DISTANCE BETWEEN TOROIDAL SURGERIES ON HYPERBOLIC KNOTS IN THE 3-SPHERE

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For a hyperbolic knot in the 3-sphere S^3, at most finitely many Dehn surgeries yield non-hyperbolic 3-manifolds. As a typical case of such an exceptional surgery, a toroidal surgery is one that yields a closed 3-manifold containing an incompressible torus. The slope corresponding to a toroidal surgery, called a toroidal slope, is known to be integral or half-integral. There are many examples of integral toroidal surgery, and Eudave-Muñoz constructed an infinite family of hyperbolic knots $k(\ell, m, n, p)$ admitting half-integral toroidal surgeries. Recently, Gordon and Luecke proved that the Eudave-Muñoz knots are the only hyperbolic knots with half-integral toroidal surgeries.

We consider the distance (the minimal geometric intersection number) between toroidal slopes on a hyperbolic knot in S^3. The figure-eight knot admits exactly three toroidal slopes $0, 4$ and -4. Note that $\Delta(-4, 4) = 8$. If a hyperbolic knot is not the figure-eight knot, then the distance between two toroidal slopes is at most 5 by Gordon's general result. (There are exactly four hyperbolic 3-manifolds which admit two toroidal slopes with distance at least 6. They all are obtained from the Whitehead link by some Dehn surgery on one component. Among those, only the figure-eight knot exterior can be embedded in S^3.) This upper bound 5 is sharp. For example, the $(-2, 3, 7)$-pretzel knot has toroidal slopes 16 and 37/2 with $\Delta(16, 37/2) = 5$. The purpose of this talk is to show that we can reduce the upper bound when both of toroidal slopes are integral.

Theorem 1. *Let K be a hyperbolic knot in S^3, which is not the figure-eight knot. If α and β are two integral toroidal slopes for K, then $\Delta(\alpha, \beta) \leq 4$.*

This is sharp. For example, the twist knot $C[2n, 2]$ in Conway's notation with $n \geq 1$ admits two integral toroidal slopes 0 and 4.

Corollary 2. *If a hyperbolic knot K in S^3 admits two toroidal slopes α and β with $\Delta(\alpha, \beta) = 5$, then K is the Eudave-Muñoz knot $k(2, -1, n, 0)$ for some integer $n \neq 1$, and $\{\alpha, \beta\} = \{25n - \frac{37}{2}, 25n - 16\}$.*

It is conjectured that a hyperbolic knot in S^3 admits at most three toroidal surgeries. This holds for Eudave-Muñoz knots. By Gordon's result stated above, a hyperbolic knot admits at most 6 toroidal surgeries. Our main theorem also gives an improvement of this upper bound.

Corollary 3. *A hyperbolic knot in S^3 admits at most 5 toroidal surgeries.*

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