

# Abstracts for Arithmetic and Algebraic Geometry 2019

– in honour of Professor Tomohide Terasoma's 60th birthday –

**Date** January 21 (Mon) – 25 (Fri), 2019  
**Location** The University of Tokyo, Komaba I Campus  
**Venue** Auditorium in Graduate School of Mathematical Sciences building  
**Website** <http://www.math.kobe-u.ac.jp/HOME/tani/workshop1901tokyo/>

## TIMETABLE

	21 (Mon)	22 (Tue)	23 (Wed)	24 (Thu)	25 (Fri)
10:00-11:00	Kaji	Mieda	Esnault	S. Saito	Pearlstein
11:30-12:30	Shioda	Li	Levine	Geisser	Kimura
14:30-15:30	Schütt	Zagier	**	Gangl	Hanamura
16:00-17:00	Shimada	Terasoma	**	Furusho	T. Saito
18:00-		PARTY			

### ◆ Hélène Esnault (Berlin)

#### Vanishing theorems for étale sheaves

We'll explain Scholze's vanishing theorem for étale sheaves and the proof we gave of it. This leads to questions of new type in particular on the Galois invariant loci in the character variety, which we started investigating with Moritz Kerz.

### ◆ Hidekazu Furusho (Nagoya)

#### Kontsevich's eye, Lie graphs and associators

After I will review on the notion of Kontsevich's eye and his weights associated with Lie graphs, I will discuss on the new associator constructed by Alekseev and Torossian.

### ◆ Herbert Gangl (Durham)

#### Towards a quadruple ratio

In his celebrated proof of Zagier's polylogarithm conjecture for weight 3 Goncharov has introduced as a crucial new ingredient the so-called triple ratio which is a projective invariant generalising the well known cross ratio. Recent work of Goncharov and Rudenko prove the weight 4 case of the conjecture with a somewhat indirect method where they circumvent the need to define a corresponding notion of 'higher cross ratio'. We propose a definition of such a notion by giving an explicit albeit complicated candidate (joint work with S. Charlton and D. Radchenko).

◆ Thomas Geisser (Rikkyo)

### Comparing the Brauer group and the Tate-Shafarevich group

Artin and Grothendieck proved that for a regular proper surface  $X$  over a complete regular curve  $C$  over a finite field, the Brauer group of  $X$  is finite if and only if the Tate-Shafarevich group of the Jacobian of the generic fiber of  $X/C$  is finite. Several authors gave formulas relating their order under certain hypothesis. I will give the proof of a formula in the general case.

◆ Masaki Hanamura (Tohoku)

### Explicit Hodge complexes of smooth varieties

The usual Hodge complex for a smooth variety (according to Beilinson-Deligne) consists of complexes which are “abstract” in nature, involving Godement resolutions of sheaves for example.

We give the construction of a Hodge complex for a pair of smooth varieties, which is explicit in the sense that its  $Q$ -part is a complex of topological chains, and its  $C$ -part is made out of the complex of differential forms. The Cauchy-Stokes formula, which vastly generalizes the classical Cauchy formula, is employed to give the comparison map between the  $Q$ -part and the  $C$ -part.

◆ Hajime Kaji (Waseda)

### Degree Formula for Grassmann Bundles, II

A Grassmann bundle associated to a vector bundle on an algebraic variety is by definition a variety parametrizing subbundles of the vector bundle with fixed rank. If the base variety is projective and if the determinant bundle of the vector bundle is very ample, then the Grassmann bundle is naturally embedded into a projective space. In this talk we discuss degree of Grassmann bundles as projective varieties. An explicit formula and its proof is given in *Journal of Pure and Applied Algebra* 219 (2015) by Terasoma and Kaji. I explain here another proof of the formula. This is a joint work with Tomohide Terasoma.

◆ Kenichiro Kimura (Tsukuba)

### The Abel-Jacobi map for higher Chow cycles (in progress)

I will explain about an attempt to describe the Abel-Jacobi map for higher Chow cycles as period integrals. The main ingredient is what we call the complex of admissible chains defined in terms of semi-algebraic triangulation of complex varieties.

◆ Marc Levine (Duisburg-Essen)

### Curve counting with quadratic forms

Developments in motivic homotopy theory have enabled refinements of classical integral solutions to enumerative problems to yield invariants in a suitable Grothendieck-Witt ring with the rank recovering the integral invariant. Over the reals, the signature gives information on the real solutions to the problem. We give an introduction to this theory as well as examples such as counting lines on hypersurfaces and counting rational curves on del Pezzo surfaces.

◆ Zhonghua Li (Tongji)

#### Sum of interpolated multiple $q$ -zeta values and sum of their truncations

Interpolated multiple  $q$ -zeta values are deformation of multiple  $q$ -zeta values. The truncated interpolated multiple  $q$ -zeta values are called the interpolated finite multiple harmonic  $q$ -series. In this talk, I will first talk about the joint work with N. Wakabayashi on the sums of the interpolated multiple  $q$ -zeta values with fixed weight, depth and  $i$ -height. A generating function of these sums is represented by basic hypergeometric functions, which implies some relations for interpolated multiple  $q$ -zeta values. Then I will talk about the joint work with E. Pan on the sums of the interpolated finite multiple harmonic  $q$ -series with fixed weight, depth and  $i$ -height. A similar generating function is computed and some relations of the interpolated finite multiple harmonic  $q$ -series at roots of unity are obtained.

◆ Yoichi Mieda (Univ. Tokyo)

#### Cohomology of perfectoid spaces and their reductions, with application to the local Langlands correspondence

Under some conditions, I will compare the compactly supported  $l$ -adic cohomology of a perfectoid space and that of the reduction of its formal model. I will also explain how it can be used to study relation between the local Langlands correspondence and geometry.

◆ Gregory Pearlstein (Texas)

#### Jumps in the Archimedean Height

To pair of normal functions on a complex manifold  $U$  one can associate a canonically metrized line bundle  $(B, h)$ . If  $U$  is Zariski open in a complex manifold  $S$  one can ask two natural questions:

- (a) Is there a canonical choice of extension of  $B$  to a line bundle on  $S$ ?
- (b) Does the metric  $h$  extend to a metric on  $S$ ?

For normal functions which are admissible in the sense of M. Saito, in joint work with P. Brosnan, we give a positive answer to (a) and determine an obstruction to (b) which can be expressed as a pairing on intersection cohomology.

◆ Shuji Saito (Univ. Tokyo)

#### Theory of motives with modulus

An overview of recent development of theory of motives with modulus based on joint works with B. Kahn, T. Yamazaki, F. Binda, and K. Rülling. I explain the construction of a triangulated category of *motives with modulus* that enlarges Voevodsky's category of motives whose construction is based on  $\mathbb{A}^1$ -invariance. I explain how this new category encompasses non- $\mathbb{A}^1$ -invariant phenomena such as wild ramification and connections with irregular singularities. As an application, I explain canonical extensions of cohomology theories on smooth schemes to those on modulus pairs.

◆ Takeshi Saito (Univ. Tokyo)

#### Characteristic cycle of an $\ell$ -adic sheaf

For an  $\ell$ -adic sheaf on a smooth variety over a perfect field of characteristic  $p \geq 0$ , its characteristic cycle is defined as a cycle supported on the singular support defined by Beilinson as a closed conical subset of the cotangent bundle. We state the axioms characterizing the characteristic cycle and discuss some basic properties.

- ◆ Matthias Schütt (Leibniz)

#### Equicharacteristic models of nodal Enriques surfaces

We propose a new approach for explicit models of nodal Enriques surfaces in terms of genus one fibrations. This covers all characteristics directly at the same time, including classical nodal Enriques surfaces in characteristic 2 while preserving the computational advantages of genus one fibrations. Applications include maximal root types on Enriques surfaces (also for supersingular Enriques surfaces in characteristic 2) and integral models.

- ◆ Ichiro Shimada (Hiroshima)

#### Enriques involutions on singular K3 surfaces of small discriminants (joint work with Davide Cesare Veniani (Mainz))

We classify all Enriques involutions, up to conjugation in the automorphism group, on singular K3 surfaces with transcendental lattice of discriminant smaller than or equal to 36. In particular, we investigate the structure of the two most algebraic Enriques surfaces.

- ◆ Tetsuji Shioda (Rikkyo, Emeritus)

#### Algebraic cycles on the Fermat surfaces and the cyclotomic models

First we review some known results about algebraic cycles on the Fermat surfaces. Then we discuss some new results on the exceptional Hodge characters which can be induced from the Mordell-Weil lattice of certain elliptic Delsarte surfaces (named the “cyclotomic models” for short).

- ◆ Tomohide Terasoma (Univ. Tokyo)

#### Algebraic correspondences and Period integrals

In modern mathematics, period integrals are formulated using comparison theorem between singular cohomologies and de Rham cohomologies. Hodge theory is a strong tool to explain relations between period integrals. Hodge conjecture claims that relations of Hodge structure are always reincarnations of algebraic correspondences. In this talk, we discuss some evidences of this philosophy which appears in classical and semi-classical mathematics.

- ◆ Don Zagier (Max Planck)

#### Knots, number theory, and $q$ -series

Quantum invariants of 3-manifolds and knots have been much studied in recent years, in particular because of the famous “volume conjecture” relating the asymptotic growth of the Kashaev invariant of a hyperbolic knot to its volume. More recently, many surprising connections with number theory have also appeared, related to such different-seeming topics as algebraic  $K$ -theory, units in algebraic number fields, modular forms, and  $q$ -hypergeometric series. The talk will report on some of these developments, mostly from recent joint work with Frank Calegari and Stavros Garoufalidis.