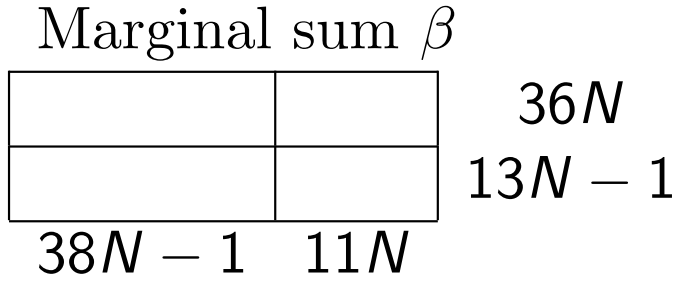
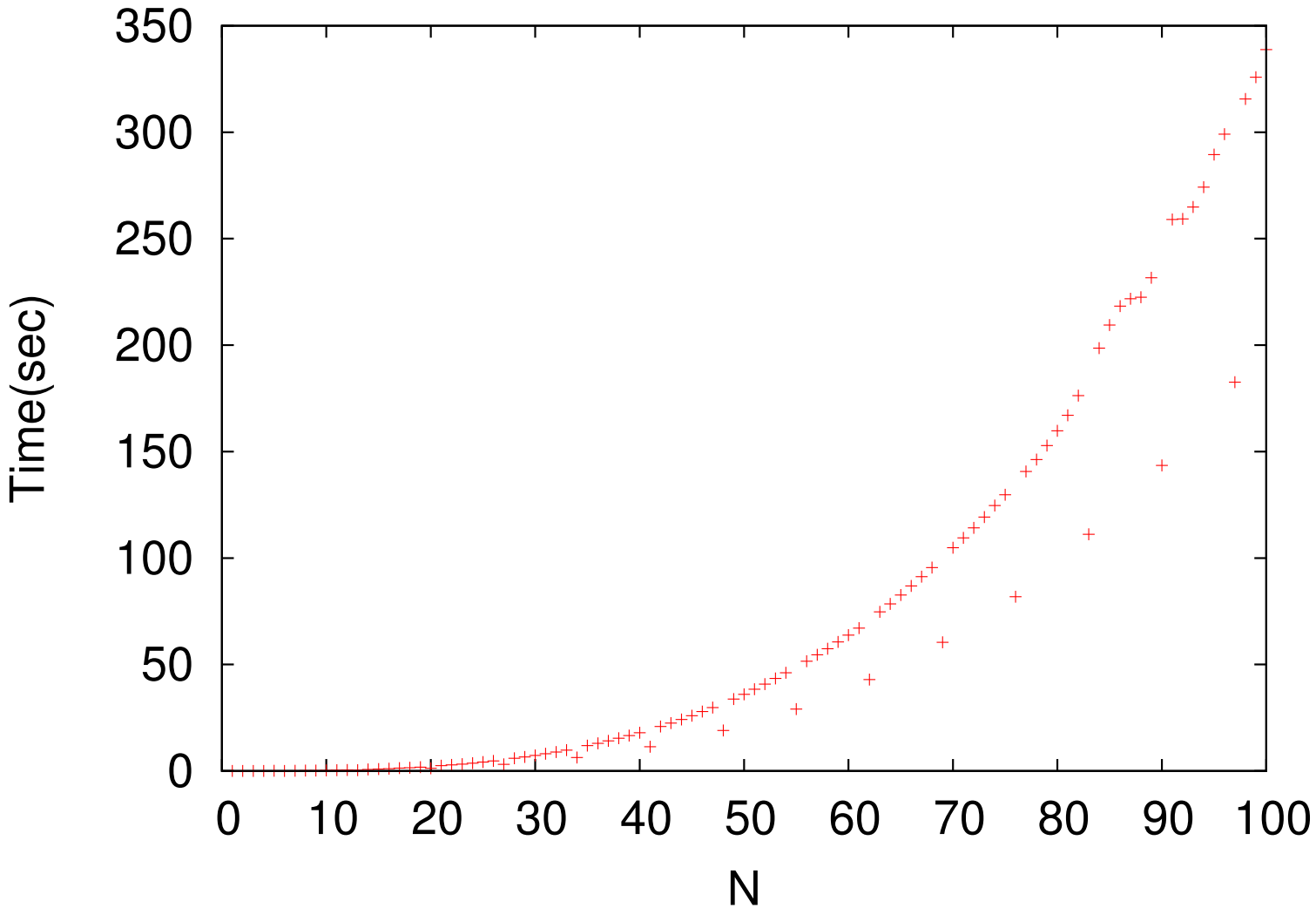
Evaluating the normalized constant of 2×2 with the following marginal sums by the modular method.



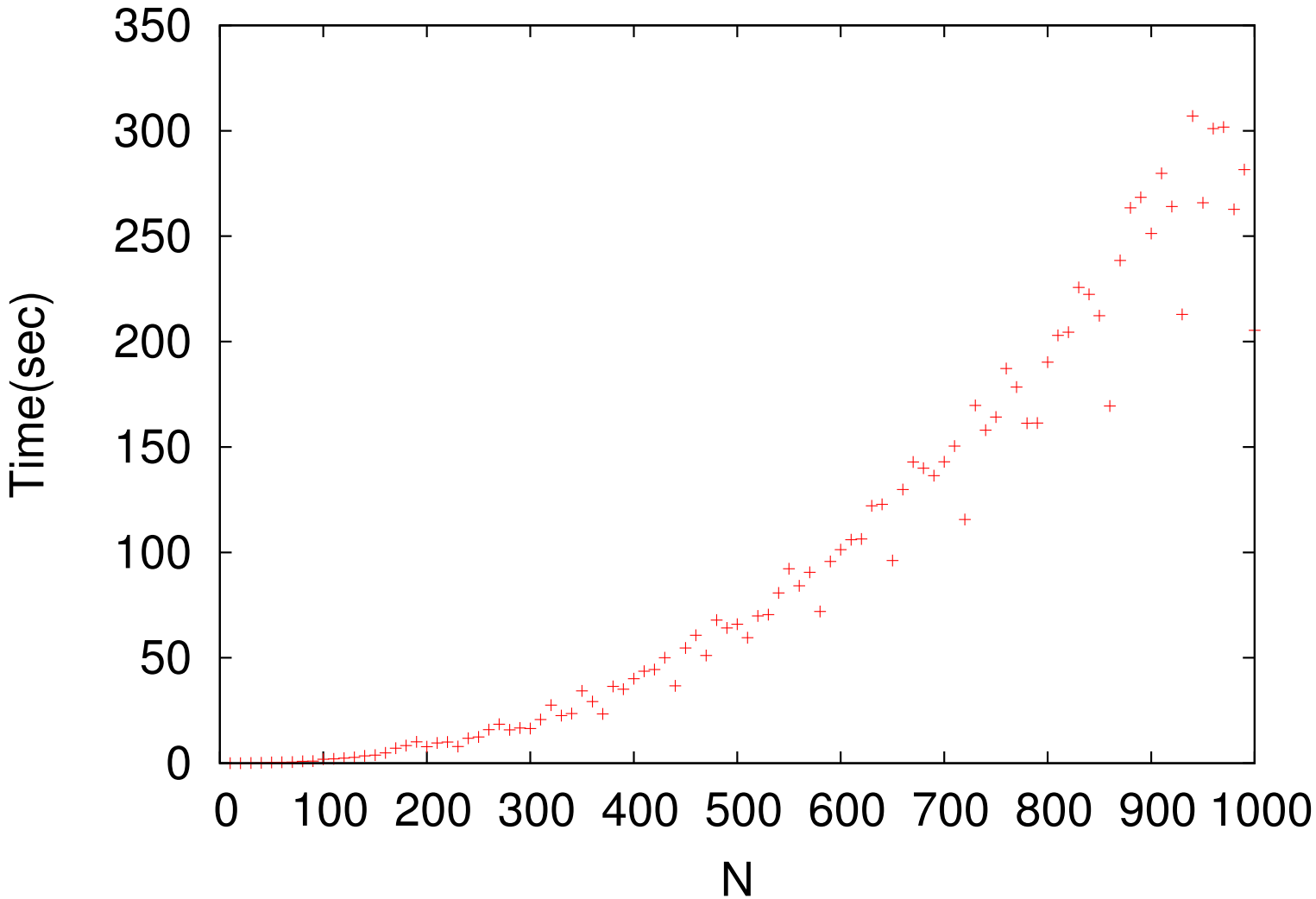
cpu	Intel(R) Xeon(R) CPU E5-4650 2.70GHz
the number of cpu's	32
the number of cores	8
os	debian 7.8
memory	256 GB
language	Risa/Asir version20150126

Data 1: Naive rational arithmetic



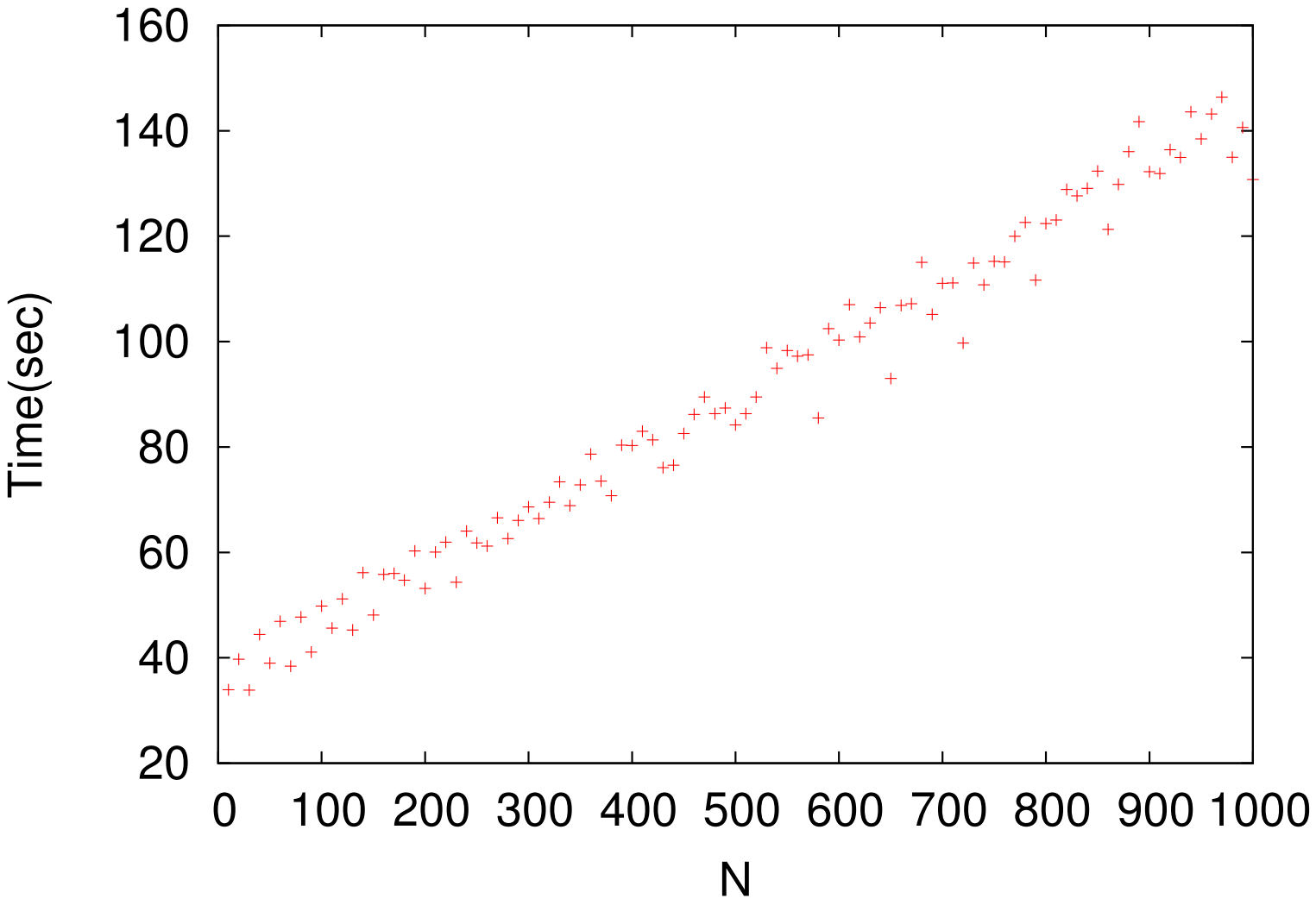
Data 2: Integer arithmetic

Numerators and denominators are evaluated separately.

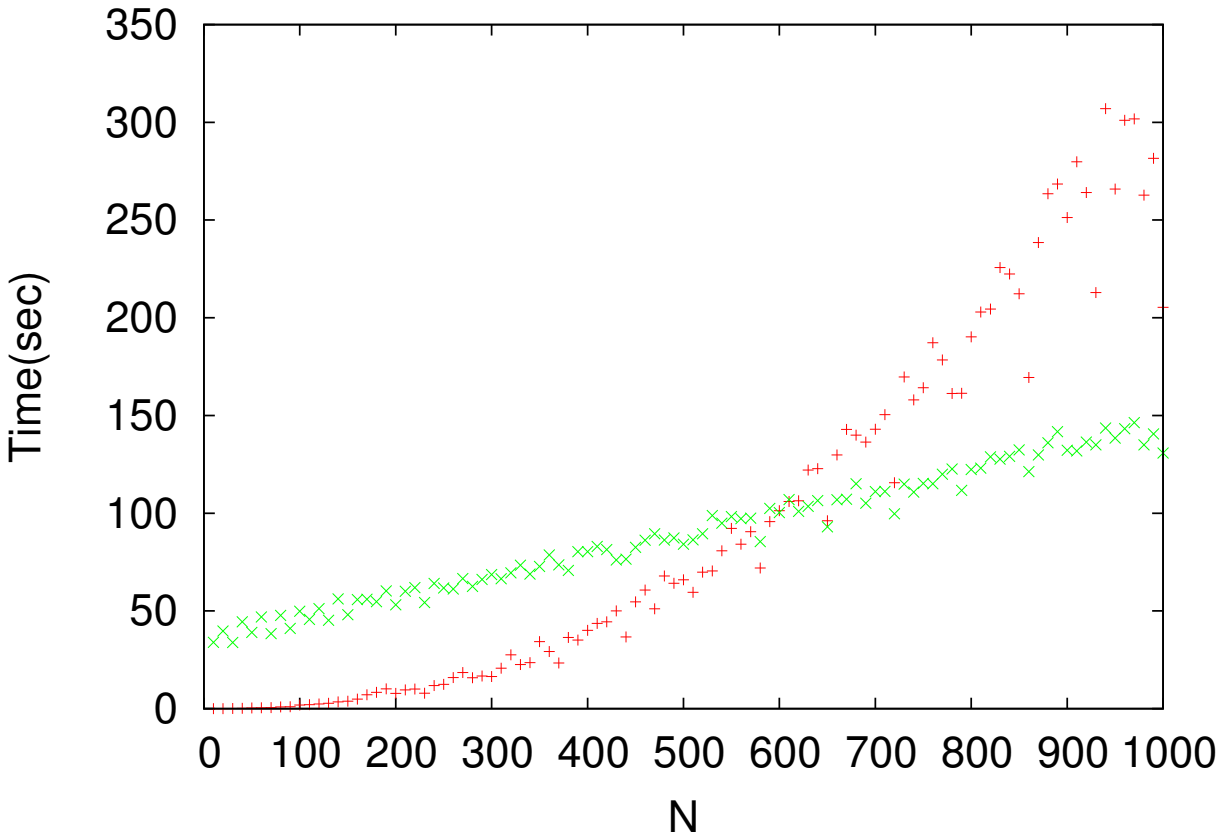


Data 3: Modular method

16 processes, 1000 primes of 100 digits.



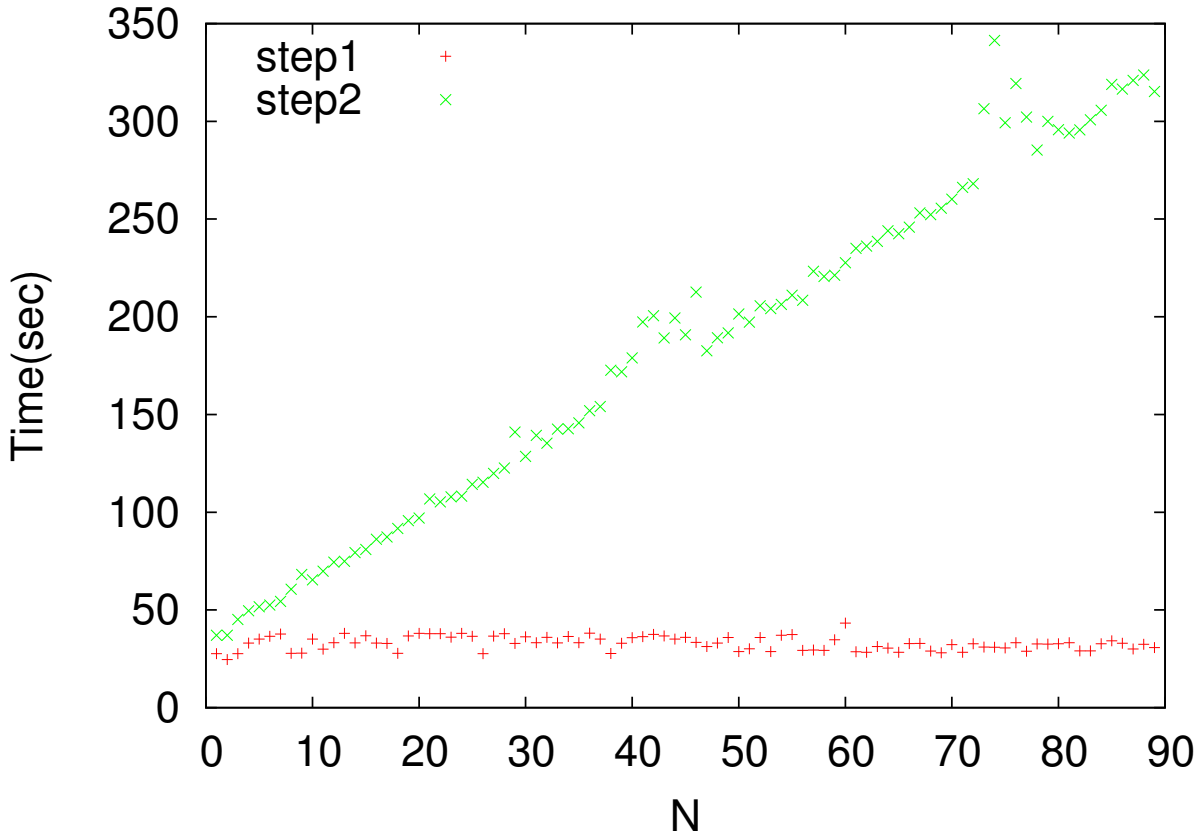
Super impose of data 2 and data 3



The modular method is useful for exact evaluation of the normalizing constant and its derivatives.

5×5 contingency tables

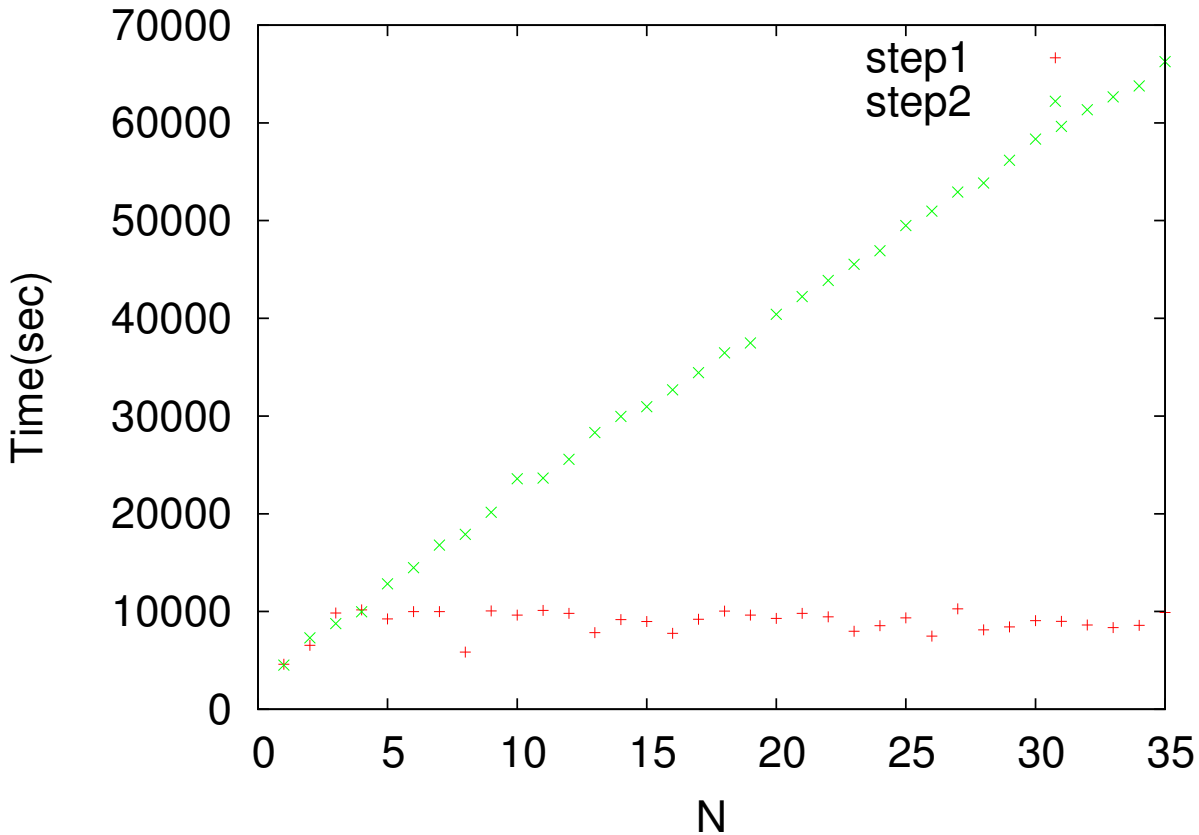
Marginal sums are
$\left[\left[4N \quad 4N \quad 4N \quad 4N \quad 4N \right] \quad \left[2N \quad 3N \quad 5N \quad 5N \quad 5N \right] \right]$
200 prime numbers of 100 digits, 8 processes. Failure at $N = 90$.



7×7 contingency tables

$$\beta = \begin{bmatrix} \begin{bmatrix} 4N & 4N & 5N & 5N & 5N & 5N & 5N \end{bmatrix} \\ \begin{bmatrix} 6N & 3N & 4N & 4N & 4N & 6N & 6N \end{bmatrix} \end{bmatrix}$$

200 prime number of 100 digits. 16 processes. Failure at $N = 35$.



setup of gtt_ekn.rr

Options.

1. `nps` : number of processes.
2. `nprm` : number of primes)
3. `minp` : use primes more than minp.
4. `report=1` : show the current option values.

Default values are nps=1, nprm=10, minp=10^{10}.

Example

```
[2163] gtt_ekn.setup(|nps=4,nprm=200,minp=10^100);
Number of process = 4.
Number of prime = 200.
Min of plist = 100000...000267.
0
[2164] gtt_ekn.report();
Number of process = 4.
Number of prime = 200.
Min of plist = 100000...000267.
0
```

gtt_ekn.nc (normalizing constant)

Web:
`http://asir2.math.kobe-u.ac.jp/cgi-bin/cgi-cmle.sh`

input β =[[row sums],[column sums]], p: probability

return normalizing constant Z and its derivatibes($(k+1)\times(n+1)$)

```
[2162] gtt_ekn.nc([[5,5],[3,3,4]],[[1,1,1],[1,1/2,1/3]]);
[947/62208,[ 2779/186624 2179/93312 589/15552 ]
[ 359/11664 4165/93312 179/2592 ]]
```

marginal sums β

			5
			5
3	3	4	

probability p

1	1	1
1	1/2	1/3

gtt_ekn.expectation

input β =[[row sums],[column sums]], p: probability

return expectation of each cells.

```
[2163] gtt_ekn.expectation([[5,5],[3,3,4]],
                           [[1,1,1],[1,1/2,1/3]]);
[ 2779/2841 4358/2841 2356/947 ]
[ 5744/2841 4165/2841 1432/947 ]
```

marginal sums β

			5
			5
3	3	4	

probability p

1	1	1
1	1/2	1/3

option — crt

When crt=1, the modular method is used by utilizing crt = chinese reminder theorem.

```
[2166] gtt_ekn.setup()$
Number of processes = 1.
Number of primes = 10.
Min of plist = 10000000019.

[2167] gtt_ekn.nc([[5,5],[3,3,4]],
                  [[1,1,1],[1,1/2,1/3]]|crt=1);
[947/62208,[ 2779/186624 2179/93312 589/15552 ]
[ 359/11664 4165/93312 179/2592 ]]

[2168] gtt_ekn.expectation([[5,5],[3,3,4]],
                           [[1,1,1],[1,1/2,1/3]]|crt=1);
[ 2779/2841 4358/2841 2356/947 ]
[ 5744/2841 4165/2841 1432/947 ]
```

Comparison with the Markov chain Monte Carlo (3×3contingency tables)

.

u_{11}	u_{12}	u_{13}	400
u_{21}	u_{22}	u_{23}	410
u_{31}	u_{32}	u_{33}	1011
910	411	500	1821

probability p

1	1/1000	1/1001
1	1/2	1/3
1	1	1

Compare our exact result with values by MCMC.

```
[2719] Beta = [[400,410,1011],[910,411,500]]$
[2720] P = [[1,1/1000,1/1001],[1,1/2,1/3],[1,1,1]]$
[2721] Expect=gtt_ekn.expectation(Beta,P)$
[2722] ExpectMCMC=expectT([[400,0,0],[410,0,0],[100,411,500
[2723] eRatio(Expect,ExpectMCMC); // ratios with the exact
[ 1.00045 0.821255 0.855683 ]
[ 0.999222 0.997529 1.00455 ]
[ 0.999952 1.00115 0.999172 ]
```

MCMC :
Burn-in $= 10^5$, Number of steps $= 10^5$.

```
[2724] Expect[0][0];
```

```
21465911405418907647281605696499755266148553200038206872361455316958743172476403024795926679503805825488531140624592210723252264183355960549595951421888693949967026245818776113719783268852950478133427795044374747600837657576543817160213973561467717042624372969512858469416556730008249622602790383274729836822514803725904381407982128056973554424518285605679108610873979248646925608080230805430948159646640955980102617248856151061778754097894805591379296154547037096333665851485740801299081987084246119008372124224686144235221387354781058704230326263619240727362812787840389323238013702999849128431173709444889561622609406544004420508726239056651840066765865363486592597997646552145410833587286575118758161914174673908095312504213840212957967695334123926455064618791979995358670449029408214170252801473974929221995015042388897331096588107419426060720634373071470312035025415854241676133918921970584014741404041311202398838839224697340887230011468831034584981273304218007134616476111866043118753096869215648576850692020693752530258771455114709656261282021290839725459821689179705962456690433383094422853871608387510756459695208435195846597476807617291762183309328166892721460556985988434455051597244707653388706974061236571088552302438061842208048511446704724475391290759687736461253128897934383536999658875293562637533933101575325014769572404503767719038356702230212524694393883569165168862612872943075131410310601319557440514383694741266155690403858409024277011847006355005733381471897395133718602942733785351174822735915368906583882643861101550119404198215864167338901502358419701237153096102176845859971587730624315950214630494552180908644269475765051298171981870445142942858737759734127235103182002639154924772725722969700461115584278312774022004301538100861908075680059629205271458661693728378412201428706119051807564400544491680504430366101346214063167511324249630707182506599903361986375535502076732600446782199997410188426222013100208484739440924853317420308430900009428010046199403943055674534639385801896408692409244645197830488610925294881232145577725797064488500004850281286436188102129193633326883145550636911353525948650042607799473321917976984178798515465394205200729114467966650277205251918511980045973275325609765533086008683725521372698489358719606365476767416606838688984062114993925929375729477590285844196861922951006922678253005762856754483529328066309779557165566278039087196660946964745273969480832351963743122696468646750967674919587449488942983734513164421154985285468362516994481502833609166554652419434301785496820485075767848871048428577932117385045294635655818185902817602882136899192410832190239891825324861213169358165751525698291826760970042654001947031283449042777428121381280753471712738481942060757441244911125369961819472512613074037509298855264168030149246113069929893212517163758584997387461996371137640874084235458509190645543537375070397645145508401045402740855240882731828350645649572780714141247707313071128882600262279178893293139050393634981639435460625756036343629542444032545704904654026892034391278675885414774884124093143814017146668144245075000/537909525471669567070951580950290105541547580030867305561430733444502853041961644020985835792149006474196409012680042001767514212525308049354183623992255139231539386359816374602887805184205448793004091228070952554535958116027160182568758762117264011913754408639071566200319832160791777214271859245010718864427648975779210031075674250954707591730133164003970623946818396801829360947500160260244275367077148677370743072607580841450317074374017599730668788572102565881747679939337628393616588204436025357688435015479331222982165967820377101203746209046340265505450153774196044887069664913352090346100750903111705754902802167783694917680859057534172382397046097471005281421543561438762029307801389641143234861107427073918091982000736544567517354146673830676578325373003775135053254040345263988858374311850380015342138827814391416738423414580552795389435523998829734880604910708075663691410228740550279670694496433649393558206150546800430195778623614598964567093206746780821060665992251421490616061453471075491399855957125609159176686894809708461705652698902557020694735747792837691571216763208403944171678293337422681150023989013706303686651565275525904892728025848068300006309528865398695946723494165734841696142635085428388279438519526632396081274954457090523970357807305957153138772410652239862390123200490520892813899365054864509521897742065503388807995645676998344393467600659663276588969809539079712108683404115887138318054344400547233852027845088855646218118746559124077194955551353916384371489125776280787378939756696159729991880711623086617275309939689628404966095221194101508494950925773625968261477839008390826088599880747402291273363514803114705699848416512964538192034914404486051805394773566448971689617859430705625044803432069096621189461830650387718082466143655705739808734427601284315939792080619468228025423180267365779506705640116809291323846790150255939755816413353074083038748092123722304384222315074843874413683082952302967313631512090781953391637964768620804181145363717708373780510536548711392994344594333234556830770428211254985314968140215687733868359333566562071817715612642795875866471643370533656846616625203193791657261173179295451975135874377964534791972098634843957632447101133218375043222678085256698876966192882386948644145225306864158417578249527331167457547018420614244393757546149541469777080326250961597049567043442670410036760274702010136136012937402363712358822751840100183932920997069820365571768394957442644851330254898877408078951827054274989264428508410208067177786442734554573778537401893421933022885009669351466234442513470081144211941251825932029316530556845194000724887303339424329670717863051196729873840540661280676332944780643183109066003907810927240665539671172056403319449790923956530165348245589821859422995995834899033037373959045965178364351836106245118112067887992449695559787169276978966016251516449789468250458297777399624394962218069953650846512894774920114207826621854853894044891364282926012727160420453653196241698980550071584637072432175639837601298649589762906960158302623931838872924925379502900825553677215763235782774963528634
```

```
[2724] Expect[0][0];
21465911405418907647281605696499755266148553200038206872361
43747476008376575765438171602139735614677170426243729695128
10261724885615106177875409789480559137929615454703709633366
09406544004420508726239056651840066765865363486592597997646
88973310965881074194260607206343730714703120350254158542416
25877145511470965626128202129083972545498216891797059624566
43806184220820848511446704724475391290759687736461253128897
12661556904038584090242770118470063550057333814718973951337
64426947576505129817198187044514294285873775973412723510318
01346214063167511324249630707182506599033619863755355020760
12321455777257970644885000048502812864361881021291936333268
48935871960636547676741660683868984062114993925929375729477
49488942983734513164421154985285468362516944815028336091660
67609700426540019470312834490427774281213812807534717127384
37507039764514550840104540274085524088273182835064564957278
17146668144245075000/53790952547166956707095158095029010550
18420544879300409122807095255453595811602716018256875876211
60244275367077148677370743072607580841450317074374017599730
```