THE UNIQUENESS THEOREM FOR EUCLIDEAN TWO-BRIDGE KNOT CONE-MANIFOLDS

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A *3-dimensional Euclidean cone-manifold* is a metric space obtained as the quotient space of a disjoint union of a collection of geodesic 3-simplices in the 3-dimensional Euclidean space $\mathbb{E}^3$ by an isometric pairing of faces in such a combinatorial fashion that the underlying topological space is a manifold. *Hyperbolic* and *spherical cone-manifolds* are defined similarly.

Such a space possesses a Riemannian metric of constant sectional curvature on the union of the top-dimensional cells and the dimension-2 cells. On each dimension-1 cell, the structure is completely described by an angle, which is the sum of the dihedral angles around all of the dimension-1 simplicial faces which are identified to give the cell. The *singular set* of a cone-manifold is the closure of all the dimension-1 cells for which this angle, called the *cone angle*, is not 2π (the Riemannian metric may be extended smoothly over all cells whose angle is 2π).

Let $(\mathbb{S}^3, (p/q)_\alpha)$ be a cone-manifold whose underlying space is the 3-sphere and singular set is a 2-bridge knot p/q with a cone angle α.

The main result of this report is the following theorem.

The Uniqueness Theorem. *If a cone-manifold $(\mathbb{S}^3, (p/q)_\alpha)$ admits the Euclidean structure, then there is unique cone angle $0 < \alpha_e < \pi$ such that the cone-manifold $(\mathbb{S}^3, (p/q)_{\alpha_e})$ is Euclidean.*

This theorem is proved by using results of S. Kojima [1], C.D. Hodgson, S.P. Kerckhoff [2], G.D. Mostow [3] and J. Porti [4].

References

[1] Kojima S. *Deformations of hyperbolic 3-cone-manifolds* // J. Differential Geom. — 1998. — V. 49. — P. 469–516.

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[4] Porti J. *Regenerating hyperbolic and spherical cone structures from Euclidean ones* // Topology. — 1998. — V. 37, N. 2. — P. 365–392.

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