

KNOTS, LINKS AND LEHMER'S QUESTION

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We describe joint work with Susan G. Williams that applies techniques of symbolic and algebraic dynamical systems to the study of knots and links.

The Mahler measure of an integral polynomial of several variables is the geometric mean of its absolute value over the multiplicative d -dimensional torus. For a monic 1-variable polynomial, the definition reduces to the product of the absolute values of those zeros that lie outside the unit circle. In 1933, D.H. Lehmer discovered a remarkable polynomial $L(x)$ of degree 10 with Mahler measure approximately equal to 1.17628. He asked if there is a polynomial with measure closer but not equal to 1. Lehmer's Question remains open despite extensive computer searches.

If ℓ is a d -component link, then the Mahler measure of its d -variable Alexander polynomial has a topological interpretation in terms of homology of finite abelian covers. Hyperbolic links with small volume often have Alexander polynomials with small Mahler measure. Lehmer's polynomial, for example, arises as the Alexander polynomial of the $(-2, 3, 7)$ -pretzel knot. We discuss examples and possible explanations for the behaviour.

Lehmer's Question is equivalent to a question about suitably defined growth rates of Lefschetz numbers of surface homeomorphisms.

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