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On The Group Of Fuchsian Equations

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PREFACE

It has been 10 years since I first gave a talk on the group of fuchsian equations before a public audience. Although I do not believe that the study came to a completion or came nearly to it, I decided to give a systematic account of what we know and we cannot achieve. I had to leave the materials on Saalshutzian series for the next occasion.

In chapter I, we reduce a single higher order fuchsian equation to a system of first order equations with linear coefficients. We give a proof of our basic formula (extended Gauss-formula) in the next Chapter. With this tool we discuss monodromy representations of the system and will show that the group is generated by some sort of reflections in complex space. Existence of Hermitian invariants is an important implication of the structure of the covering surfaces of solutions. But we could not give full description of the geometric structures, yet.

The following Chapter IV was a result of our friendly collaboration which had been neglected because of my personal situations. I am grateful for the permission given to me by Professor K.Takano to include the material here. It will be published somewhere else.

In chapter V, we discuss reducibility and irreducibility. This topic is very important and the results we obtained are not sufficient to claim that we have given a systematic account of fuchsian equations. Because the reduction we used in the first chapter reduces an irreducible lower order equation to reducible systems with more number of dependent equations than the order of the original equation. Moreover, we always encounter a reducible equation in studying Stokes phenomena.

Our appendix is a list of systems whose monodromy representation can be computed algebraically. There is an interseting relation with the theory of automorphic functions and the field in which our matrix elements belong.

Namely, if we are to deal with fuchsian equations only, we are no longer concerned with groups of the most general representation GL(d,C). Moreover we may restrict ourselves on groups with hermitian invariants. The restricted Riemann Problem of, say, U(d) or GL(d,K) for some K, may well be of sense.

If one thinks this survey to be boring, I recommend the articles [9] and [15] for short review, especially the later. This survey gives the general background for the Sasai's paper [15] wishing to enrich our vocaburary of the so-called special functions.

Finally, I wish to express my heartful gratitude to my teacher Masuo Hukuhara, Professor Emeritus of Tokyo University, without whose encouragement I never thought of publishing the material in this form. I dedicate this paper to him. I, also, extend my indebtedness to my friend Professor Shinji Yamashita, who accused me of my being idle for such a long period.

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