyama** (Toyo University)

Title : Chebyshev's Bias for Ramanujan's τ -function via the Deep Riemann Hypothesis

Abstract : A reason for the emergence of Chebyshev's bias is elucidated. The Deep Riemann Hypothesis (DRH) enables us to reveal that the bias is a natural phenomenon in the sense that it makes a well-balanced disposition of the whole sequence of primes. This new formulation of the bias is by virtue of the behavior of the Euler products of L -functions in the critical strip. By means of a weighted counting function of primes, we succeed in expressing magnitudes of the deflection by a certain asymptotic formula under the assumption of DRH.

As an application we prove under DRH that a weighted sum of Ramanujan's τ -function has a bias to being positive.