On the inverse problem of the discrete calculus of variations

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One of the most powerful tools in Mathematical Physics since Euler and Lagrange is the *calculus of variations*. The variational formulation of mechanics where the equations of motion arise as the minimum of an *action functional* (the so-called *Hamilton's principle*), is fundamental in the development of theoretical mechanics and its foundations are present in each textbook on this subject [1, 3, 6]. Beside this, the application of calculus of variations goes beyond mechanics as many important mathematical problems, e.g. the isoperimetrical problem and the catenary, can be formulated in terms of calculus of variations.

An important problem regarding the calculus of variations is to determine which system of differential equations are Euler–Lagrange equations for some variational problem. This problem has a long and interesting history, see e.g. [4]. The general case of this problem remains unsolved, whereas several important results for particular cases were presented during the 20th century.

In this talk we present some conditions on the existence of a Lagrangian in the discrete scalar setting. We will introduce a set of differential operators called annihilation operators. We will use these operators to reduce the functional equation governing of existence of a Lagrangian for a scalar difference equation of arbitrary even order $2k$, with $k > 1$ to the solution of a system of linear partial differential equations. Solving such equations one can either find the Lagrangian or conclude that it does not exist.

We comment the relationship of our solution of the inverse problem of the discrete calculus of variation with the one given in [2], where a result analogous to the homotopy formula [5] for the continuous case was proven.

References

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