Abstracts for Ehime Number Theory Workshop

February 17th – 20th, 2009
at Ehime University
`http://www.math.kobe-u.ac.jp/HOME/tani/workshop0902ehime/`

Gombodorj Bayarmagnai (Univ. of Tokyo)

On the family of cyclic Galois extensions

We discuss how Galois cohomology leads to a descent-generic cyclic polynomial of degree n, with minimal number of parameters, under certain conditions of base field. Then we prove that the splitting field extension of that polynomial is a versal element for the first Galois cohomology functor with coefficient cyclic group of order n.

Anna Cadoret (Univ. de Bordeaux 1)

A uniform open image theorem for p-adic representations of etale fundamental groups of curves
(joint work with Akio Tamagawa - R.I.M.S.)

Let k be a finitely generated field of characteristic 0, X a smooth, separated, geometrically connected curve over k with generic point η. Fix a prime p. A representation $\rho : \pi_1(X) \to \mathrm{GL}_d(\mathbb{Z}_p)$ is said to be geometrically strictly nonabelian (GSNA for short) if $\mathrm{Lie}(\rho(\pi_1(X_{\overline{k}})))^{ab} = 0$ or, equivalently, any open subgroup of $\rho(\pi_1(X_{\overline{k}}))$ has finite abelianization. Typical examples of such representations are the representations $\rho_{A,p}$ arising from the action of $\pi_1(X)$ on the generic Tate module $T_p(A_\eta)$ of an abelian scheme A over X. Let G denote the image of ρ. Any k -rational point x on X induces a splitting $x : \Gamma_k \to \pi_1(X)$ of the canonical restriction epimorphism $\pi_1(X) \to \Gamma_k$ so one can define the closed subgroup $G_x := \rho \circ x(\Gamma_k) \subset G$. The main result I am going to di scuss is the following uniform open image theorem. *For any GSNA representation $\rho : \pi_1(X) \to \mathrm{GL}_d(\mathbb{Z}_p)$ the set X_ρ of all $x \in X(k)$ such that G_x is not open in G is finite and there exists an integer $B_\rho \geq 1$ such that $[G : G_x] \leq B_\rho,\ x \in X(k) \setminus X_\rho$.*

Applied to the action of $\pi_1(X)$ on the generic Tate module $T_p(A_\eta)$ of an abelian scheme A over X, this result yields uniform bounds for the (twisted) p-primary torsion of the A_x, $x \in X(k)$ as well as uniform bounds for the order of the Γ_k-invariant p-primary subgroups of the A_x, $x \in X(k) \setminus X_{\rho_{A,p}}$.

Using additionnal technics for estimating the gonality along projective systems of curves, one can deduce from the above uniform open image theorem the following stronger statement: *For any GSNA representation $\rho : \pi_1(X) \to \mathrm{GL}_d(\mathbb{Z}_p)$ and for any integer $d \geq 1$ the set $X_{\rho,d}$ of all closed points $x \in X$ such that $[k(x) : k] \leq d$ and G_x is not open in G is finite and there exists an integer $B_{\rho,d} \geq 1$ such that $[G : G_x] \leq B_{\rho,d},\ x \notin X_{\rho,d}$ with $[k(x) : k] \leq d$.*

During the first talk, I will describe the frame of our work, state the main results and explain the links with several standard conjectures such as Mumford-Tate conjecture and torsion conjectures for abelian varieties. The second talk will be devoted to sketching the proof of the uniform open image theorem and its application to uniform boundededness of p-primary torsion of abelian varieties. If time allows, I will also indicate briefly how our uniform open image theorem and our results about gonality imply the strong variant.

Kohei Katata (Ehime Univ.)

An example of the Ihara zeta function associated to a normal covering of a graph

There is a zeta function associated to a graph. This function relates to the eigenvalues of the adjacency matrix of the graph. In 1988, the notion of the Ramanujan graphs was introduced by Lubotzky, Phillips and Sarnak. Ramanujan graphs are "good" graphs not only in communication network but also in number theory, since they have proper eigenvalues. There is a probrem in constructing a family of Ramanujan graphs with a fixed regularity. We will construct a family of graphs with a fixed regularity, and study their Ramanujency.

Taku Ishii (Seikei Univ.)

Archimedean Whittaker functions and archimedean zeta integrals

We will report recent progress on explicit formulas of Whittaker functions over archimedean local fields and their application to computation of archimedean zeta integrals for certain automorphic L-functions.

Tetsushi Ito (Kyoto Univ.)

On the Sato-Tate conjecture for elliptic curves over number fields which are not necessarily totally real

Recently, based on a joint work with Clozel, Harris and Shepherd-Barron, Taylor proved the Sato-Tate conjecture for elliptic curves over totally real fields whose j-invariants are not algebraic integers. In this talk, I give an argument on the automorphy of ℓ-adic Galois representations which enables us prove few more cases of the Sato-Tate conjecture for elliptic curves over number fields. The base fields are not necessarily totally real, but both the elliptic curves and the base fields should satisfy some very restrictive conditions. This result might be "well-known" for specialists, but it seems of some interest for some other applications concerning analytic properties of L-functions (e.g. the Tate conjecture for Hilbert modular surfaces, etc.).

Yoichi Mieda (Kyushu Univ.)

Non-cuspidality outside the middle degree of ℓ-adic cohomology of the Lubin-Tate tower

The Lubin-Tate space is the moduli space of deformations of a fixed one-dimensional formal O-module of finite height, where O is the ring of integers of a non-archimedean local field F. By using Drinfeld level structures, we may construct a projective system of moduli spaces over the Lubin-Tate space, which is called the Lubin-Tate tower. The representation obtained as the vanishing cycle cohomology of the Lubin-Tate tower is very interesting. In fact, it realizes the local Langlands correspondence (of the general linear group) and the local Jacquet-Langlands correspondence over F.

In this talk, we give an easy and direct proof of the fact that no supercuspidal representation appears as a subquotient of such representations unless they are obtained from the cohomology of the middle degree. This fact has previously been proved by Boyer and Faltings by using the relation between the Lubin-Tate tower and an integral model of a certain Shimura variety.

Hiro-aki Narita (Kumamoto Univ.)

Fourier expansion of Arakawa lifting and central L-values

We have an explicit formula for Fourier coefficients of Arakawa lifting, i.e. a theta lift from a pair of an elliptic cusp form f and an automorphic form f' on a definite quaternion algebra to a cusp form on $GSp(1;1)$ generating quaternionic discrete series at the

Archimedean place. Up to an elementary constant, such Fourier coefficient is a product of toral integrals of f and f' with respect to a Hecke character χ of an imaginary quadratic field. A well-known formula of Waldspurger says that square norms of such toral integrals are proportional to the central values of χ-twisted L-functions for quadratic base change of f or f'. Our next task is to explicitly determine the constant of proportionality for the square norms of the toral integrals and the central L-values. This leads to an explicit constant of proportionality for the square norm of a Fourier coefficient of Arakawa lift and a product of the central L-values attached to f and f'. In this talk we will report the recent progress of our study for that. This is a joint work with Atsushi Murase.

Nicole Raulf (Univ. de Sciences et Technologies de Lille)

Class number asymptotics for fundamental discriminants

Since the early studies of Gauß class numbers have fascinated mathematicians. One of the important questions concerning class numbers is the following: Let h_d be the class number of primitive binary quadratic forms of discriminant d whose coefficients belong to $\mathbb{Z}$. What is the behaviour of the mean value of h_d? So far mathematicians have not been able to solve this problem. There only exist results that give the asymptotic behaviour of

$$\sum_{0<d\leq N} h_d \log \epsilon_d$$

as $N \to \infty$. Here ϵ_d is the fundamental solution of Pell 's equation $t^2 - du^2 = 4$. Sarnak showed that the problem of separating the class number from the regulator disappears if we order the terms of the sum by the size of the regulator. In this talk we will show how to obtain a similar result if we restrict ourselves to discriminants that belong to a given progression or are fundamental discriminants.

Jeehoon Park (McGill Univ.)

(1) Introduction to Iwasawa theory

This is a preparation of the second talk. We will introduce the basic concepts in Iwasawa theory and explain the Iwasawa main conjecture for $\mathbb{Q}$ (the simplest case and the original conjecture of Iwasawa) for non-experts.

(2) Iwasawa main conjecture for CM elliptic curves at supersingular primes

We generalize the Pollack-Rubin proof of the Iwasawa main conjecture for CM elliptic curves over $\mathbb{Q}$ at supersingular primes to CM elliptic curves over an abelian extension of the imaginary quadratic field given by CM. We will explain the precise set-up, how to construct plus/minus algebraic p-adic L-functions and plus/minus analytic p-adic L-functions and how they coincide. This is a joint work with Byoung Du Kim and Bei Zhang.

Henrik Russell (Univ. of Duisburg-Essen / Kyoto Univ.)

Generalized Albanese and duality

A generalized Albanese for a singular projective variety over an algebraically closed field was constructed by Esnault, Viehweg and Srinivas as a universal regular quotient of a relative Chow group of zero cycles modulo rational equivalence. This is a smooth connected commutative algebraic group (not in general an abelian variety), satisfying a universal mapping property. For the talk, we suppose the characteristic of the base field is zero. We ask for a dual of this gen. Albanese and a functorial description.

Takashi Taniguchi (Kobe Univ.)

Orbit structures of exceptional type prehomogeneous vector spaces

The notion of prehomogeneous vector space (PV) was introduced by M. Sato. Many intersting PVs appear as nilpotent orbits of certain simple algebraic groups. For example, the space of binary quadratic forms $(\mathrm{GL}(2), \mathrm{Sym}^2 k^2)$ arises as nilpotent orbits for the Sigel parabolic subgroup of Sp(4) (the symplectic group of rank 2). Among them, PVs those related to exceptional groups have extremely rich orbit structures from arithmetic point of view. In this talk we explain such orbit structures and possible applications to number theory.

Takahiro Tsushima (Univ. of Tokyo)

Epsilon factor of Fermat curves

Coleman gave a stable model of Fermat curve and calculated the Jacobi sum - Hecke character explicitly in 1988 using rigid geometry and CM theory. In my talk, I would like to formulate three conjectures about refined conductor formula, and twist formula and mod p formula for epsilon factor. In the Fermat curve case, we will prove these conjectures. We introduce another proof of a special case of Coleman's result by using ℓ-adic etale cohomology and Kato's ramification theory.

Yoshinori Yamasaki (Ehime Univ.)

Evaluations of higher depth determinants of Laplacians

For a "good" operator T, higher depth determinants of T are defined by derivatives at non-positive integers of the spectral zeta function attached to T. In this talk, we will evaluate the higher depth determinants of Laplacians on both compact Riemann surfaces with negative constant curvature and higher dimensional spheres (the former is a joint work with Nobushige Kurokawa and Masato Wakayama).

Shuji Yamamoto (Univ. of Tokyo)

Zeta functions and cone decompositions for totally real fields

To investigate zeta functions of a totally real number field F, Shintani introduced a cone decomposition of the totally positive part of $F \otimes \mathbb{R}$ which is compatible with the action of units. When one uses this method, some combinatorial arguments about these cones enter into the study of zeta functions. To illustrate this point, I want to explain some properties of Shintani's ray class invariants (constructed from partial zeta functions) and their proofs.

Masahiko Yoshinaga (Kobe Univ.)

Periods and computational complexity of real numbers

Kontsevich and Zagier introduced the notion of "Periods", which contains all algebraic numbers and several transcendental quantities and considered to be an important class of complex numbers. In this talk, I will give a few remarks on periods from the view point of computational complexity of Cauchy sequences.