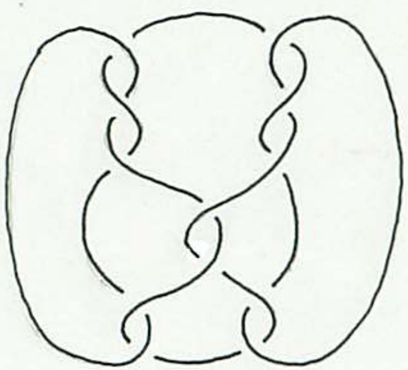
結び目と3次元多様体

ー幾何構造とファイバー構造を中心としてー

恩師の細川藤次先生と田尾鶏三先生の
思い出に捧げます



広島大学　作間誠

24 problems proposed by Thurston (Bull. A.M.S. 1982)

- The geometrization conjecture for 3-manifolds

  Perelman, Kleiner-Lott, Morgan-Tian, Cao-Ge

  Bessieres-Besson-Boileau-Maillot-Porti

- The geometrization conjecture for 3-orbifolds

  Boileau-Porti, Boileau-Leeb-Porti, Cooper-Hodgson-Kerckhoff

  MSJ Regional Workshop "Cone-manifolds and Hyperbolic Geometry" 1999

- Kleinian groups

  – Density Conjecture, Ending Lamination Conjecture, Tameness Conjecture, Ahlforse Measure Conjecture –

  Agol, Brock-Bromberg, Bowditch, Calegari-Gabai, Canary-Minsky, Brock-Canary-Minsky, Namazi-Souto, Ohshika, Soma, ...

- Virtual fibering conjecture

  Agol, Kahn-Markovic, Bergeron-Wise, Haglund-Wise

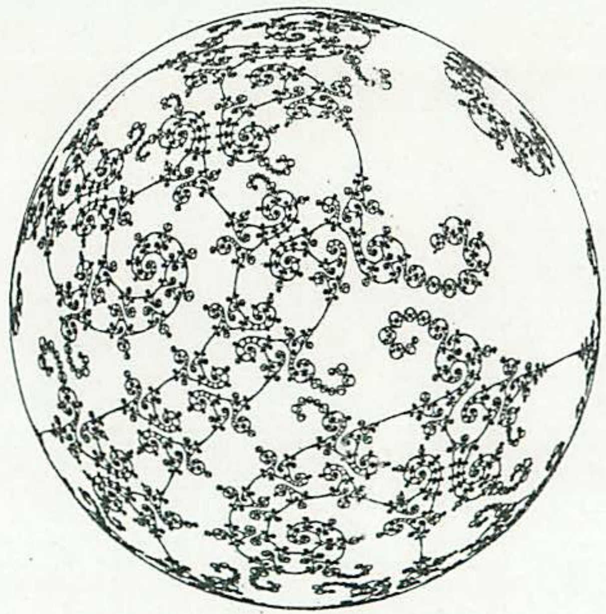
  Scott, Sageev

My personal encounter with Thurston

at the Japan-US low-dimensional conference, Hawaii 1982

I was deeply impressed and shocked by

- the announcement of the uniformization theorem of 3-orbifolds
- the picutures of the sphere-filling curves, associated with hyperbolic surface bundles over S^1
- Thurston's message "3-manifolds are not isolated; they are connected to each other."



Plan of the talk

(1) 2-dimensional geometry

Teichmüller space and Thurston compactification

Nielsen-Thurston theory

(2) Geometrization of 3-manifolds

(3) Hyperbolization Theorem for surface bundles

"See" the hyperbolic structure by using Wada's OPTi

(4) Gueritaud's work on Cannon-Thurston maps
and veering triangulations

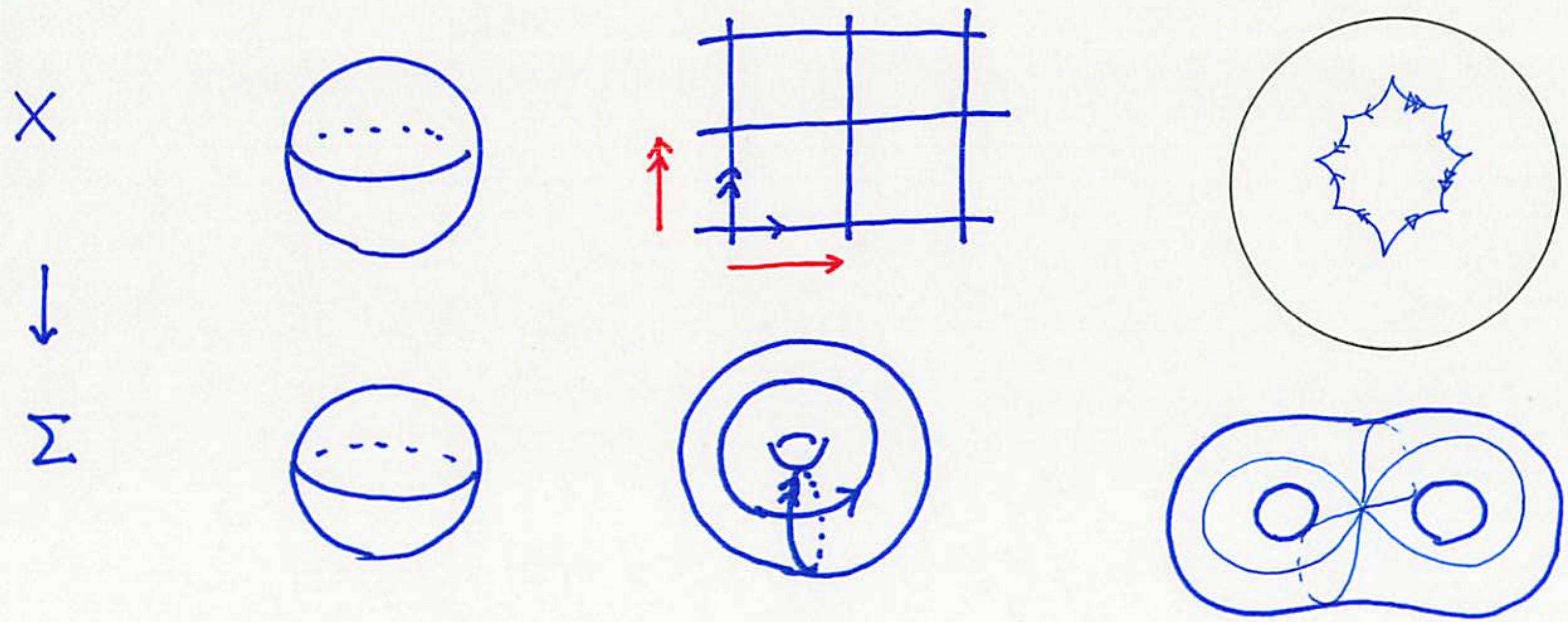
(5) My dogma

Fact : Any closed orientable surface Σ admits a spherical, Euclidean or hyperbolic structure, i.e.

$$\Sigma \cong X/\Gamma \quad \text{where } X = \mathbb{S}^2, \ \mathbb{E}^2 \text{ or } \mathbb{H}^2$$

$$\Gamma < \mathrm{Isom}(X) \ \text{ st } \ \Gamma \curvearrowright X \ \text{ free.}$$

Thus the universal cover $\tilde{\Sigma}$ is identified with X, and the fundamental group $\pi_1(\Sigma)$ is identified with $\Gamma < \mathrm{Isom}(X)$.



Def A Riemann surface is a surface obtained by gluing open sets in $\mathbb{C}$ by holomorphic homeomorphisms.

Uniformization Theorem (Poincaré, Koebe)

A simply connected Riemann surface is conformally equivalent to

$$\hat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}, \quad \mathbb{C}, \quad \text{or} \quad \mathbb{H}^2 = \{ z \in \mathbb{C} \mid \operatorname{Im} z > 0 \}$$

$$\cong \quad \Delta = \{ z \in \mathbb{C} \mid |z| < 1 \}$$

Cor Any Riemann surface S is conformally equivalent to X/Γ, where $X = \hat{\mathbb{C}}$, $\mathbb{C}$ or $\mathbb{H}^2$ and $\Gamma < \mathrm{Conf}(X)$ st $\Gamma \curvearrowright X$ is free

Remark $\mathrm{Conf}(\hat{\mathbb{C}}) \cong PSL(2, \mathbb{C})$, $\mathrm{Conf}(\mathbb{C}) \cong \mathrm{Isom}^+(\mathbb{E}^2) \ltimes \mathbb{R}^*$

$\mathrm{Conf}(\mathbb{H}^2) \cong PSL(2, \mathbb{R}) \cong \mathrm{Isom}^+(\mathbb{H}^2)$

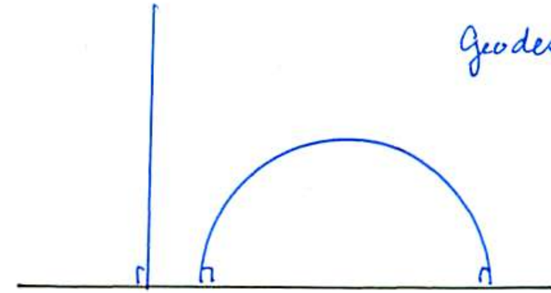
Crash course in Hyperbolic geometry

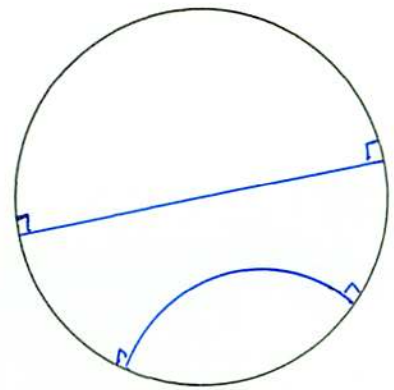
- Hyperbolic plane

$$\mathbb{H}^2 = \left(\{ z = x + iy \mid y > 0 \},\ \frac{dx^2 + dy^2}{y^2} \right)$$

$$\cong$$

$$\Delta = \left(\{ z \in \mathbb{C} \mid |z| < 1 \},\ \frac{4(dx^2 + dy^2)}{(1 - |z|^2)^2} \right)$$

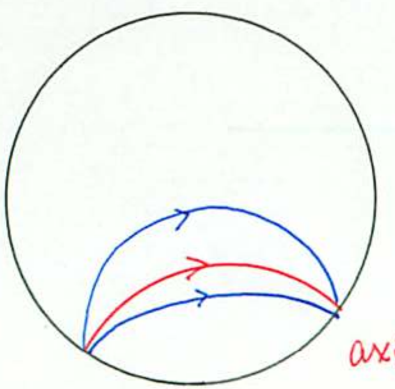


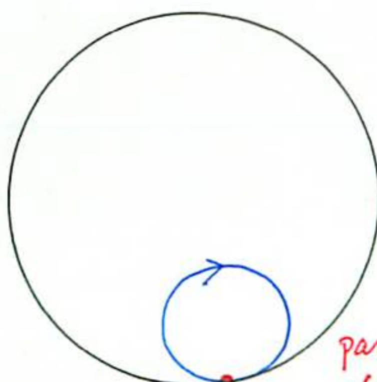


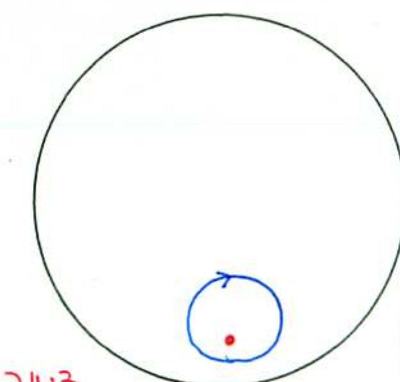
- $PSL(2,\mathbb{R}) \cong \mathrm{Conf}(\mathbb{H}^2) \cong \mathrm{Isom}^+(\mathbb{H}^2)$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \leftrightarrow A(z) = \frac{az+b}{cz+d}$$

- Hyperbolic: $\mathrm{tr} A \in \mathbb{R} \setminus [-2,2]$ — parabolic: $\mathrm{tr} A = \pm 2$ — elliptic: $\mathrm{tr} A \in (-2,2)$

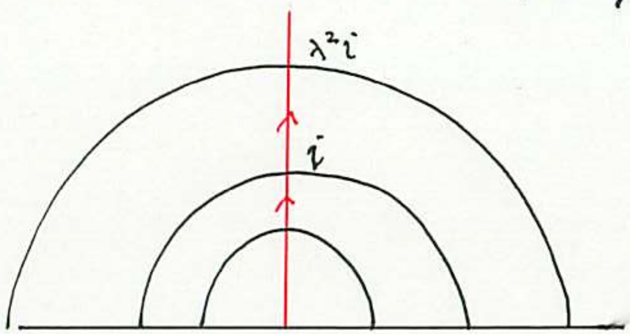


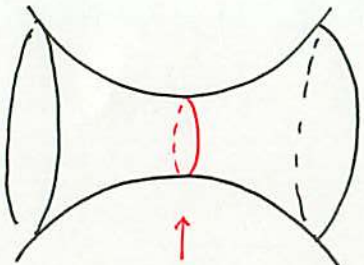




Hyperbolic surfaces with elementary fundamental group

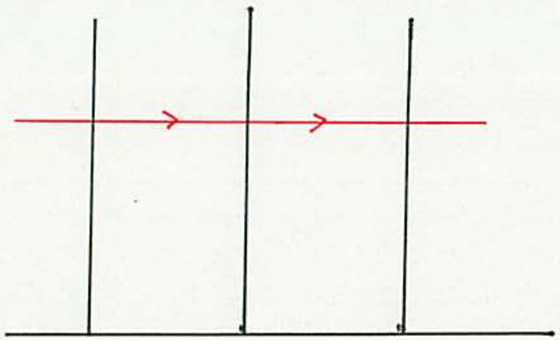
(1) $\Gamma = \langle A \rangle$ $\quad A = \begin{pmatrix} \lambda & 0 \\ 0 & 1/\lambda \end{pmatrix}$ hyperbolic

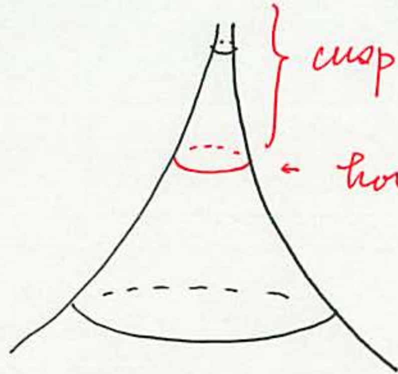




unique closed geodesic of length $\log |\lambda|^2$

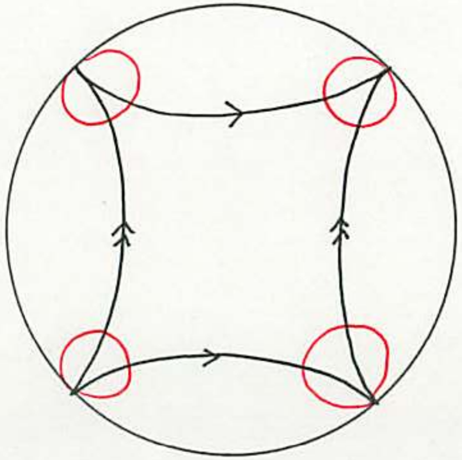
(2) $\Gamma = \langle A \rangle$ $\quad A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ parabolic

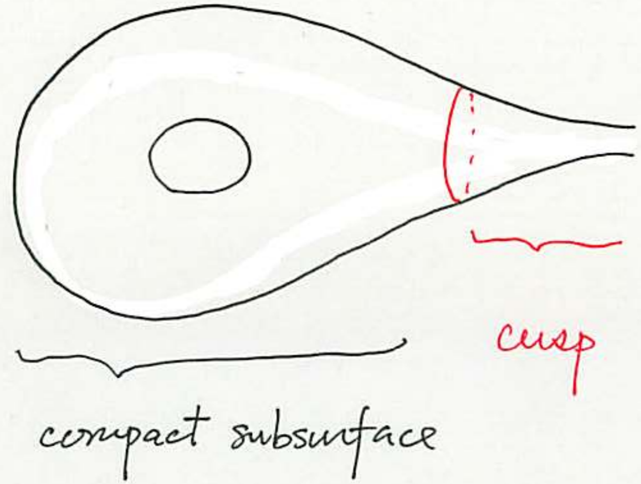




horocycle $\neq$ geodesic

Hyperbolic structure on punctured surface $\Sigma_{g,n}$ of finite area





Holonomy representation is "type-preserving".

$$\begin{array}{ccc}
\rho : \pi_1(\Sigma_{g,n}) & \longrightarrow & PSL(2,\mathbb{R}) = \mathrm{Isom}^+ \mathbb{H}^2 \\
\rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\
\gamma & \longmapsto & \text{parabolic transformation}
\end{array}$$

peripheral loop

i.e. loop around a puncture

Teichmüller space

$$\mathrm{Teich}(\Sigma_{g,n}) = \{\text{hyperbolic structures on } \Sigma_{g,n} \text{ of finite area}\} / \text{isotopy}$$

$$\cong \left\{ \rho : \pi_1(\Sigma_{g,n}) \to PSL(2,\mathbb{R}) \;\middle|\; \begin{array}{l}\text{type. pres.}\\ \text{faithful discrete}\end{array} \right\} / \text{conjugacy}$$

Mapping class group

$$MCG^+(\Sigma_{g,n}) = \{\text{orientation-preserving homeo of } \Sigma_{g,n}\} / \text{istopy}$$

Mapping class group acts on the Teichmüller space

$$\begin{array}{ccc} MCG^+(\Sigma_{g,n}) \times \mathrm{Teich}(\Sigma_{g,n}) & \longrightarrow & \mathrm{Teich}(\Sigma_{g,n}) \\ \in \quad\quad\quad \in & & \in \\ (f, \rho) & \longmapsto & \rho \circ f \end{array}$$

Fenchel-Nielsen coordinate

$$\mathrm{Teich}(\Sigma_{g,n}) \cong \mathbb{R}_+^{3g-3+n} \times \mathbb{R}^{3g-3+n} \cong \mathring{B}^{6g-6+2n}$$

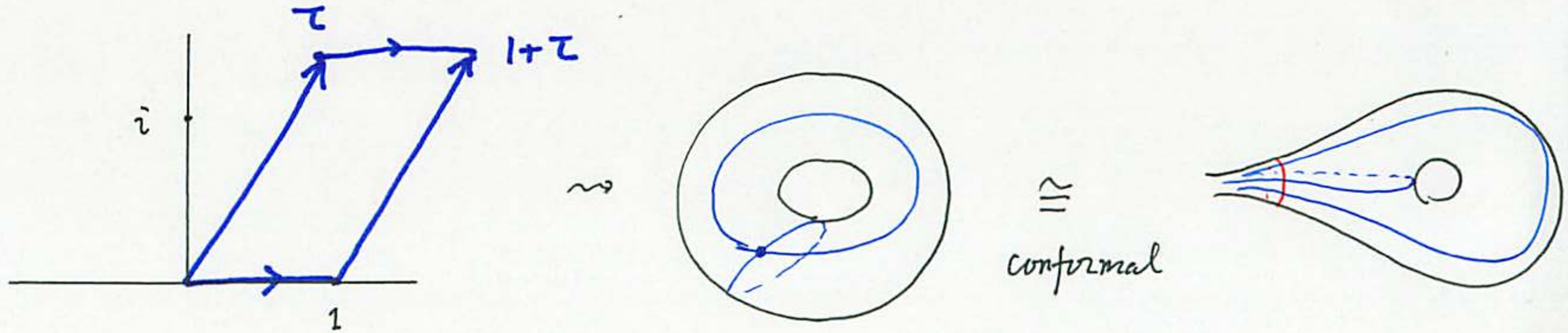
Thurston compactification

$$\overline{\mathrm{Teich}(\Sigma_{g,n})} = \mathrm{Teich}(\Sigma_{g,n}) \sqcup \mathcal{PML}(\Sigma_{g,n})$$
$$\cong \mathring{B}^{6g-6+2n} \sqcup S^{6g-6+2n-1} \cong B^{6g-6+2n}$$

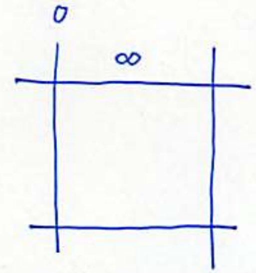
The action of $MCG^+(\Sigma_{g,n})$ on $\mathrm{Teich}(\Sigma_{g,n})$ extends to that on $\overline{\mathrm{Teich}(\Sigma_{g,n})}$

Punctured torus case

$$\mathrm{Teich}(\Sigma_{1,1}) \cong \{\text{Conformal structure on } \Sigma_{1,1}\}/\text{isotopy}$$
$$\cong \{\text{Conformal structure on } \Sigma_1\}/\text{isotopy}$$
$$\cong \mathbb{H}^2 = \{\tau \in \mathbb{C} \mid \mathrm{Im}\,\tau > 0\}$$



$\mathcal{S} := \{ \text{essential simple loop on } \Sigma_{1,1} = (\mathbb{R}^2 - \mathbb{Z}^2)/\mathbb{Z}^2 \} / \text{isotopy}$



$\updownarrow$ $\qquad \alpha_r$: the image of a line in $\mathbb{R}^2 - \mathbb{Z}^2$ of "slope r"

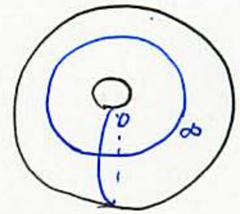
$\hat{\mathbb{Q}} \ni r$

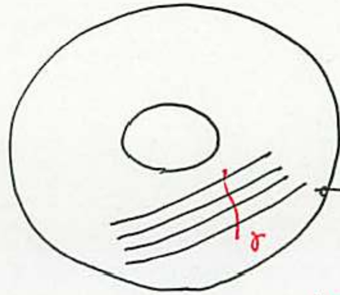
$\cap$

$\hat{\mathbb{R}} \ni \lambda$

$\updownarrow$

$PML(\Sigma_{1,1}) \qquad \omega = dx - \lambda dy$

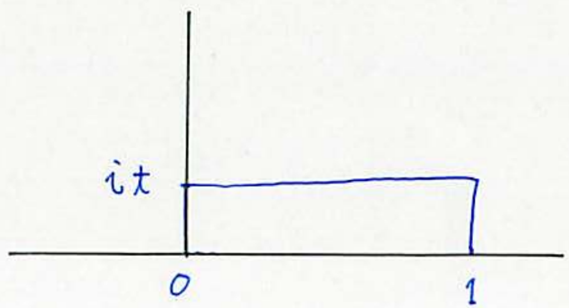




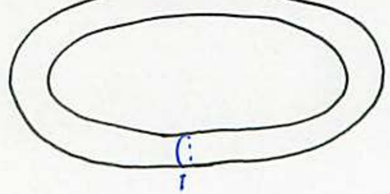
slope λ line foliation

$\gamma \mapsto \left| \int_\gamma \omega \right|$

$\overline{\mathrm{Teich}}(\Sigma_{1,1}) = \mathbb{H}^2 \sqcup \partial \mathbb{H}^2 = \mathbb{H}^2 \sqcup \hat{\mathbb{R}}$



$\leftrightarrow$



short loop (slope 0)

- $MCG^+(\Sigma_{1,1}) \cong SL(2,\mathbb{Z})$

  $\varphi_A \longleftrightarrow A$

$$\begin{tikzcd}
\mathbb{R}^2 \arrow[r, "A"] \arrow[d] & \mathbb{R}^2 \arrow[d] & \text{linear map} \\
\mathbb{R}^2/\mathbb{Z}^2 \arrow[r] & \mathbb{R}^2/\mathbb{Z}^2 & \\
\Sigma_{1,1} \arrow[u, phantom, "\cup"] \arrow[r, "\varphi_A"] & \Sigma_{1,1} \arrow[u, phantom, "\cup"] &
\end{tikzcd}$$

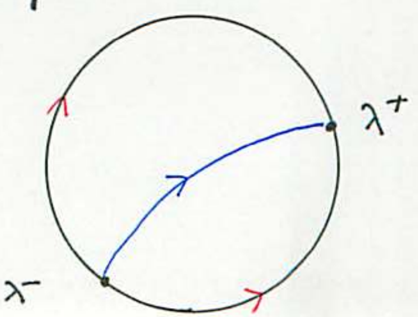
- Action of $MCG^+(\Sigma_{1,1}) \cong SL(2,\mathbb{Z})$ on $\overline{\mathrm{Teich}(\Sigma_{1,1})} = \overline{\mathbb{H}^2}$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z}), \quad \tau \in \overline{\mathbb{H}^2} \rightsquigarrow A\cdot\tau = \frac{a\tau+b}{c\tau+d}$$

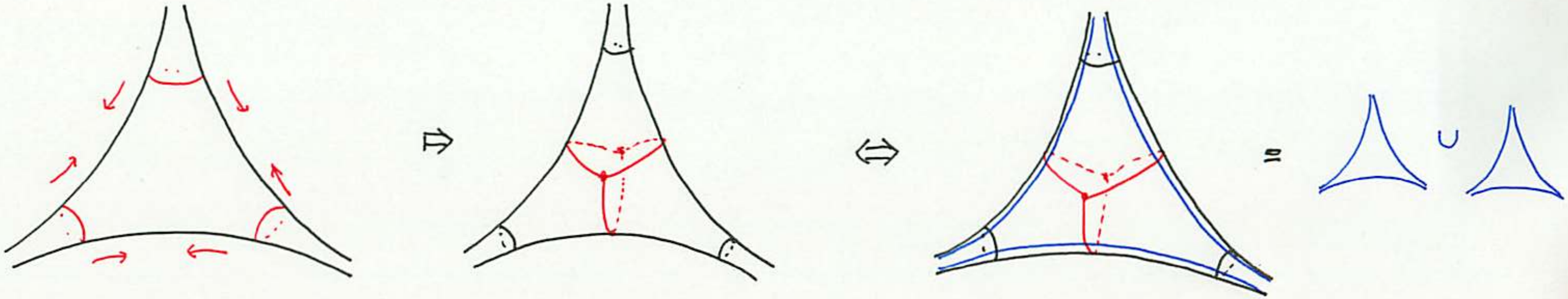
- **Nielsen - Thurston classification**
  - $|\mathrm{Tr}A| \leqq 1 \Rightarrow \varphi_A$ has a (unique) fixed point in $\mathrm{Teich}(\Sigma_{1,1})$, and φ_A is periodic
  - $|\mathrm{Tr}A| = 2 \Rightarrow \varphi_A$ fixes an essential simple loop, and φ_A is reducible
  - $|\mathrm{Tr}A| \geqq 3 \Rightarrow \varphi_A$ has a unique attractive/repulsive fixed point $\lambda^+, \lambda^- \in \partial\overline{\mathrm{Teich}}$,

    which spans a unique invariant "Teichmüller geodesic".

    φ_A is (pseudo) Anosov.



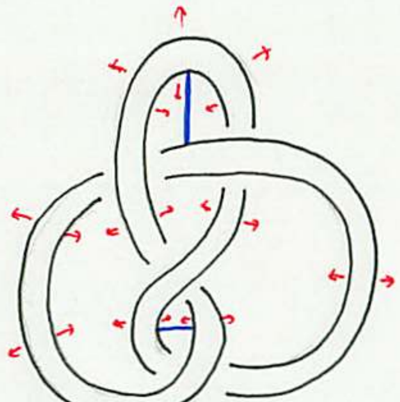
Epstein-Penner decomposition of cusped hyperbolic manifolds

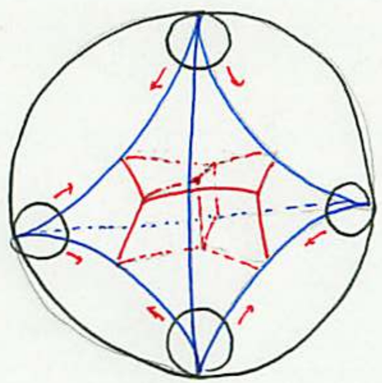


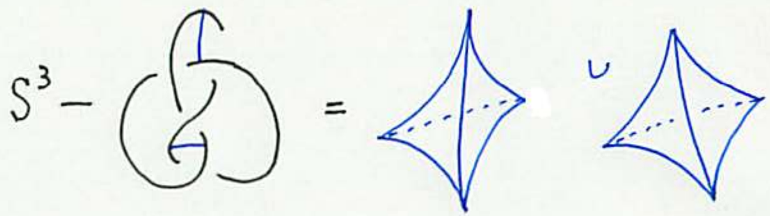
Expand the cusps

Ford complex
=
collision locus of
the expanding cusps

Epstein-Penner decomposition
=
geometric dual to the Ford complex







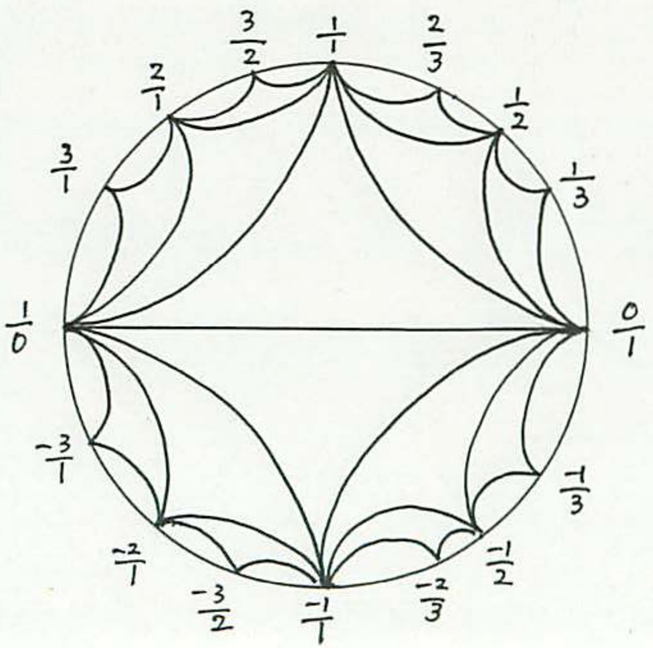
The upper/lower tunnels
are the shortest geodesic
between cusps

[Bowditch - Epstein]

Epstein - Penner decomposition gives a natural decomposition of $\mathrm{Teich}(\Sigma_{g,n})$ $(n \neq 0)$ by :

$$\begin{array}{ccl} \mathrm{Teich}(\Sigma_{g,n}) & \xrightarrow{T} & \{\text{ideal cell decomposition of } \Sigma_{g,n}\} / \text{isotopy} \\ \cup & & \\ T^{-1}(\text{an ideal cell decomposition}) & & \end{array}$$

Bowditch - Epstein decomposition of $\mathrm{Teich}(\Sigma_{1,1}) \cong \mathbb{H}^2$ is the Farey tessellation



$\{\text{Vertices}\} \leftrightarrow \hat{\mathbb{Q}}$

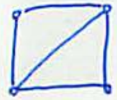
$\leftrightarrow \{\text{essential simple loops in } \Sigma_{1,1}\}$

$\leftrightarrow \{\text{essential simple arcs in } \Sigma_{1,1}\}$

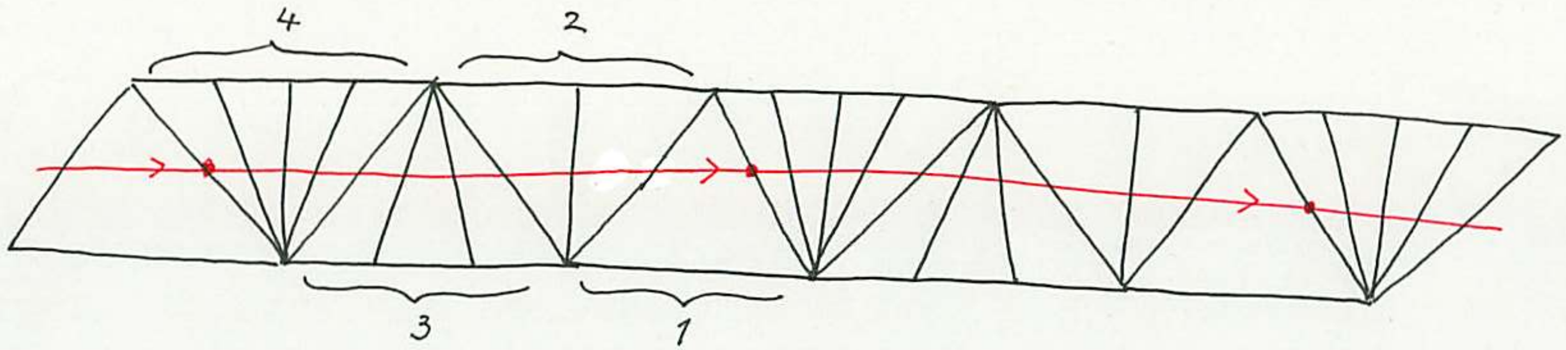
$\{\text{Edges}\} \leftrightarrow \{\text{pairs of mutually disjoint arcs}\}$

$\{\text{triangles}\} \leftrightarrow \{\text{triples of mutually disjoint ars}\}$

$\leftrightarrow \{\text{ideal triangulation of } \Sigma_{1,1}\}$



- The conjugacy class of a pseudo Anosov map $\varphi_A \in MCG(\Sigma_{1,1})$ is determined by the intersection of the axis of φ_A with the Farey tessellation.



$$\varphi_A = R^4 L^3 R^2 L, \quad R = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad L = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

Geometrization Theorem [Thurston, Perelman, ⋯]

Every compact orientable 3-manifold admits a natural splitting into geometric manifolds.

First decomposition: Unique prime decomposition (Kneser-Milnor) by 2-spheres (unique up to homeomorphism).

Second decomposition: Jaco-Shalen-Johansonn decomposition by tori (unique up to isotopy)

Remark: JSJ decompositions for finitely presented groups are given by:

Sela, Rips-Sela, Dunwoody-Sageev, Fujiwara-Papasoglu.

- A Geometric structure on M is a pair

$$(X, \ \rho : \pi_1(M) \hookrightarrow \mathrm{Isom}(X)), \text{ st}$$

X : complete Riemannian manifold, which is homogeneous and simply connected

$\rho : \pi_1(X) \xrightarrow{\cong} \Gamma < \mathrm{Isom}(X)$

st $\Gamma \curvearrowright X$ freely and $M \cong X/\Gamma$

In this case, the universal cover $\widetilde{M}$ is identified with X, and the covering transformation group $\mathrm{Cov}(\widetilde{M}) \cong \pi_1(M)$ is identified with Γ

- Classification of 3-dim. geometries

Stabilzer		Geometry
$SO(3)$	:	$\mathbb{S}^3$, $\mathbb{E}^3$, $\mathbb{H}^3$
$SO(2)$	:	$\mathbb{S}^2 \times \mathbb{E}^1$, $\mathbb{E}^2 \tilde{\times} \mathbb{E}^1 = Nil$, $\mathbb{H}^2 \times \mathbb{E}^1$, $\mathbb{H}^2 \tilde{\times} \mathbb{E}^1 = \widetilde{SL_2}$
$SO(1)$	:	$\mathbb{E}^2 \ltimes \mathbb{E}^1 = Sol$

Classification of 3-manifolds with X-structure ($X \neq \mathbb{H}^3$)

- Seifert fibered spaces (ie manifolds foliated by circles), which are classified by the base orbifold, data for singular fibers, and the Euler number
- torus bundles with Anosov monodromy

  torus semi-bundles (Sappire space)

  Remark : The first example of a pair of (mutually non-homeomorphic) 3-manifolds with the same homology is a pair of torus bundles given by Poincaré.

Remark The Teichmüller spaces of Seifert fibered spaces are determined by Ohshika.

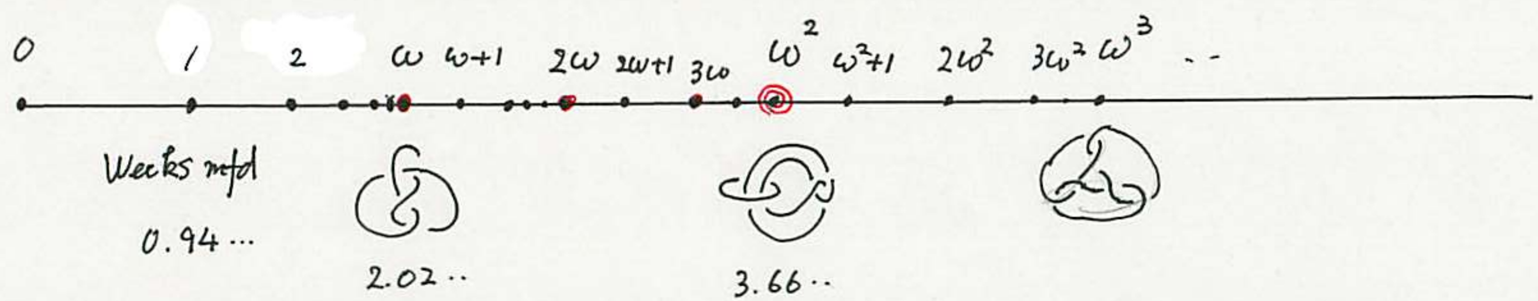
Mostow - Prasad Rigidity

Suppose $M = \mathbb{H}^n/\Gamma$ and $M' = \mathbb{H}^n/\Gamma'$ are finite volume hyperbolic manifolds with $n \geqq 3$. If there exist an isomorphism $\phi : \pi_1(M) = \Gamma \to \pi_1(M') = \Gamma'$, then ϕ is induced by a unique isometry $f : M \to N$.

Jorgensen - Thurston theory

$\mathrm{vol} : \{\text{finite volume hyp 3-mfds}\} \to \mathbb{R}_+$ is finite to one,

and $\mathrm{Image}(\mathrm{vol}) \cong \omega^\omega$ as an ordered set.



Small volum manifolds are studied by

Adams, Agol, Meyerhoff, Cao-Meyerhoff, Gabai-Meyerhoff-Milley, Yoshida, ..

Hyperbolization Theorem for surface bundles

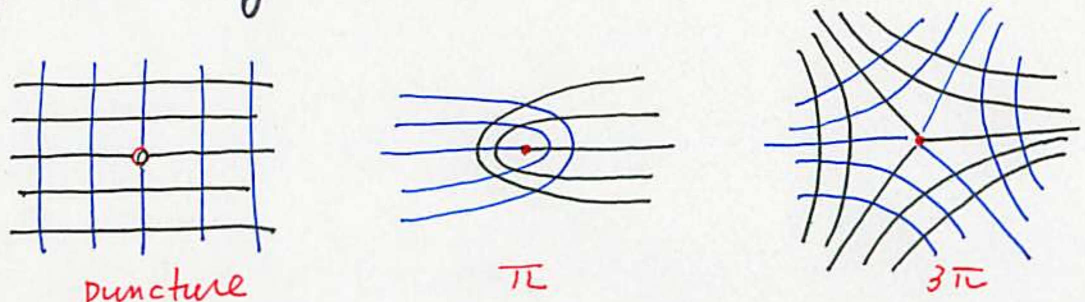
$$M = \Sigma \times \mathbb{R} \big/ (x,t) \sim (\varphi(x), t+1)$$: Σ-bundle over S^1 with monodromy φ

Then M admits a (complete) hyperbolic structure of finite volume

iff φ is pseudo-Anosov,

i.e. $\exists$ singular Euclidean metric on Σ, st

(i) the only singularities are punctures or cone points of cone angle $n\pi$

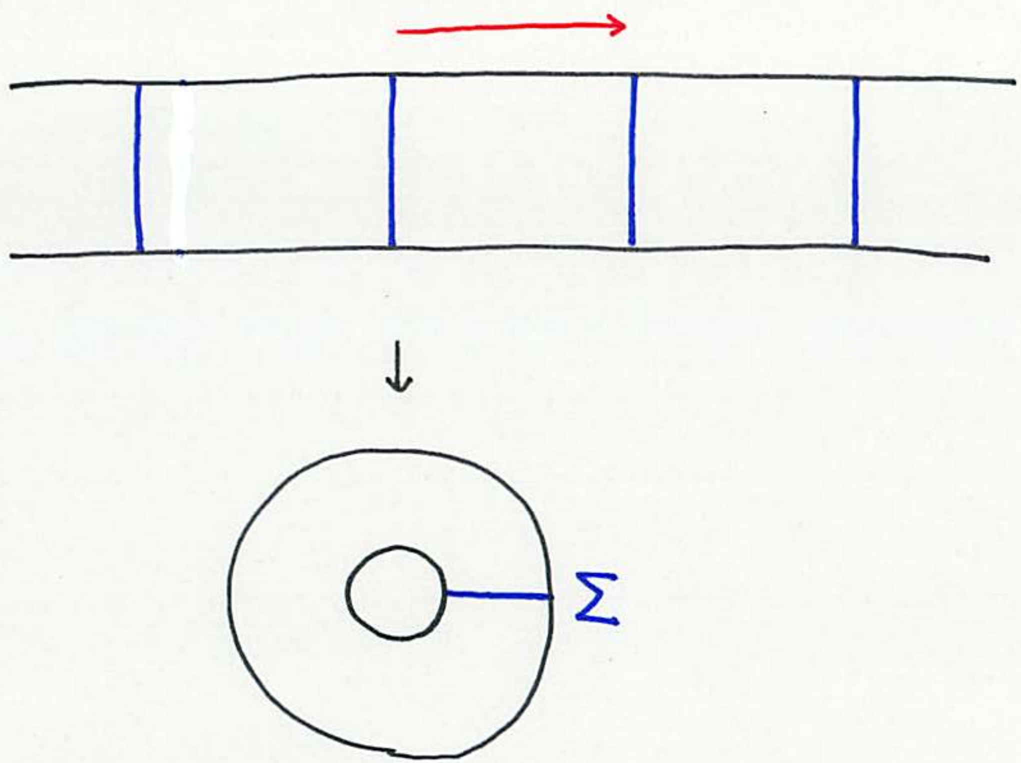


(ii) φ has a local expression $(x,y) \mapsto \pm(Kx, \frac{1}{K}y)$ $(K>1)$

Hyperbolic structure on M_φ

$\Leftrightarrow$ Hyperbolic structure on $\Sigma \times \mathbb{R}$, st

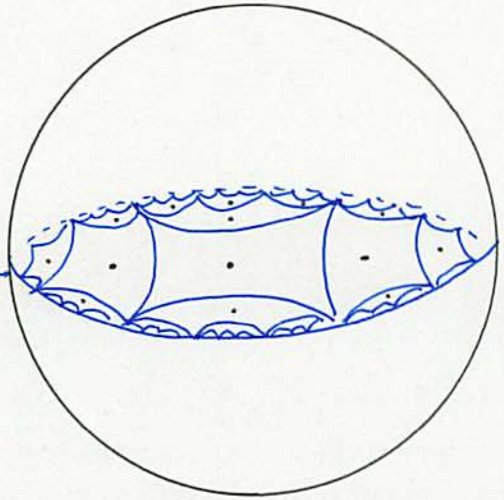
$(x, t) \mapsto (\varphi(x), t+1)$ is an isometry ?

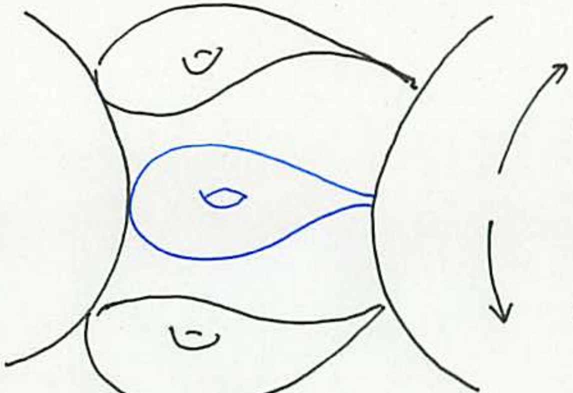


Can such structure exist ?

Example of a hyperbolic structure on $\Sigma \times \mathbb{R}$

Fuchsian group $\rho : \pi_1(\Sigma) \xrightarrow[\cong]{} \Gamma < \mathrm{Isom}^+(\mathbb{H}^2) < \mathrm{Isom}^+(\mathbb{H}^3)$





Limit set $\Lambda(\Gamma) := \{\text{accummulation points of } \Gamma \cdot x \text{ in } \overline{\mathbb{H}^3}\} \subset \partial \mathbb{H}^3 = \hat{\mathbb{C}}$

$(x \in \mathbb{H}^3)$

Γ: Fuchsian group $\Leftrightarrow$ $\Lambda(\Gamma)$ is a round circle

Γ: quasi-Fuchsian group $\Leftrightarrow$ $\Lambda(\Gamma)$ is a topological circle

$\Leftrightarrow$ **Domain of discontinuity** $\Omega(\Gamma) := \hat{\mathbb{C}} \setminus \Lambda(\Gamma)$

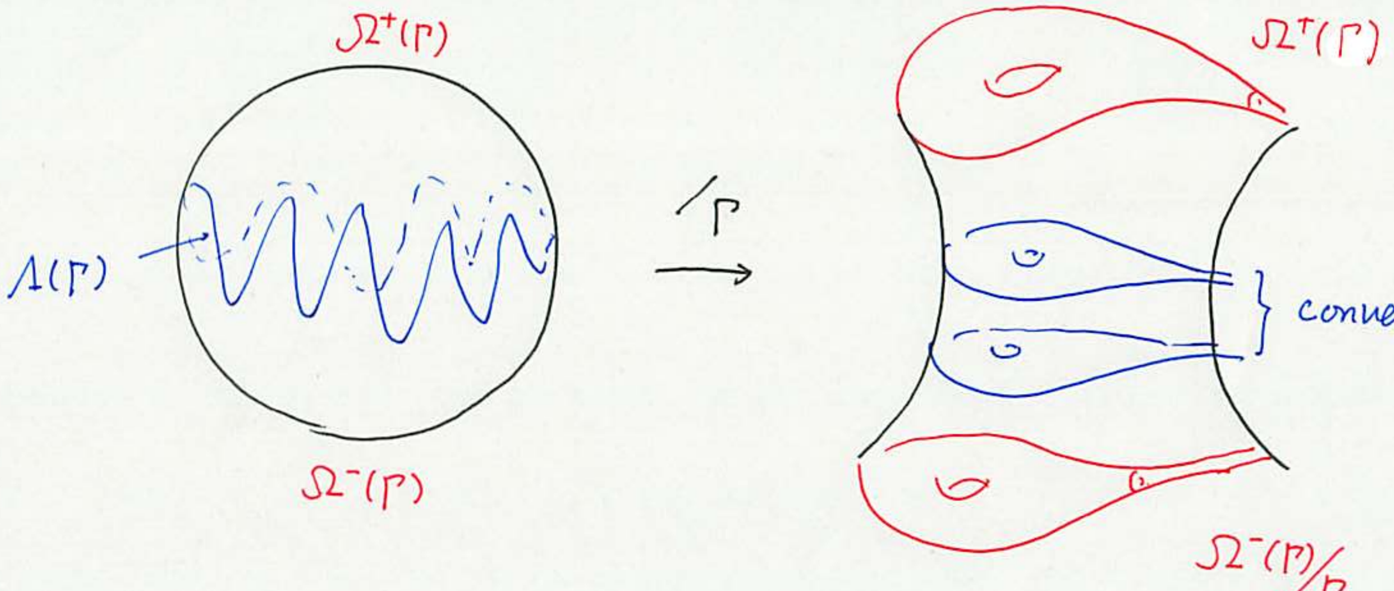
consists of two open disks $\Omega^+(\Gamma)$ and $\Omega^-(\Gamma)$.

Ber's simultaneous uniformization theorem

$$QF(\Sigma) = \left\{ \begin{array}{l} \text{hyperbolic structures on } \Sigma \times \mathbb{R} \\ \text{given by quasi-fuchsian groups} \end{array} \right\} \xrightarrow{\cong} \mathrm{Teich}(\Sigma) \times \mathrm{Teich}(\Sigma)$$

$$\mathbb{H}^3/\Gamma \longmapsto \left(\Omega^+(\Gamma)/\Gamma \,,\, \Omega^-(\Gamma)/\Gamma \right)$$

where $\Omega(\Gamma) = \Omega^+(\Gamma) \sqcup \Omega^-(\Gamma)$



convex core $= \mathrm{Hull}(\Lambda(\Gamma))/\Gamma$

All closed geodesics are contained in the convex core.

The space of hyperbolic structures on $\Sigma \times \mathbb{R}$

$$\mathcal{D}(\Sigma) := \{ \text{complete hyperbolic structures on } \Sigma \times \mathbb{R} \} / \text{isotopy}$$

$$= \left\{ \rho : \pi_1(\Sigma) \to PSL(2,\mathbb{C}) \;\middle|\; \begin{array}{l} \text{type-preserving} \\ \text{discrete, faithful} \end{array} \right\} / \text{conjugacy}$$

[Minsky] Ending lamination conjecture and density conjecture for $T = \Sigma_{1,1}$

- $\mathcal{D}(T) = \overline{\mathcal{QF}(T)}$
- A hyperbolic structure on $T \times \mathbb{R}$ is determined by its end invariant $\nu(\rho)$

i.e. $\nu : \mathcal{D}(T) \to \overline{\mathbb{H}^2} \times \overline{\mathbb{H}^2} - \mathrm{diag}(\partial \mathbb{H}^2)$ is a bijection.

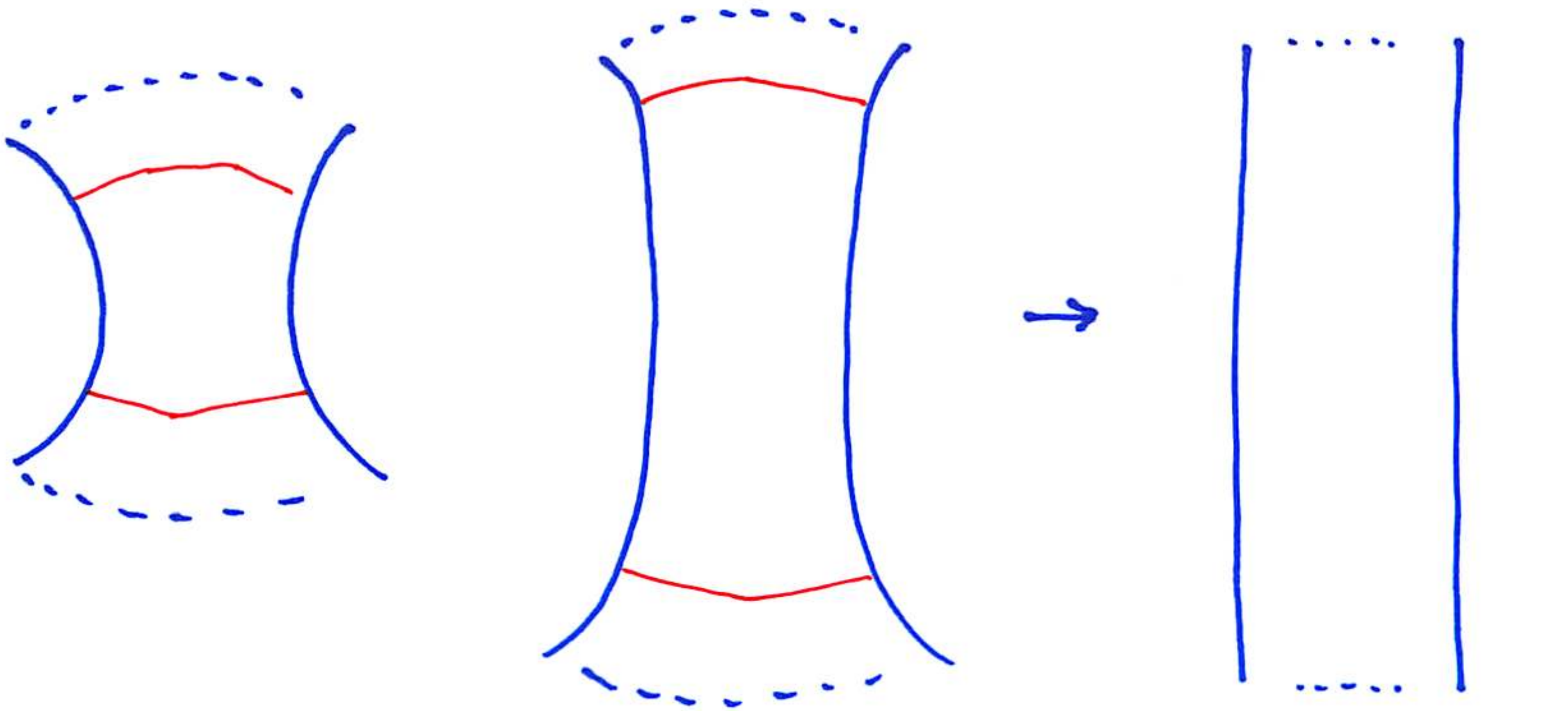
Remark Though ν^{-1} is continuous, ν is not continuous.

Thus the topology of $\mathcal{D}(T)$ is very complicated.

Double limit theorem for punctured torus groups

$$(z_n^-, z_n^+) \in \mathbb{H}^2 \times \mathbb{H}^2$$
$$\downarrow$$
$$(\lambda^-, \lambda^+) \in \partial\mathbb{H}^2 \times \partial\mathbb{H}^2 \qquad \lambda^- \neq \lambda^+$$

Then $\rho_n := \nu^-(z_n^-, z_n^+)$ converges to $\rho_\infty := \nu^+(\lambda^-, \lambda^+)$



For $A \in SL(2,\mathbb{Z}) = MCG^+(T)$, with $|\mathrm{Tr}\, A| \geqq 3$,

the end invariant (ending lamination) of the infinite cyclic cover

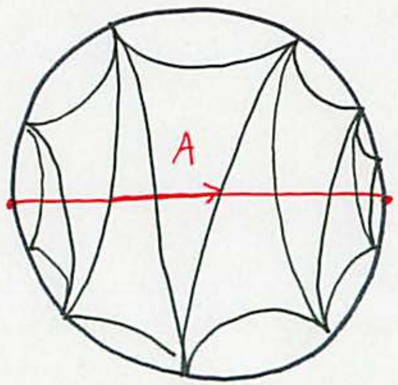
$\tilde{M}_A \cong T \times \mathbb{R}$ of the T-bundle M_A with monodromy A is

$$(\nu^+, \nu^-) \in \hat{\mathbb{R}} \times \hat{\mathbb{R}} = \partial\mathbb{H}^2 \times \partial\mathbb{H}^2$$

where $\nu^\pm$ is the attractive/repulsive fixed point of the action of A on $\overline{\mathrm{Teich}(T)} = \mathbb{H}^2 \cup \partial\mathbb{H}^2$

ie the slopes of the eigen spaces of $A \in SL(2,\mathbb{Z})$.

Example $\quad A = RL = \begin{pmatrix}1&1\\0&1\end{pmatrix}\begin{pmatrix}1&0\\1&1\end{pmatrix} = \begin{pmatrix}2&1\\1&1\end{pmatrix}$

$\frac{-1-\sqrt{5}}{2} = \nu^-$ 

$\nu^+ = \frac{-1+\sqrt{5}}{2}$

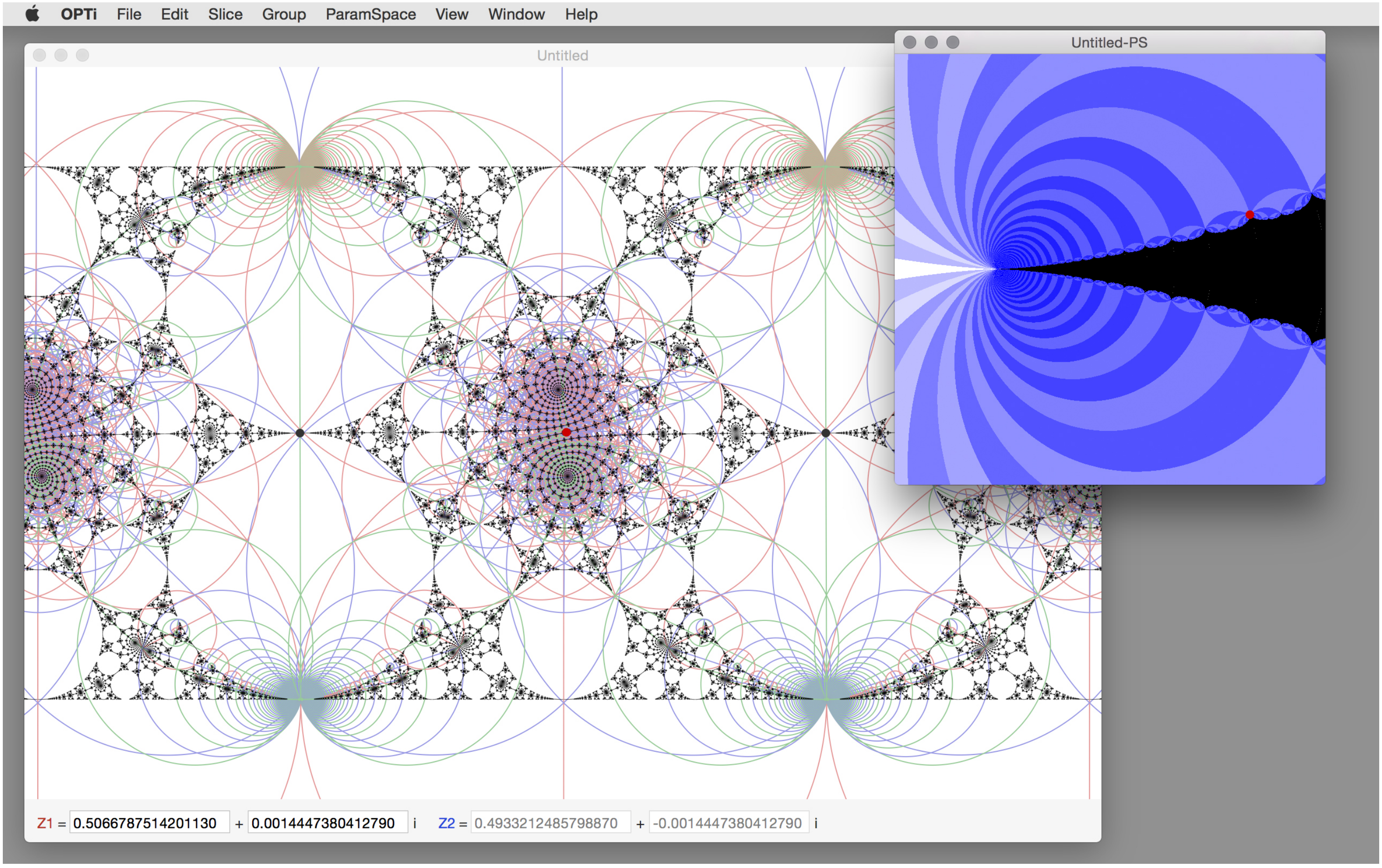
$M_A \cong S^3 -$ 

OPTi Home Page

Last update: 2016.5.17

What is OPTi ?

"OPTi" is a Macintosh application program which visualizes quasi-conformal deformations of the once-punctured-torus groups. It interactively draws the isometric circles, the Ford region, the limit set, etc.



[Click for a large picture](#)

What's new ?

OPTi 4.0.2 fixes a bug where OPTi did not show windows properly under Mac OS X 10.9.

OPTi 4.0 released. [Download OPTi 4.0 from the Mac App Store](#). OPTi 4.0 runs under Mac OS X 10.9 and later. (2016.5.18)

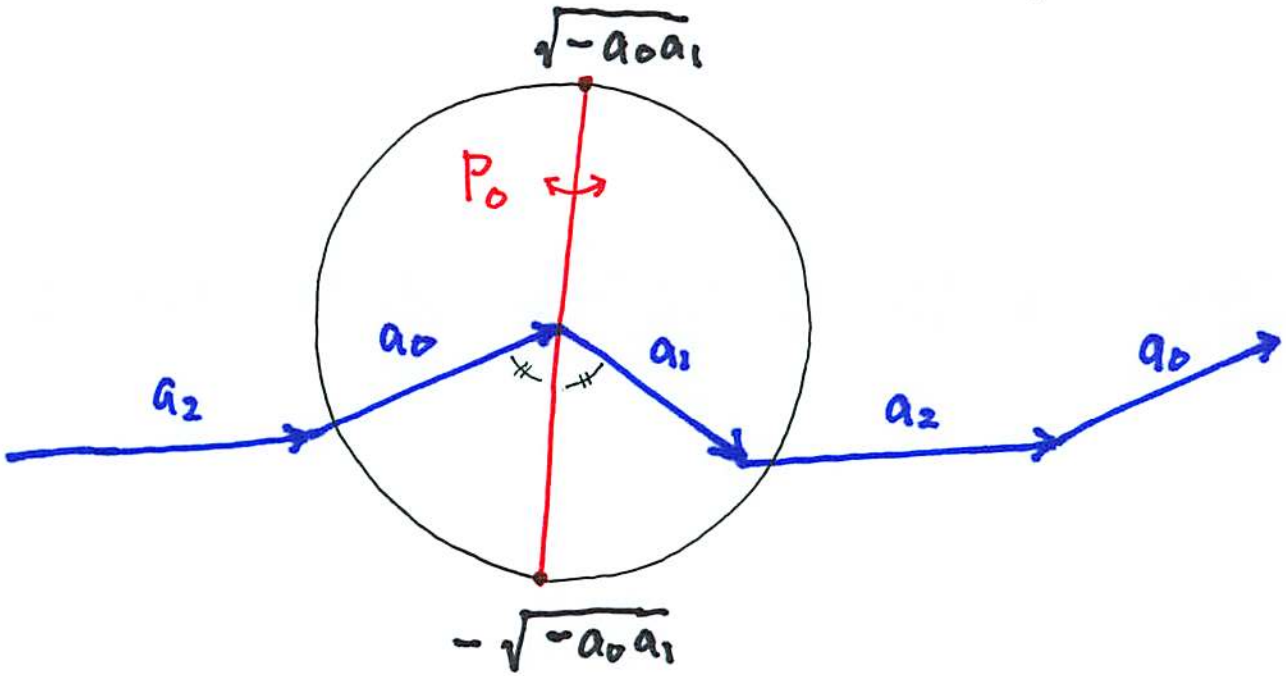
Corrected the link to OPTi 3.61. (2011.6.7)

There is nothing really new, but I have packaged OPTi 3.61 with 3.60 documents in zip format, rather than sit format. 2010.12.4.

Jorgensen's construction of $\rho : \pi_1(T) \to PSL(2.\mathbb{C})$

$(a_0, a_1, a_2) \in (\mathbb{C}^*)^3$ st $a_0 + a_1 + a_2 = 1$

(complex probability)

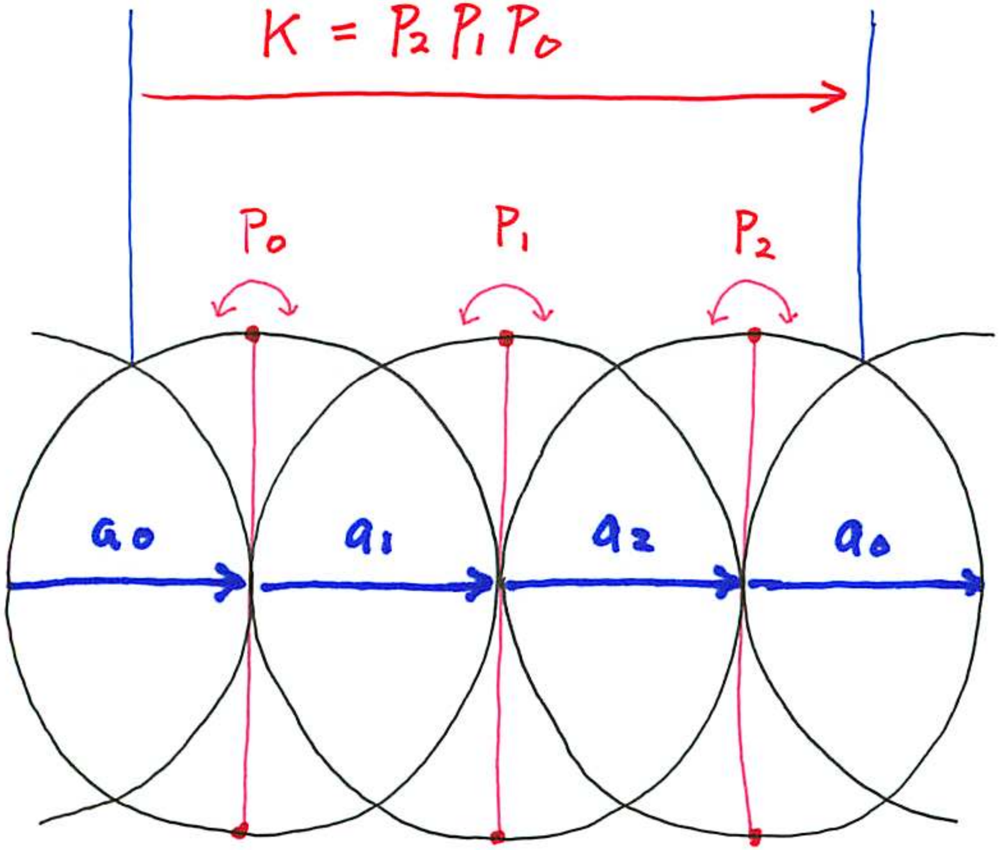


P_i : π-rotation around the geodesic with endpoints $\pm\sqrt{-a_0 a_1}$

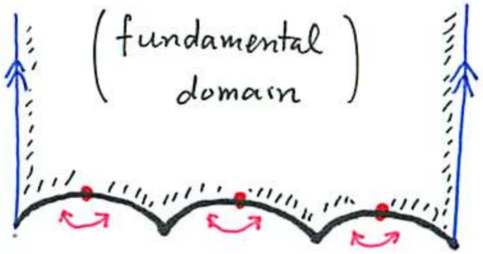
Consider the group $\langle P_0, P_1, P_2 \rangle < PSL(2.\mathbb{C})$

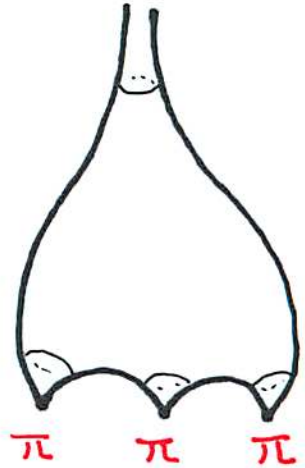
Then $K := P_2 P_1 P_0 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

Example $(a_0, a_1, a_2) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$



$$\mathcal{O} := \mathbb{H}^2 / \langle P_0, P_1, P_2 \rangle$$





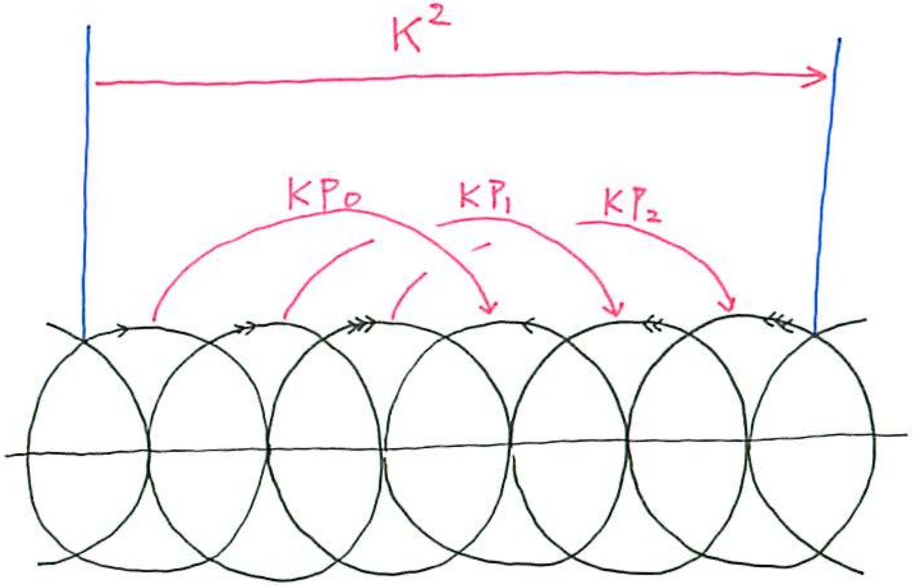
$$\pi_1(\mathcal{O}) = \langle P_0, P_1, P_2 \mid P_0^2 = P_1^2 = P_2^2 = 1 \rangle$$

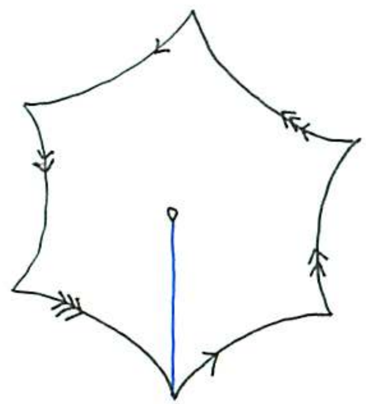
$$\pi_1(\mathcal{O}) = \langle P_0, P_1, P_2 \mid P_0^2 = P_1^2 = P_2^2 = 1 \rangle \ni K = P_2 P_1 P_0$$

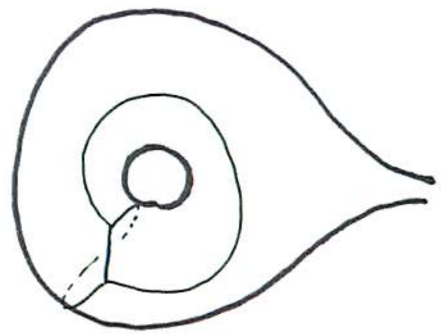
$$\nabla$$

$$\pi_1(T) = \operatorname{Ker}\left[\pi_1(\mathcal{O}) \to \mathbb{Z}/2\mathbb{Z}\right]$$

$$= \langle KP_0, KP_1 \mid - \rangle$$







[Cannon-Thurston, Minsky, Alperin-Dicks-Porti, Bowditch, Mj, ...]

Fix $\rho_0 \in \mathcal{F}(T)$.

Then for any $\rho \in \mathcal{D}(T)$, there is a continuous surjective map

$$CT : \Lambda(\rho_0) = \hat{\mathbb{R}} \longrightarrow \Lambda(\rho) \qquad \text{(Cannon-Thurston map)}$$

which is $\pi_1(T)$-equivariant.

i.e.

$$\begin{tikzcd} \Lambda(\rho_0) \arrow[r, "CT"] \arrow[d, "\rho_0(\alpha)"'] & \Lambda(\rho) \arrow[d, "\rho(\alpha)"] \\ \Lambda(\rho_0) \arrow[r, "CT"] & \Lambda(\rho) \end{tikzcd} \qquad (\forall \alpha \in \pi_1(T))$$

In particular, if ρ corresponds to $\widetilde{E}(\;)$,

then $CT : \Lambda(\rho_0) = \hat{\mathbb{R}} \longrightarrow \Lambda(\rho) = \hat{\mathbb{C}}$ is a

$\pi_1(T)$-equivariant sphere filling curve!

Two tessellations of $\mathbb{C}$ associated with M_A $(|\mathrm{Tr} A| \geqq 3)$

$\rho_A : \pi_1(T) \to PSL(2, \mathbb{C})$ holonomy representation for the fiber group

ρ_A (peripheral loop) $= \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ ie ∞ is a parabolic fixed point

(1) Epstein-Penner's ideal triangulation of M_A

$\rightsquigarrow$ Triangulation of the cusp torus $S^1 \times S^1$

$\rightsquigarrow$ Triangulation of the universal cover $\mathbb{C}$ of $S^1 \times S^1$

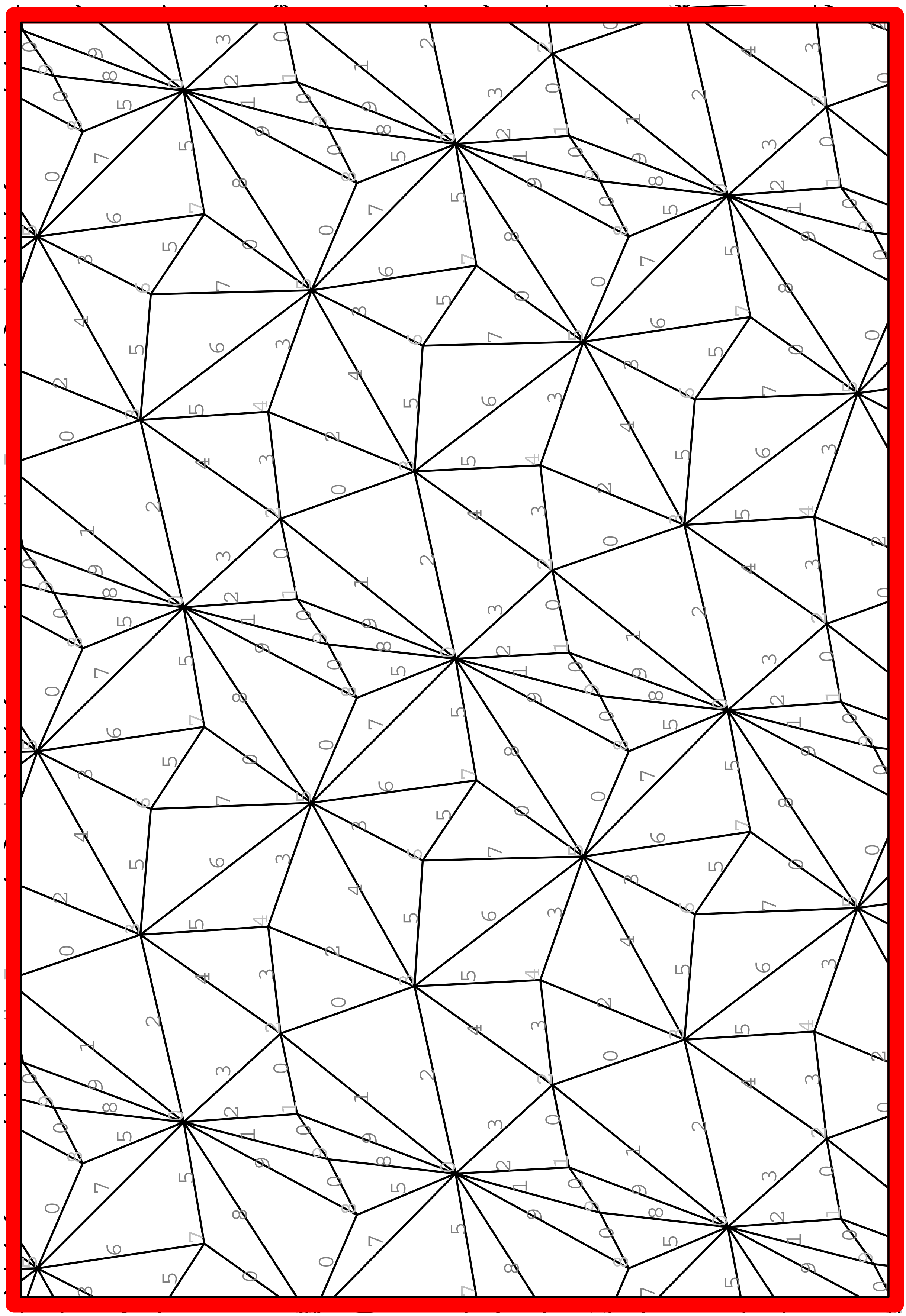
Δ_A

(2) Cannon-Dicks fractal tessellation CW_A associated with the Cannon-Thurston map of the fiber group $\rho_A(\pi_1(T))$

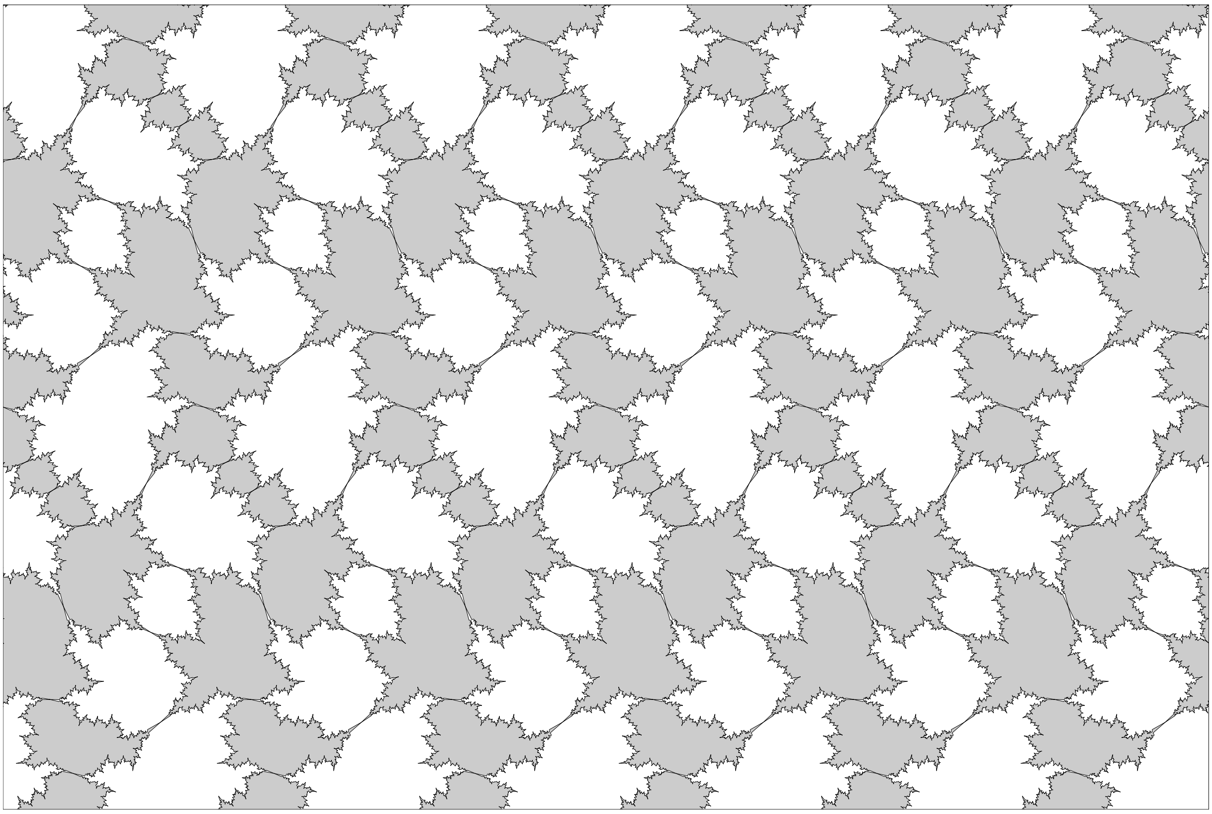
[Dicks - S]

Δ_A and CW_A are intimately related.

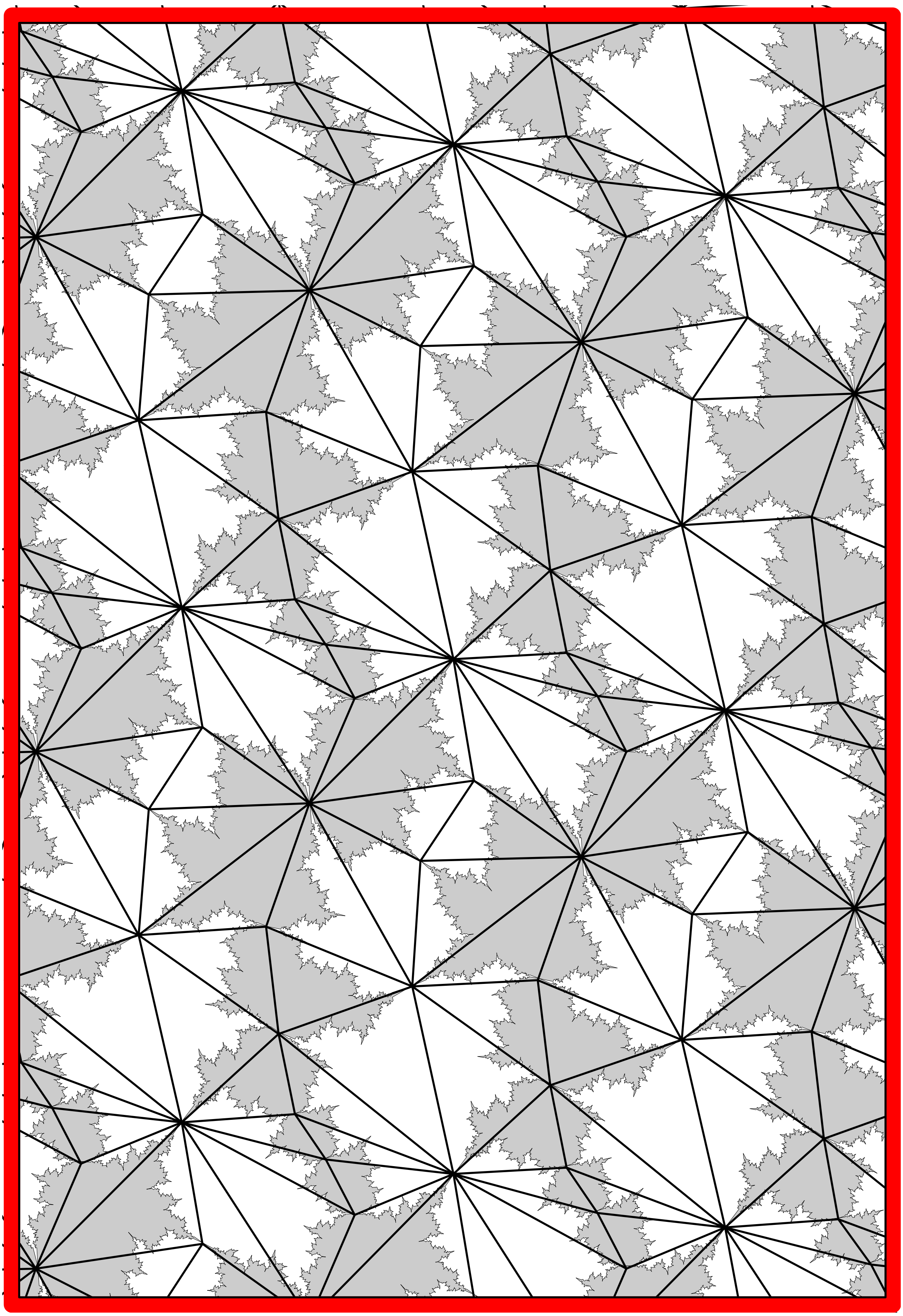
In particular, they share the same vertex set.



Snappea's labelled cusp triangulation for LRRLLLRRRR.



The Cannon-Thurston tessellation for RLLRRRLLLL.



The Cannon-Thurston tessellation for RLLRRRLLLL scaled, rotated, trimmed, and placed under Snappea's cusp triangulation for LRRLLLRRRR.

[Gueritaud]

$\varphi : \Sigma \to \Sigma$: p. A homeo, which is fully punctured

i.e the associated singular Euclidean metric of Σ

has singularities only at the punctures.

(1) One can construct a natural topological ideal triangulation

of $M_\varphi = \Sigma \times \mathbb{R} / (x,t) \sim (\varphi(x), t+1)$, by using the

singular Euclidean structure of Σ.

(2) The ideal triangulation is isotopic to Agol's veering ideal triangulation.

(3) Cannon - Dicks fractal tessellation exists in this setting,

and it is intimately related to the lifted cusp triangulation

induced by the veering ideal triangulation.

[Gueritaud] + [Sakata] $\rightsquigarrow$ nice result for hyperbolic 2-bridge link

with slope $[2, 2, 2, \cdots, 2]$

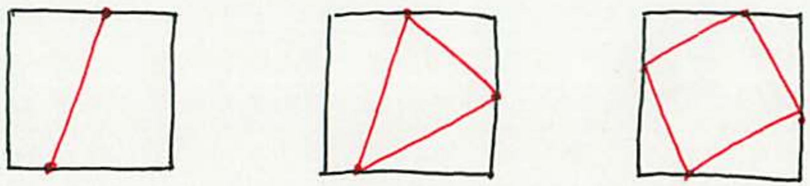
(Idea) Delaunay construction with respect to the singular Euclidean metric

g_0 : singular Euclidean metric on Σ, s.t. $\varphi(x, y) = (Kx, \frac{1}{K}y)$

$g_t := \begin{pmatrix} e^t & 0 \\ 0 & e^{-t} \end{pmatrix} g_0$,

$\overline{\Sigma}_t$ = metric completion of the universal cover of (Σ, g_t)

τ_t : triangulation of $\overline{\Sigma}_t$ obtained from maximal, singularity-free squares



(non-generic)

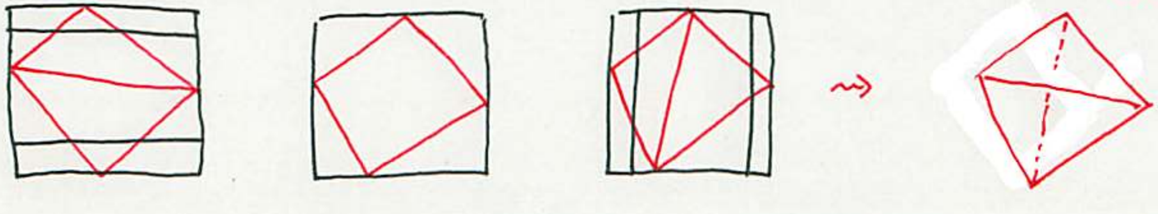
square

= ball w.r.t. the distance

"$d((x_1, y_1), (x_2, y_2)) = \max(d(x_1, x_2), d(y_1, y_2))$"

Generically τ_t is a triangulation, and non-generic τ_t gives a Whitehead move :

tetrahedron



[Minsky - Taylor]

Veering triangulations have nice properties and are very useful !

- The edges of a veering triangulation form a totally geodesic subspace of the arc complex of Σ.
- Veering triangulations do not depend on particular fibrations. They depend only on the corresponding fiber face, i.e they depend only on the flow determined by the fibration.

My dogma

Surface bundles and Heegaard splittings are intimately related. (Please see Section 7)

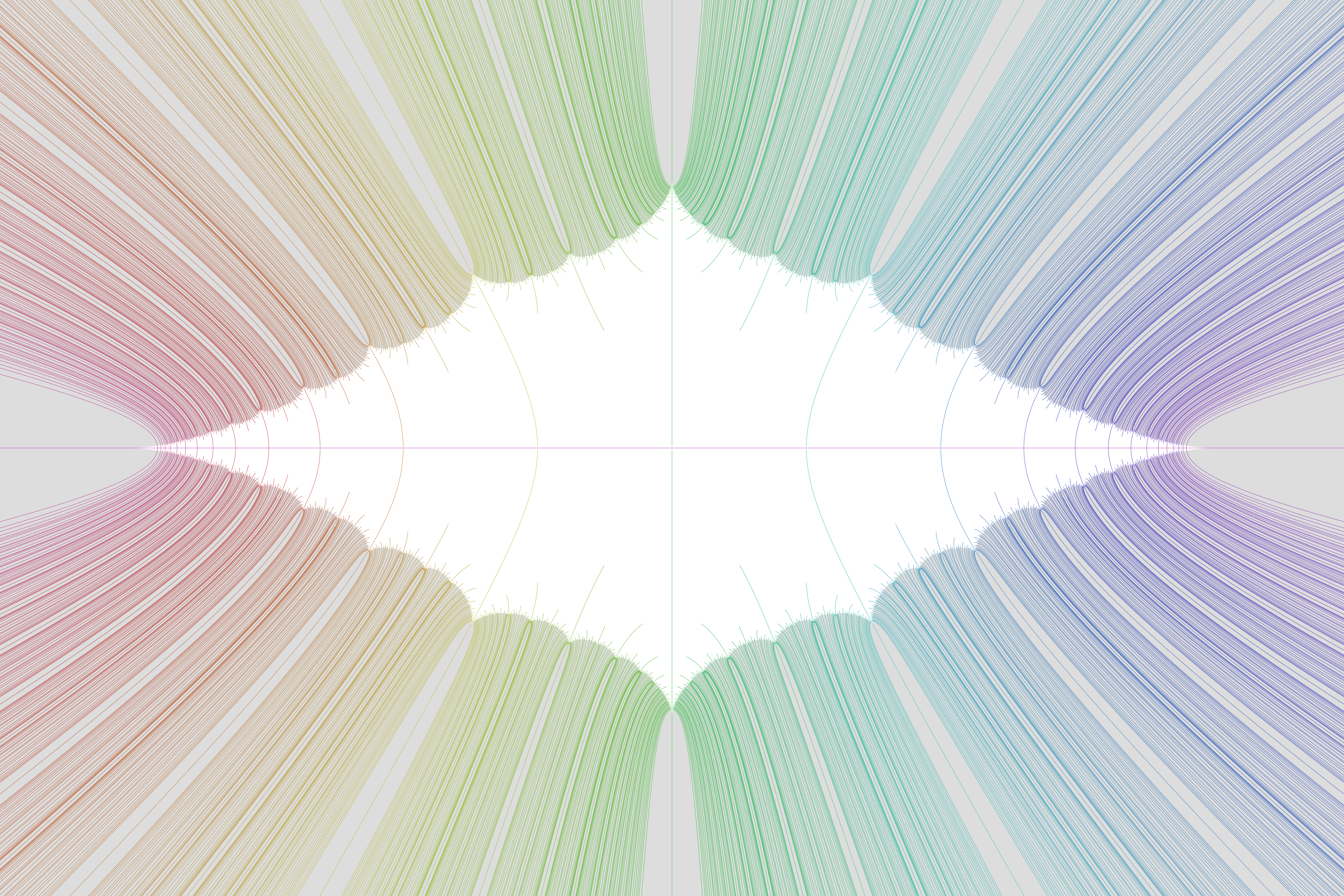
伊藤若冲の世界

象 と 鯨

Surface bundle & Heegaard splitting

Question

Is there an analogy of veering triangulations for bridge decompositions of links?



ご清聴

ありがとうございました