Direct sampler

Direct sampler by contiguity relations of GKZ hypergeometric systems
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1 About this document

This document explains Risa/Asir functions for direct sampler by contiguity relations of GKZ hypergeometric systems
Loading the package:

```
import("gtt_ds.rr");
```

References cited in this document.

- [MT2021v2] S.Mano, N.Takayama, Algorithm for direct sampling from conditional distributions of toric models, version 2, https://arxiv.org/abs/2110.14992.

2 Simplicial complex to A matrix

2.1 `gtt_ds.getA`

`gtt_ds.getA(`*Levels*`,`*Facets*`)`

:: It returns a matrix A standings for a simplicial complex defined by *Facets* and the set of levels of each vertex *Levels*.

return Matrix A to define a GKZ system.

Levels Levels of each vertex of the simplicial complex *Facets*.

Facets The set of facets of a simplicial complex. A facet is expressed by a 0, 1 vector. 1 stands for the vertex of the position belongs to the facet and 0 stands for the vertex of the position does not belong to the facet.

- The following is an explanation of how facets are expressed by the varible *Facets*. Consider the simplicial complex of 4 vertices [123][134] (square with one diagonal line)

```
1      2
*------*
|\     |
| \    |
|  \   |
|   \  |
|    \ |
*------*
4      3
```

  The facet 12 is expressed as `[1,1,0,0]`, The facet 23 is expressed as `[0,1,1,0]`, the facet 13 is expressed as `[1,0,1,0]`. The set of all facets is `[[1,1,0,0],[0,1,1,0],[1,0,1,0],[1,0,0,1],[0,0,1,1]]`.
- The variable *Levels* is a set of levels of each vertex by pressed by numbers 1, 2, 3, ... For example, if we have four vertices and *Levels* is `[2,2,3,4]`, then the vertex 1 has the levels 1,2, the vertex 2 also has the levels 1,2, the vertex 3 has the levels 1,2,3, and the vertex 4 ahs the levels 1,2,3,4.
- The rows of the matrix A is indexed by all levels of all facets. See page 4 of [MT2021v2], $\mathcal{I}_F$. Let us show an example by our four vertices simplicial complex above. Let F be the facet [13] and *Levels* `[2,2,3,4]`. Then, all levels of the facet F is `[1,0,1,0],[1,0,2,0],[1,0,3,0],[2,0,1,0],[2,0,2,0],[2,0,3,0]`. The rows of the matrix A is indexed by such index that runs over all facets.
- For a graphical model, the set of facets are the set of cliques.
- The columns of the matrix A is indexed by all possible combinations of levels. See page 4 of [MT2021v2], $calI_V$. For example, if `Levels=[2,4]`, then the column index is `[1,1],[1,2],[1,3],[1,4],[2,1],[2,2],[2,3],[2,4]`.
- The matrix A is a 0-1 matrix. If the row index and the column index matches on the facet F underlying the row index, the the entry of A is 1.

Example: Consider the simplicial complex [1][2] (independent two points) with `Levels=[2,2]`. Then, the rows of the matrix A is indexed by `[1,0],[2,0],[0,1],[0,2]` and the columns of the matrix A is indexed by `[1,1],[1,2],[2,1],[2,2]`. The matrix A is $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$ For example, [1,0],[1,1] matches (1?,11, replace 0 by the wild card ? and regard it a regular expression), [1,0],[1,2] also matches (1?,12), but [1,0],[0,1] does not match (1?,01).

```
[2122] import("gtt_ds.rr");
[4432] gtt_ds.getA([2,2],[[1,0],[0,1]]);
[ 1 1 0 0 ]
[ 0 0 1 1 ]
[ 1 0 1 0 ]
[ 0 1 0 1 ]
```

Example: Consider the simplicial complex [1][2] (independent two points) with `Levels=[2,3]`. Then, the rows of the matrix A is indexed by `[1,0],[2,0],[0,1],[0,2],[0,3]` and the columns of the matrix A is indexed by `[1,1],[1,2],[1,3],[2,1],[2,2],[2,3]`.
The matrix A is $\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$

```
[2122] import("gtt_ds.rr");
[4432]  gtt_ds.getA([2,3],[[1,0],[0,1]]);
[ 1 1 1 0 0 0 ]
[ 0 0 0 1 1 1 ]
[ 1 0 0 1 0 0 ]
[ 0 1 0 0 1 0 ]
[ 0 0 1 0 0 1 ]
```

Example: Consider the simplicial complex [123][134] with `Levels=[2,2,3,4]`. For example, the row index [1,2,2,0] and the column index [1,1,1,1]. They do not match (122? and 1111). The row index [1,2,2,0] and the column index [1,2,2,4]. matches (122? and 1114).

The row index [1,1,1,0] and the column index [1,1,1,1]. matches (111? and 1111). The row index [1,1,1,0] and the column index [1,1,1,2]. matches (111? and 1112). ... The row index [1,1,1,0] and the column index [1,2,1,1]. does not matche (111? and 1211). ... The row index [1,1,1,0] and the column index [2,2,3,4]. does not matche (111? and 2234).

```
[2122] import("gtt_ds.rr");
[4432] gtt_ds.getA([2,2,3,4],[[1,1,1,0],[1,0,1,1]]);
[ 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
                          0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

```
                            0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
--- snip ---
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
                            0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1 ]
```

Example: See pages 20, 21 of [MT2021v2].

```
[2123] import("mt_mm.rr");
[4432] gtt_ds.getA([2,2,2,2],[[1,1,1,0],[1,1,0,1]]);
[ 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 ]
[ 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 ]
```

`Refer to` Section 3.1 [gtt_ds.direct_sampler], page 5,

3 Direct sampler of two way contingency table

3.1 gtt_ds.direct_sampler

`gtt_ds.direct_sampler(`*`MarginalSum`*`,`*`CellProb`*`)`

:: It returns a sample contingency table.

return A sample contingency table.

MarginalSum

Marginal sums of a two way contingency table.

CellProb A list of probabilities of each cell of the table.

- Repeating to call this function generates samples with integral entries with given marginal sums *MarginalSum*.
- The function utilizes the package `gtt_ekn3.rr` to obtain contiguity relations for the GKZ hypergeometric systems (Aomoto-Gel'fand system) for two way contingency tables.

Example: we consider 2×3 contingency tables with the cell probability $\begin{pmatrix} 1 & 1/2 & 1/3 \\ 1 & 1 & 1 \end{pmatrix}$, the row marginal sums $(5, 6)$, and the column marignal sums $(3, 3, 5)$.

```
[1883] import("gtt_ds.rr");
[4447] for (I=0; I<3; I++)
  printf("%a\n-----\n",gtt_ds.direct_sampler(MarginalSum,CellProb));
[ 2 0 3 ]
[ 1 3 2 ]
-----
[ 2 2 1 ]
[ 1 1 4 ]
-----
[ 2 0 3 ]
[ 1 3 2 ]
-----
```

Todo, install a function to obtain contiguity relations for a given matrix A. Call this function with optional variable `a` (A matrix), `MarginalSum` is β. Refer to `mt_mm.rr` and latest isom-project codes.

`Refer to` Section 2.1 [gtt_ds.getA], page 2,

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