PRODUCING KLEIN BOTTLES BY TWO DISTINCT DEHN FILLINGS

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Let M be a compact, connected and orientable 3-manifold whose boundary contains a 2-torus T. Suppose that M is hyperbolic, and there exist two distinct simple closed curves γ_1, γ_2 on T such that gluing along a solid torus V_i (for $i = 1$ or 2) in such a way that the boundary of a meridian disk of V_i is attached on γ_i, produce 3-manifolds, both containing a Klein bottle (in particular, not hyperbolic). We give a bound on the distance $\Delta(\gamma_1, \gamma_2)$, i.e. the minimal geometric intersection number between γ_1 and γ_2 after isotopy. We show that generically $\Delta(\gamma_1, \gamma_2) \leq 4$. More precisely, there are exactly two cases for which $\Delta(\gamma_1, \gamma_2) > 5$: either $\Delta(\gamma_1, \gamma_2) = 6$ and so M is the 3-manifold obtained by a 2/1-Dehn filling on the exterior of the Whitehead link in S^3, or $\Delta(\gamma_1, \gamma_2) = 8$ and so M is either the exterior of the figure-eight knot or the 3-manifold obtained by a $-5/1$-Dehn filling on the exterior of the Whitehead link in S^3. Moreover, there exists at most one manifold M for which $\Delta(\gamma_1, \gamma_2) = 5$.

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