大川領 a

《 (-2) blow up formula 》

・ADHM data

$Q = \mathbb{C}^2$, $\Gamma = \{\pm id_Q\} \subset SL(Q)$

$V = V_0 \oplus V_1$ $\quad W = W_0 \oplus W_1$ $\quad$ = graded vector sp. (Γ-rep)

$Q^\vee = \mathrm{Hom}_{\mathbb{C}}(Q, \mathbb{C})$

$$\mu = \mathbb{M} = \mathrm{Hom}_\Gamma(Q^\vee \otimes V, V) \to \mathrm{Hom}_\Gamma(\Lambda^2 Q^\vee \otimes V, V)$$
$$\oplus\ \mathrm{Hom}_\Gamma(W, V)$$
$$\oplus\ \mathrm{Hom}_\Gamma(\Lambda^2 Q^\vee \otimes V, W)$$

$$(B, z, w) \longmapsto [B \wedge B] + zw$$

$$\begin{array}{ccccc} \Lambda^2 Q^\vee \otimes V & \hookrightarrow & Q^\vee \otimes Q^\vee \otimes V \xrightarrow{id_{Q^\vee} \otimes B} & & Q^\vee \otimes V \\ {\scriptstyle w}\downarrow & & & & \downarrow{\scriptstyle B} \\ W & & \xrightarrow[\quad z \quad]{} & & V \end{array}$$

Def. ADHM data on (W, V) is an element of $\mu^{-1}(0)$

$G = GL(V_0) \times GL(V_1)$

$\zeta = (\zeta_0, \zeta_1) \in \mathbb{R}^2$

$\zeta(V_0 \oplus V_1) = \zeta_0 \dim V_0 + \zeta_1 \dim V_1$

<u>Def.</u> $(B,z,w) \in \mu^{-1}(0)$ is ζ-semistable if (1), (2) hold

$\overset{\text{def.}}{\Longleftrightarrow}$

(1) $S \subset V = \Gamma$ - subrep.

$B(Q^\vee \otimes S) \subset S$ $\Rightarrow$ $\zeta(V/S) \geq 0$

$\mathrm{Im}\, z \subset S$

(2) $T \subset V = \Gamma$ - subrep.

$B(Q^\vee \otimes T) \subset T$ $\Rightarrow$ $\zeta(T) \leq 0$

$\Lambda^2 Q^\vee \otimes T \subset \ker w$

$\mu^{-1}(0)^\zeta := \{ (B,z,w) \in \mu^{-1}(0) \mid \zeta\text{-semistable} \}$

$M^\zeta(w,v) := \mu^{-1}(0)^\zeta / G$

$w = (\dim W_0, \dim W_1)$
$v = (\dim V_0, \dim V_1)$

<u>Fact</u> (Nakajima) ζ = generic $\Rightarrow$ $M^\zeta(w,v)$ = smooth

<u>Integration</u> $T = \mathbb{C}^*$, $\mathbb{T} = T^2 \times T^r \curvearrowright M^\zeta(w,v)$ $\circlearrowleft GL(Q) \times GL(W)$

$(r = \dim W_0 + \dim W_1)$

<u>Fact.</u> $M^\zeta(w,v)^{\mathbb{T}}$ = finite set $\underset{t=(t_1,t_2)}{\underset{\psi}{T^2}} \longrightarrow \underset{e=(e_1,\dots,e_r)}{T^r}$

$\psi \in A^*_{\mathbb{T}}(M^\zeta(w,v))$ = $\mathbb{T}$ - equivariant Chow ring.

$$\int_{M^\zeta(w,v)} \psi = \sum_{p \in M^\zeta(w,v)^{\mathbb{T}}} \frac{\psi|_p}{e(T_p M^\zeta(w,v))} \in \mathrm{Frac}\, A^*_{\mathbb{T}}(pt) = \mathbb{Q}(\vec{\varepsilon}, \vec{a})$$

equiv. Euler class.

$\vec{\varepsilon} = (\varepsilon_1, \varepsilon_2)$, $\varepsilon_1 = c_1(t_1)$, $\varepsilon_2 = c_1(t_2)$; $\vec{a} = (a_1, \dots, a_r)$, $a_1 = c_1(e_1)$, $a_r = c_r(e_r)$

Integuands.

$k = -v_0 + v_1 - \frac{w_1}{2}$

$\mathcal{V}_i = [\mu^{-1}(0)^\zeta \times V_i / G] \to M^\zeta(w,v)$

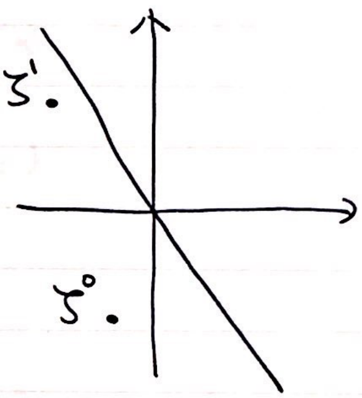
$\mathcal{W}_i = [\mu^{-1}(0)^\zeta \times W_i / G] \to M^\zeta(w,v)$

$$\Psi_+ = 2c_1(\mathcal{V}_0) - 2c_1(\mathcal{V}_1) + c_1(\mathcal{W}_1) + (\varepsilon_1+\varepsilon_2)\left(2k + \frac{w_1}{2}\right)$$

$$\Psi_- = 2k(\varepsilon_1+\varepsilon_2) - \Psi_+$$

Main Thm

$$\int_{M^{\zeta'}(w,v)} (\Psi_\pm)^d = \int_{M^{\zeta^0}(w,v)} (\Psi_\pm)^d$$



when $\begin{cases} d \leq 2-4k & \text{for } \Psi_+ \\ d \leq 2r+2-4k & \text{for } \Psi_- \end{cases}$

Remark $\hat{Z}^k(\vec{\varepsilon}, \vec{a}, q, t) = \sum q^{v_0 + \frac{w_1}{4}} \int_{M^{\zeta^k}(w,v)} \exp(-\Psi_+, t)$

fixed $k = -v_0 + v_1 - \frac{w_1}{2}$ $\quad (k=0,1)$

$w = (w_0, w_1)$

k=1

$M^{\zeta'}(w,v)$ is isom. to moduli of framed sheaves on $|\mathcal{O}_{\mathbb{P}^1}(-2)|$

$Z'(\vec{\varepsilon}, \vec{a}, q, t)$ is written in terms of Hirota diff. of Nekrasov function

$$Z_{\mathbb{P}^2}(\vec{\varepsilon}, \vec{a}, q) = \sum_{n=0}^{\infty} q^n \int_{M(r,n)} 1$$

$M(r,n)$ = moduli of framed sheaves on $\mathbb{P}^2$

$$\int_{M(r,n)} 1 = \sum_{p \in M(r,n)^T} \frac{1}{e(T_p M(r,n))} \in A_\pi^*(pt)$$

$$\sum \frac{1}{\Lambda_{-1} T_p M(r,n)} \in K_\pi(pt)$$