1 Introduction

1.1 Organization of the Manual

This manual is organized as follows.

1. Introduction
   Organization of the Manual, notation and how to get Risa/Asir
2. Risa/Asir
   Summary of Asir, Installation
3. Types
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4. Asir user language
   Description of Asir user language
5. Debugger
   Description of the debugger of Asir user language
6. Built-in function
   Detailed description of various built-in functions
7. Distributed computation
   Description of functions for distributed computation
8. Groebner bases
   Description of functions and operations for Groebner basis computation
9. Algebraic numbers
   Description of functions and operations for algebraic numbers
10. Finite fields
    Description of functions and operations on finite fields
11. Appendix
    Syntax in detail, description of sample files, interfaces for input from keyboard, changes, references

1.2 Notation

In this manual, we shall use several notations, which is described here

- The name of a function is written in a typewriter type
  gcd(), gr()
- For the description of a function, its argument is written in a slanted type.
  int, poly
- A file name is written in a ‘typewriter type with single quotes’
  ‘dbxinit’, ‘asir_plot’
- An example is indented and written in a typewriter type.
  
  [0] 1;
  1
  [1] quit;
• References are made by a typewriter type bracketed by [].
[Boehm, Weiser]

• Arguments (actual parameters) of a function are optional when they are bracketed by []’s. The repeatable items (including non-existence of the item) are bracketed by [\*]’s.
setprec([u]),  diff(rat[\*], varn[\*])

• The prompt from the shell (csh) is denoted, as it is, by %. The prompt, however, is denoted by #, when you are assumed to be working as the root, for example, at the installation.
  
  % cat afo
  afo
  bfo
  % su
  Password:XXXX
  # cp asir /usr/local/bin
  # exit
  %

• The rational integer ring is denoted by $\mathbb{Z}$, the rational number field by $\mathbb{Q}$, the real number field by $\mathbb{R}$, and the complex number field by $\mathbb{C}$.

1.3 How to get Risa/Asir

Risa/Asir is available via http from
  
  http://www.math.kobe-u.ac.jp/Asir

1.4 Asir-contrib packages

Asir-contrib is a collection of packages developed by user communities. Documents can be found at http://www.math.kobe-u.ac.jp/OpenXM/Current/doc/index-doc.html This page also contains links to a manual on experimental functions of Risa/Asir and documents on the internal structure of Risa/Asir. As for bugs on Asir-contrib, please contact directly to the developer of the package. If you do not know who develops the package or there is no maintener on the package now, please send a mail to asir2005@math.kobe-u.ac.jp.
2 Risa/Asir

2.1 Risa and Asir

Risa is the name of whole libraries of a computer algebra system which is under development at FUJITSU LABORATORIES LIMITED. The structure of Risa is as follows.

- **The basic algebraic engine**
  This is the part which performs basic algebraic operations, such as arithmetic operations, to algebraic objects, e.g., numbers and polynomials, which are already converted into internal forms. It exists, like ‘libc.a’ of UNIX, as a library of ordinary UNIX system. The algebraic engine is written mainly in C language and partly in assembler. It serves as the basic operation part of Asir, a standard language interface of Risa.

- **Memory Manager**
  Risa employs, as its memory management component (the memory manager), a free software distributed by Boehm (gc-6.1alpha5). It is proposed by [Boehm, Weiser], and developed by Boehm and his colleagues. The memory manager has a memory allocator which automatically reclaims garbages, i.e., allocated but unused memories, and refreshes them for further use. The algebraic engine gets all its necessary memories through the memory manager.

- **Asir**
  Asir is a standard language interface of Risa’s algebraic engine. It is one of the possible language interfaces, because one can develop one’s own language interface easily on Risa system. Asir is an example of such language interfaces. Asir has very similar syntax and semantics as C language. Furthermore, it has a debugger that provide a subset of commands of dbx, a widely used debugger of C language.

2.2 Features of Asir

As mentioned in the previous section, Asir is a standard language interface for Risa’s algebraic engine. Usually, it is provided as an executable file named asir. Main features supported for the current version of Asir is as follows.

- A C-like programming language
- Arithmetic operations (addition, subtraction, multiplication and division) on numbers, polynomials and rational expressions
- Operations on vectors and matrices
- List processing operations at the minimum
- Several Built-in functions (factorization, GCD computation, Grobner basis computation etc.)
- Useful user defined functions (e.g., factorization over algebraic number fields)
- A dbx-like debugger
- Plotting of implicit functions
- Numerical evaluation of mathematical expressions including elementary transcendental functions at arbitrary precision. This feature is in force only if PARI system (see Section 6.1.14 [pari], page 41).
2.3 Installation

Any questions and any comments on this manual are welcome by e-mails to the following address.

noro@math.kobe-u.ac.jp

2.3.1 UNIX binary version

A file ‘asir.tgz’ suitable for the target machine/architecture is required. After getting it, you have to unpack it by gzip. First of all, determine a directory where binaries and library files are installed. We call the directory the library directory. The following installs the files in ‘/usr/local/lib/asir’.

```
# gzip -dc asir.tgz | ( cd /usr/local/lib; tar xf - )
```

In this case you don’t have to set any environment variable.

You can install them elsewhere.

```
% gzip -dc asir.tgz | ( cd $HOME; tar xf - )
```

In this case you have to set the name of the library directory to the environment variable ASIR_LIBDIR.

```
% setenv ASIR_LIBDIR $HOME/asir
```

Asir itself is in the library directory. It will be convenient to create a symbolic link to it from ‘/usr/local/bin’ or the user’s search path.

```
# ln -s /usr/local/lib/asir/asir /usr/local/bin/asir
```

Then you can start ‘asir’.

```
% /usr/local/bin/asir
```

This is Risa/Asir, Version 20000821.
Copyright (C) FUJITSU LABORATORIES LIMITED.
1994-2000. All rights reserved.

2.3.2 UNIX source code version

First of all you have to determine the install directory. In the install directory, the following subdirectories are put:

- bin
  executables of PARI and Asir
- lib
  library files of PARI and Asir
- include
  header files of PARI

These subdirectories are created automatically if they does not exist. If you can be a root, it is recommended to set the install directory to ‘/usr/local’. In the following the directory is denoted by TARGETDIR.

Then, install PARI library. After getting ‘pari.tgz’, unpack and install it as follows:
% gzip -dc pari.tgz | tar xvff -
% cd pari
% ./Configure --prefix=TARGETDIR
% make all
% su
  # make install
  # make install-lib-sta

While executing 'make install', the procedure may stop due to some error. Then try the following:

% cd Oxxx
% make lib-sta
% su
  # make install-lib-sta
  # make install-include
  # exit
%

In the above example, xxx denotes the name of the target operating system. Although GP is not built, the library necessary for building asir2000 will be generated.

After getting 'asir2000.tgz', unpack it and install necessary files as follows.

% gzip -dc asir.tgz | tar xvf -
% cd asir2000
% ./configure --prefix=TARGETDIR --with-pari --enable-plot
% make
% su
  # make install
  # make install-lib
  # make install-doc
  # exit

2.3.3 Windows version

The installers are ‘asirwin32.msi’ (32bit version) and ‘asirwin64.msi’ (64bit version). These installers set the installed folder to the environmental variable ‘ASIR_ROOTDIR’. (If you install Risa/Asir by the zip version, you need to set the environmental variable ‘ASIR_ROOTDIR’ by hand. You also need to unlock the security block before unzipping. The zip version is for developpers.) Double click the “asirgui” on the desktop, then the asir starts. To use a unified environment with text editors, follow the instruction in ‘%ASIR_ROOTDIR%\share\editor’. If the folder name to which you install asir or your home folder name contain non-ascii characters, some functions of asir may not work properly. For example, in the Japense locale, names which may cause troubles can be checked by “damemoji checker”.

2.4 Command line options

Command-line options for the command ‘asir’ are as follows.
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-heap number

In Risa/Asir, 4KB is used as an unit, called block, for memory allocation. By default, 16 blocks (64KB) are allocated initially. This value can be changed by giving an option -heap a number parameter in unit block. Size of the heap area is obtained by a Built-in function heap(), the result of which is a number in Bytes.

-adj number

Heap area will be stretched by the memory manager, if the size of reclaimed memories is less than 1/number of currently allocated heap area. The default value for number is 3. If you do not prefer to stretch heap area by some reason, perhaps by restriction of available memories, but if prefer to resort to reclaiming garbages as far as possible, then a large value should be chosen for number, e.g., 8.

-norc

When this option is specified, Asir does not read the initial file `$HOME/.asirrc`.

-quiet

Instead of the standard input, file is used as the input. Upon an error, the execution immediately terminates.

-paristack number

This option specifies the private memory size for PARI (see Section 6.1.14 [pari], page 41). The unit is Bytes. By default, it is set to 1 MB.

-maxheap number

This option sets an upper limit of the heap size. The unit is Bytes. Note that the size is already limited by the value of datasure displayed by the command limit on UNIX.

2.5 Environment variable

There exist several environment variables concerning with an execution of Asir. On UNIX, an environment variable is set from shells, or in rc files of shells. On Windows, it can be set from [Editing environmetal variables].

- ASIR_LIBDIR
  This environmental variable is obsolete.

- ASIR_CONTRIB_DIR
  This environmental variable is obsolete.

- ASIRLOADPATH
  This environment specifies directories which contains files to be loaded by Asir command load(). Directories are separated by a ':' on UNIX, a ';:' on Windows respectively. The search order is from the left to the right. After searching out all directories in ASIRLOADPATH, or in case of no specification at all, the library directory will be searched. The library directories of the unix version are `$OpenXM_HOME/lib/asir-contrib`, `$OpenXM_HOME/lib/asir`, /usr/local/lib/asir-contrib, /usr/local/lib/asir. When `$OpenXM_HOME` is not set, the library directories of the Windows version are `%ASIR_ROOTDIR%\lib\asir-contrib`,
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%ASIR_ROOTDIR%\lib\asir. When ‘OpenXM_HOME’ is set, the library directories are same with the unix version. In the Windows version, the private folder ‘%APPDATA%\OpenXM\lib\asir-contrib’ is also set to the library folders. In the unix version, there is no default private library folder. In the OpenXM/unix version, ‘$OpenXM_tmp/OpenXM/lib/asir-contrib’ is added to the ‘ASIRLOADPATH’ by a script in ‘OpenXM/rc/’. See also asir_contrib_update. See which, ctrl("loadpath"), asir2000/parse/load.c.

- HOME
  If Asir is invoked without -norc, ‘$HOME/.asirrc’, if exists, is executed. If HOME is not set, nothing is done on UNIX. On Windows, ‘.asirrc’ in Asir root directory is executed if it exists.

2.6 Starting and Terminating an Asir session

Run Asir, then the copyright notice and the first prompt will appear on your screen, and a new Asir session will be started.

[0]

When initialization file ‘$HOME/.asirrc’ exists, Asir interpreter executes it at first taking it as a program file written in Asir.

The prompt indicates the sequential number of your input commands to Asir. The session will terminate when you input end; or quit; to Asir. Input commands are evaluated statement by statement. A statement normally ends with its terminator ‘;’ or ‘$’. (There are some exceptions. See, syntax of Asir.) The result will be displayed when the command, i.e. statement, is terminated by a ‘;’, and will not when terminated by a ‘$’.

% asir
[0] A;
0
[1] A=(x+y)^5;
     x^5+5*y*x^4+10*y^2*x^3+10*y^3*x^2+5*y^4*x+y^5
[2] A;
     x^5+5*y*x^4+10*y^2*x^3+10*y^3*x^2+5*y^4*x+y^5
[3] a=(x+y)^5;
evalpv : invalid assignment
return to toplevel
[3] a;
a
[4] fctr(A);
[[1,1],[x+y,5]]
[5] quit;
%

In the above example, names A, a, x and y are used to identify mathematical and programming objects. There, the name A denotes a program variable (some times called simply as a program variable.) while the other names, a, x and y, denote mathematical objects, that is, indeterminates. In general, program variables have names which begin with capital letters, while names of indeterminates begin with small letters. As you can see in the example, program variables are used to hold and keep objects, such as numbers and expressions,
as their values, just like variables in C programming language. Whereas, indeterminates cannot have values so that assignment to indeterminates are illegal. If one wants to get a result by substituting a value for an indeterminate in an expression, it is achieved by the function \texttt{subst} as the value of the function.

### 2.7 Interruption

To interrupt the \texttt{Asir} execution, input an interrupt character from the keyboard. A \texttt{C-c} is usually used for it. (Notice: \texttt{C-x} on Windows and DOS.)

\begin{verbatim}
@ (x+y)^1000;
C-cinterrupt ?(q/t/c/d/u/w/?)
\end{verbatim}

Here, the meaning of options are as follows.

- \texttt{q} Terminates \texttt{Asir} session. (Confirmation requested.)
- \texttt{t} Returns to toplevel. (Confirmation requested.)
- \texttt{c} Resumes to continue the execution.
- \texttt{d} Enters debugging mode at the next statement of the \texttt{Asir} program, if \texttt{Asir} has been executing a program loaded from a file. Note that it will sometimes take a long time before entering debugging mode when \texttt{Asir} is executing basic functions in the algebraic engine, (e.g., arithmetic operation, factorization etc.) Detailed description about the debugger will be given in Chapter 5 [Debugger], page 31.
- \texttt{u} After executing a function registered by \texttt{register_handler()} (see Section 7.5.6 [ox-reset ox-intr register_handler], page 110), returns to toplevel. A confirmation is prompted.
- \texttt{w} Displays the calling sequence up to the interruption.
- \texttt{?} Show a brief description of options.

### 2.8 Error handling

When arguments with illegal types are given to a built-in function, an error will be detected and the execution will be quit. In many cases, when an error is detected in a built-in function, \texttt{Asir} automatically enters debugging mode before coming back to toplevel. At that time, one can examine the state of the program, for example, inspect argument values just before the error occurred. Messages reported there are various depending on cases. They are reported after the internal function name. The internal function name sometimes differs from the built-in function name that is specified by the user program.

In the execution of internal functions, errors may happen by various reasons. The UNIX version of \texttt{Asir} will report those errors as one of the following internal error's, and enters debugging mode just like normal errors.

- \texttt{SEGV}
- \texttt{BUS ERROR}

Some of the built-in functions transmit their arguments to internal operation routines without strict type-checking. In such cases, one of these two errors will
be reported when an access violation caused by an illegal pointer or a NULL pointer is detected.

**Broken Pipe**

In the process communication, this error will be reported if a process attempts to read from or to write onto the partner process when the stream to the partner process does not already exist, (e.g., terminated process.)

For UNIX version, even in such a case, the process itself does not terminate because such an error can be caught by `signal()` and recovered. To remove this weak point, complete type checking of all arguments are indispensable at the entry of a built-in function, which requires an enormous amount of re-making efforts.

### 2.9 Referencing results and special numbers

An `@` used for an escape character; rules currently in force are as follows.

- `@n` The evaluated result of `n`-th input command
- `@@` The evaluated result of the last command
- `@i` The unit of imaginary number, square root of -1.
- `@pi` The number pi, the ratio of a circumference of the circle and its diameter.
- `@e` Napier’s number, the base of natural logarithm.
- `@` A generator of GF(2^`m`), a finite field of characteristic 2, over GF(2). It is a root of an irreducible univariate polynomial over GF(2) which is set as the defining polynomial of GF(2^`m`).

`@>`, `@<=`, `@>=`, `@==`, `@&`, `@||`

First order logical operators. They are used in quantifier elimination.

```makefile
[0] fctr(x^10-1);
[[1,1],[x-1,1],[x+1,1],[x^4+x^3+x^2+x+1,1],[x^4-x^3+x^2-x+1,1]]
[1] @@[3];
[x^4+x^3+x^2+x+1,1]
[2] eval(sin(@pi/2));
1.000000000000000000000000000000000000000000000000000000000
[3] eval(log(@e),20);
0.99999999999999999999999999998
[4] @0[4][0];
x^4-x^3+x^2-x+1
[5] (1+@i)^5;
(-4-4*@i)
[6] eval(exp(@pi*@i));
-1.000000000000000000000000000000000000000000000000000000000
[7] (@+1)^9;
(10^8+@^9+@+1)
```

As you can see in the above example, results of toplevel computation can be referred to by `@` convention. This is convenient for users, while it sometimes imposes a heavy burden to
the garbage collector. It may happen that GC time will rapidly increase after computing a very large expression at the toplevel. In such cases \texttt{delete\_history()} (see Section 6.14.15 \texttt{[delete\_history]}, page 100) takes effect.
3 Data types

3.1 Types in Asir

In Asir, various objects described according to the syntax of Asir are translated to intermediate forms and by Asir interpreter further translated into internal forms with the help of basic algebraic engine. Such an object in an internal form has one of the following types listed below. In the list, the number coincides with the value returned by the built-in function type(). Each example shows possible forms of inputs for Asir’s prompt.

0 0
As a matter of fact, no object exists that has 0 as its identification number. The number 0 is implemented as a null (0) pointer of C language. For convenience’s sake, a 0 is returned for the input type(0).

1 number
1 2/3 14.5 3+2*i
Numbers have sub-types. See Section 3.2 [Types of numbers], page 14.

2 polynomial (but not a number)
x afo (2.3*x+y)^10
Every polynomial is maintained internally in its full expanded form, represented as a nested univariate polynomial, according to the current variable ordering, arranged by the descending order of exponents. (See Section 8.1 [Distributed polynomial], page 121.) In the representation, the indeterminate (or variable), appearing in the polynomial, with maximum ordering is called the main variable. Moreover, we call the coefficient of the maximum degree term of the polynomial with respect to the main variable the leading coefficient.

3 rational expression (not a polynomial)
(x+1)/(y^2-y-x) x/x
Note that in Risa/Asir a rational expression is not simplified by reducing the common divisors unless red() is called explicitly, even if it is possible. This is because the GCD computation of polynomials is a considerably heavy operation. You have to be careful enough in operating rational expressions.

4 list
[] [1,2,[3,4],[x,y]]
Lists are all read-only object. A null list is specified by []. There are operations for lists: car(), cdr(), cons() etc. And further more, element referencing by indexing is available. Indexing is done by putting [index]’s after a program variable as many as are required. For example,

[0] L = [[1,2,3],[4,[5,6]],7]$ [1] L[1][1]; [5,6]
Notice that for lists, matrices and vectors, the index begins with number 0. Also notice that referencing list elements is done by following pointers from
the first element. Therefore, it sometimes takes much more time to perform referencing operations on a large list than on a vectors or a matrices with the same size.

5 vector

\texttt{newvect(3) newvect(2,[a,1])}

Vector objects are created only by explicit execution of \texttt{newvect()} command. The first example above creates a null vector object with 3 elements. The other example creates a vector object with 2 elements which is initialized such that its 0-th element is \texttt{a} and 1st element is \texttt{1}. The second argument for \texttt{newvect} is used to initialize elements of the newly created vector. A list with size smaller or equal to the first argument will be accepted. Elements of the initializing list is used from the left to the right. If the list is too short to specify all the vector elements, the unspecified elements are filled with as many 0’s as are required. Any vector element is designated by indexing, e.g., \texttt{[index]}. \texttt{Asir} allows any type, including vector, matrix and list, for each respective element of a vector. As a matter of course, arrays with arbitrary dimensions can be represented by vectors, because each element of a vector can be a vector or matrix itself. An element designator of a vector can be a left value of assignment statement. This implies that an element designator is treated just like a simple program variable. Note that an assignment to the element designator of a vector has effect on the whole value of that vector.

\texttt{[0] A3 = newvect(3);}
\texttt{[ 0 0 0 ]}
\texttt{[1] for (I=0;I<3;I++)A3[I] = newvect(3);}
\texttt{[2] for (I=0;I<3;I++)for(J=0;J<3;J++)A3[I][J]=newvect(3);}
\texttt{[3] A3;}
\texttt{[ [ 0 0 0 ] [ 0 0 0 ] [ 0 0 0 ] ]}
\texttt{[ [ 0 0 0 ] [ 0 0 0 ] [ 0 0 0 ] ]}
\texttt{[ [ 0 0 0 ] [ 0 0 0 ] [ 0 0 0 ] ]}
\texttt{[4] A3[0];}
\texttt{[ [ 0 0 0 ] [ 0 0 0 ] [ 0 0 0 ] ]}
\texttt{[5] A3[0][0];}
\texttt{[ 0 0 0 ]}

6 matrix

\texttt{newmat(2,2) newmat(2,3,[[x,y],[z]])}

Like vector objects, matrix objects are also created only by explicit execution of \texttt{newmat()} command. Initialization of the matrix elements are done in a similar manner with that of the vector elements except that the elements are specified by a list of lists. Each element, again a list, is used to initialize each row; if the list is too short to specify all the row elements, unspecified elements are filled with as many 0’s as are required. Like vectors, any matrix element is designated by indexing, e.g., \texttt{[index][index]}. \texttt{Asir} also allows any type, including vector, matrix and list, for each respective element of a matrix. An element designator of a matrix can also be a left value of assignment statement. This implies that an element designator is treated just like a simple program variable. Note that an assignment to the element designator of a matrix has effect on the whole
value of that matrix. Note also that every row, (not column,) of a matrix can be extracted and referred to as a vector.

```
[0] M=newmat(2,3);
[ 0 0 0 ]
[ 0 0 0 ]
[1] M[1];
[ 0 0 0 ]
[2] type(@@);
```

7 string

```
"" "afo"
```

Strings are used mainly for naming files. It is also used for giving comments of the results. Operator symbol + denote the concatenation operation of two strings.

```
[0] "afo"+"take";
afotake
```

8 structure

```
newstruct(afo)
```

The type structure is a simplified version of that in C language. It is defined as a fixed length array and each entry of the array is accessed by its name. A name is associated with each structure.

9 distributed polynomial

```
2*<<0,1,2,3>>-3*<<1,2,3,4>>
```

This is the short for ‘Distributed representation of polynomials.’ This type is specially devised for computation of Groebner bases. Though for ordinary users this type may never be needed, it is provided as a distinguished type that user can operate by Asir. This is because the Groebner basis package provided with Risa/Asir is written in the Asir user language. For details See Chapter 8 [Groebner basis computation], page 121.

10 32bit unsigned integer

11 error object

These are special objects used for OpenXM.

12 matrix over GF(2)

This is used for basis conversion in finite fields of characteristic 2.

13 MATHCAP object

This object is used to express available functionalities for Open XM.

14 first order formula

This expresses a first order formula used in quantifier elimination.
15 matrix over GF($p$)

A matrix over a small finite field.

16 byte array

An array of unsigned bytes.

-1 VOID object

The object with the object identifier -1 indicates that a return value of a function is void.

3.2 Types of numbers

0 rational number
Rational numbers are implemented by arbitrary precision integers (bignum). A rational number is always expressed by a fraction of lowest terms.

1 double precision floating point number (double float)
The numbers of this type are numbers provided by the computer hardware. By default, when Asir is started, floating point numbers in a ordinary form are transformed into numbers of this type. However, they will be transformed into bigfloat numbers when the switch bigfloat is turned on (enabled) by ctrl() command.

```
[0] 1.2;
1.2
[1] 1.2e-1000;
0
[2] ctrl("bigfloat",1);
1
[3] 1.2e-1000;
1.20000000000000000513 E-1000
```

A rational number shall be converted automatically into a double float number before the operation with another double float number and the result shall be computed as a double float number.

2 algebraic number
See Chapter 9 [Algebraic numbers], page 156.

3 bigfloat
The bigfloat numbers of Asir is realized by MPFR library. A bigfloat number of MPFR has an arbitrary precision mantissa part. However, its exponent part admits only a 64bit integer. Floating point operations will be performed all in bigfloat after activating the bigfloat switch by ctrl() command. The default precision is 53 bits (about 15 digits), which can be specified by setbprec() and setprec() command.

```
[0] ctrl("bigfloat",1);
1
```
Function \texttt{eval()} evaluates numerically its argument as far as possible. Notice that the integer given for the argument of \texttt{setbprec()} guarantees the accuracy of the result according to the current rounding mode. The argument of \texttt{setbprec()} is converted to the corresponding bit length and set. (See Section 6.1.13 \texttt{eval deval}, page 40.)

4 complex number

A \texttt{complex} number of \texttt{Risa/Asir} is a number with the form \(a+b\cdot\Omega\text{i}\), where \(\Omega\text{i}\) is the unit of imaginary number, and \(a\) and \(b\) are either a \texttt{rational} number, \texttt{double float} number or \texttt{bigfloat} number, respectively. The real part and the imaginary part of a \texttt{complex} number can be taken out by \texttt{real()} and \texttt{imag()} respectively.

5 element of a small finite prime field

Here a small finite field means that its characteristic is less than \(2^{27}\). At present small finite fields are used mainly for groebner basis computation, and elements in such finite fields can be extracted by taking coefficients of distributed polynomials whose coefficients are in finite fields. Such an element itself does not have any information about the field to which the element belongs, and field operations are executed by using a prime \(p\) which is set by \texttt{setmod()}.

6 element of large finite prime field

This type expresses an element of a finite prime field whose characteristic is an arbitrary prime. An object of this type is obtained by applying \texttt{simp_ff} to an integer.

7 element of a finite field of characteristic 2

This type expresses an element of a finite field of characteristic 2. Let \(F\) be a finite field of characteristic 2. If \([F:GF(2)]\) is equal to \(n\), then \(F\) is expressed as \(F=GF(2)[t]/(f(t))\), where \(f(t)\) is an irreducible polynomial over GF(2) of degree \(n\). As an element \(g\) of GF(2)[t] can be expressed by a bit string, An element \(g\) \texttt{mod} \(f\) in \(F\) can be expressed by two bit strings representing \(g\) and \(f\) respectively. Several methods to input an element of \(F\) are provided.

- \(\@\)
  \(\@\) represents \(t \text{ mod } f\) in \(F=GF(2)[t]/(f(t))\). By using \(\@\) one can input an element of \(F\). For example \(\@^*10@+1\) represents an element of \(F\).
- \texttt{ptogf2n}
  \texttt{ptogf2n} converts a univariate polynomial into an element of \(F\).
- \texttt{ntogf2n}
  As a bit string, a non-negative integer can be regarded as an element of
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F. Note that one can input a non-negative integer in decimal, hexadecimal (0x prefix) and binary (0b prefix) formats.

- micellaneous
  - simp_ff is available if one wants to convert the whole coefficients of a polynomial.

8 **element of a finite field of characteristic \( p^n \)**

A finite field of order \( p^n \), where \( p \) is an arbitrary prime and \( n \) is a positive integer, is set by `setmod_ff` by specifying its characteristic \( p \) and an irreducible polynomial of degree \( n \) over \( \text{GF}(p) \). An element of this field is represented by a polynomial over \( \text{GF}(p) \) modulo \( m(x) \).

9 **element of a finite field of characteristic \( p^n \) (small order)**

A finite field of order \( p^n \), where \( p^n \) must be less than \( 2^{29} \) and \( n \) must be equal to 1 if \( p \) is greater or equal to \( 2^{14} \), is set by `setmod_ff` by specifying its characteristic \( p \) the extension degree \( n \). If \( p \) is less than \( 2^{14} \), each non-zero element of this field is a power of a fixed element, which is a generator of the multiplicative group of the field, and it is represented by its exponent. Otherwise, each element is represented by the reduce modulo \( p \). This specification is useful for treating both cases in a single program.

10 **element of a finite field which is an algebraic extension of a small finite field of characteristic \( p^n \)**

An extension field \( K \) of the small finite field \( F \) of order \( p^n \) is set by `setmod_ff` by specifying its characteristic \( p \) the extension degree \( n \) and \( m=[K:F] \). An irreducible polynomial of degree \( m \) over \( K \) is automatically generated and used as the defining polynomial of the generator of the extension \( K/F \). The generator is denoted by \( @s \).

11 **algebraic number represented by a distributed polynomial**

See Chapter 9 [Algebraic numbers], page 156.

Finite fields other than small finite prime fields are set by `setmod_ff`. Elements of finite fields do not have informations about the modulus. Upon an arithmetic operation, if one of the operands is a rational number, it is automatically converted into an element of the finite field currently set and the operation is done in the finite field.

### 3.3 Types of indeterminates

An algebraic object is recognized as an indeterminate when it can be a (so-called) variable in polynomials. An ordinary indeterminate is usually denoted by a string that start with a small alphabetical letter followed by an arbitrary number of alphabetical letters, digits or `_`. In addition to such ordinary indeterminates, there are other kinds of indeterminates in a wider sense in Asir. Such indeterminates in the wider sense have type **polynomial**, and further are classified into sub-types of the type **indeterminate**.

0 **ordinary indeterminate**

An object of this sub-type is denoted by a string that start with a small alphabetical letter followed by an arbitrary number of alphabetical letters, digits
or ‘\_’. This kind of indeterminates are most commonly used for variables of polynomials.

\[
\begin{align*}
[0] & \text{vtype(a),vtype(aA_12)}; \\
[0,0] \\
\end{align*}
\]

\textbf{undetermined coefficient}

The function \texttt{uc()} creates an indeterminate which is denoted by a string that begins with ‘\_’. Such an indeterminate cannot be directly input by its name. Other properties are the same as those of \texttt{ordinary indeterminate}. Therefore, it has a property that it cannot cause collision with the name of ordinary indeterminates input by the user. And this property is conveniently used to create undetermined coefficients dynamically by programs.

\[
\begin{align*}
[1] & \text{U=uc();} \\
[2] & \text{vtype(U);} \\
\end{align*}
\]

\textbf{function form}

A function call to a built-in function or to an user defined function is usually evaluated by \texttt{Asir} and retained in a proper internal form. Some expressions, however, will remain in the same form after evaluation. For example, \texttt{sin(x)} and \texttt{cos(x+1)} will remain as if they were not evaluated. These (unevaluated) forms are called ‘function forms’ and are treated as if they are indeterminates in a wider sense. Also, special forms such as \texttt{\@pi} the ratio of circumference and diameter, and \texttt{\@e} Napier’s number, will be treated as ‘function forms.’

\[
\begin{align*}
[3] & \text{V=sin(x);} \\
[4] & \text{vtype(V);} \\
[5] & \text{vars(V^2+V+1);} \\
\end{align*}
\]

\textbf{functor}

A function call (or a function form) has a form \texttt{fname(args)}. Here, \texttt{fname} alone is called a \texttt{functor}. There are several kinds of \texttt{functors}: built-in functor, user defined functor and functor for the elementary functions. A functor alone is treated as an indeterminate in a wider sense.

\[
\begin{align*}
[6] & \text{vtype(sin);} \\
\end{align*}
\]
4 User language \textit{Asir}

\textit{Asir} provides many built-in functions, which perform algebraic computations, e.g., factorization and GCD computation, file I/O, extract a part of an algebraic expression, etc. In practice, you will often encounter a specific problem for which \textit{Asir} does not provide a direct solution. For such cases, you have to write a program in a certain user language. The user language for \textit{Asir} is also called \textit{Asir}. In the following, we describe the Syntax and then show how to write a user program by several examples.

4.1 Syntax — Difference from C language

The syntax of \textit{Asir} is based on C language. Main differences are as follows. In this section, a variable does not mean an indeterminate, but a program variable which is written by a string which begins with a capital alphabetical letter in \textit{Asir}.

- No types for variables.
  As is already mentioned, any object in \textit{Asir} has their respective types. A program variable, however, is type-less, that is, any typed object can be assigned to it.

  \begin{verbatim}
  [0] A = 1;
  1
  [1] type(A);
  1
  [2] A = [1,2,3];
  [1,2,3]
  [3] type(A);
  4
  \end{verbatim}

- Variables, together with formal parameters, in a function (procedure) are all local to the function by default. Variables can be global at the top level, if they are declared with the key word \texttt{extern}. Thus, the scope rule of \textit{Asir} is very simple. There are only two types of variables: global variables and local variables. A name that is input to the \textit{Asir}’s prompt at the top level is denotes a global variable commonly accessed at the top level. In a function (procedure) the following rules are applied.

1. If a variable is declared as global by an \texttt{extern} statement in a function, the variable used in that function denotes a global variable at the top level. Furthermore, if a variable in a function is preceded by an \texttt{extern} declaration outside the function but in a file where the function is defined, all the appearance of that variable in the same file denote commonly a global variable at the top level.

2. A variable in a function is local to that function, if it is not declared as global by an \texttt{extern} declaration.

\begin{verbatim}
% cat afo
def afo() { return A;}
extern A$
def bfo() { return A;}
end$
% asir
[0] load("afo")$
\end{verbatim}
Program variables and algebraic indeterminates are distinguished in Asir. The names of program variables must begin with a capital letter; while the names of indeterminates and functions must begin with a small letter. This is a unique point that differs from almost all other existing computer algebra systems. The distinction between program variables and indeterminates is adopted to avoid the possible and usual confusion that may arise in a situation where a name is used as an indeterminate but, as it was, the name has been already assigned some value. To use different type of letters, capital and small, was a matter of syntactical convention like Prolog, but it is convenient to distinguish variables and indeterminates in a program.

No switch statements, and goto statements. Lack of goto statement makes it rather bothering to exit from within multiple loops.

Comma expressions are allowed only in $A$, $B$ and $C$ of the constructs for $(A;B;C)$ or while($A$).

This limitation came from adopting lists as legal data objects for Asir.

The above are limitations; extensions are listed as follows.

- Arithmetic for rational expressions can be done in the same manner as is done for numbers in C language.
- Lists are available for data objects.
  - Lists are conveniently used to represent a certain collection of objects. Use of lists enables to write programs more easily, shorter and more comprehensible than use of structure like C programs.
- Options can be specified in calling user defined functions.
  - See Section 4.2.12 [option], page 27.

### 4.2 Writing user defined functions

#### 4.2.1 User defined functions

To define functions by an user himself, ‘def’ statement must be used. Syntactical errors are detected in the parsing phase of Asir, and notified with an indication of where Asir found the error. If a function with the same name is already defined (regardless to its arity,) the new definition will override the old one, and the user will be told by a message,

```plaintext
afo() redefined.
```

on the screen when a flag verbose is set to a non-zero value by ctrl(). Recursive definition, and of course, recursive use of functions are available. A call for an yet undefined function in a function definition is not detected as an error. An error will be detected at execution of the call of that yet undefined function.
Chapter 4: User language \texttt{Asir}

/* \texttt{X!} */
\begin{verbatim}
def f(X) {
    if ( !X )
        return 1;
    else
        return X * f(X-1);
}
\end{verbatim}

/* \texttt{iCj ( 0 \leq i \leq N; 0 \leq j \leq i ) */
\begin{verbatim}
def c(N) {
    A = newvect(N+1); A[0] = B = newvect(1); B[0] = 1;
    for ( K = 1; K <= N; K++ ) {
        A[K] = B = newvect(K+1); B[0] = B[K] = 1;
        for ( P = A[K-1], J = 1; J < K; J++ )
            B[J] = P[J-1]+P[J];
    }
    return A;
}
\end{verbatim}

/* \texttt{A+B */
\begin{verbatim}
def add(A,B)
"add two numbers."
{
    return A+B;
}
\end{verbatim}

In the second example, \texttt{c(N)} returns a vector, say \texttt{A}, of length \texttt{N+1}. \texttt{A[I]} is a vector of length \texttt{I+1}, and each element is again a vector which contains \texttt{iCj} as its elements.

References
Section 6.14.4 [help], page 94.

In the following, the manner of writing \texttt{Asir} programs is exhibited for those who have no experience in writing \texttt{C} programs.

4.2.2 variables and indeterminates

variables (program variables)
A program variable is a string that begins with a capital alphabetical letter followed by any numbers of alphabetical letters, digits and ‘\_’.

A program variable is thought of a box (a carrier) which can contain \texttt{Asir} objects of various types. The content is called the ‘value’ of that variable. When an expression in a program is to be evaluated, the variable appearing in
the expression is first replaced by its value and then the expression is evaluated to some value and stored in the memory. Thus, no program variable appears in objects in the internal form. All the program variables are initialized to the value 0.

```
[0] X^2+X+1;
1
[1] X=2;
2
[2] X^2+X+1;
7
```

### Indeterminates
An indeterminate is a string that begins with a small alphabetical letter followed by any numbers of alphabetical letters, digits and `_`.

An indeterminate is a transcendental element, so-called variable, which is used to construct polynomial rings. An indeterminate cannot have any value. No assignment is allowed to it.

```
[3] X=x;
x
[4] X^2+X+1;
x^2+x+1
[5] A='Dx'*(x-1)+x*y-y;
(Dx)*x-y-Dx
[6] function foo(x,y);
[7] B=foo(x,y)*x^2-1;
foo(x,y)*x^2-1
```

### 4.2.3 Parameters and Arguments

```python
def sum(N) {
    for ( I = 1, S = 0; I <= N; I++ )
        S += I;
    return S;
}
```

This is an example definition of a function that sums up integers from 1 to N. The N in `sum(N)` is called the (formal) parameter of `sum(N)`. The example shows a function of the single argument. In general, any number of parameters can be specified by separating by commas (`,`). A (formal) parameter accepts a value given as an argument (or an actual parameter) at a function call of the function. Since the value of the argument is given to the formal parameter, any modification to the parameter does not usually affect the argument (or actual parameter). However, there are a few exceptions: vector arguments and matrix arguments.

Let A be a program variable and assigned to a vector value `[a, b]`. If A is given as an actual parameter to a formal parameter, say V, of a function, then an assignment in the function to the vector element designator V[1], say V[1]=c;, causes modification of the actual parameter A resulting A to have an altered value `[a c]`. Thus, if a vector is given to a formal parameter of a function, then its element (and subsequently the vector itself) in the calling side is modified through modification of the formal parameter by a vector
element designator in the called function. The same applies to a matrix argument. Note that, even in such case where a vector (or a matrix) is given to a formal parameter, the assignment to the whole parameter itself has only a local effect within the function.

```python
def clear_vector(M) {
    /* M is expected to be a vector */
    L = size(M)[0];
    for ( I = 0; I < L; I++ )
        M[I] = 0;
}
```

This function will clear off the vector given as its argument to the formal parameter \( M \) and return a 0 vector.

Passing a vector as an argument to a function enables returning multiple results by packing each result in a vector element. Another alternative to return multiple results is to use a list. Which to use depends on cases.

### 4.2.4 Comments

The text enclosed by `/*` and `*/` (containing `/*` and `*/`) is treated as a comment and has no effect to the program execution as in C programs.

```python
/*
 * This is a comment.
 */
```

A comment can span to several lines, but it cannot be nested. Only the first `/*` is effective no matter how many `/*`'s in the subsequent text exist, and the comment terminates at the first `*/`.

In order to comment out a program part that may contain comments in it, use the pair, `#if 0` and `#endif`. (See Section 4.2.11 [preprocessor], page 26.)

```python
#if 0
    def bfo(X) {
        /* empty */
    }
#endif
```

### 4.2.5 Statements

An user function of Asir is defined in the following form.

```python
def name(parameter, parameter, ..., parameter) {
    statement
    statement
    ...
    statement
}
```

As you can see, the statement is a fundamental element of the function. Therefore, in order to write a program, you have to learn what the statement is. The simplest statement is the simple statement. One example is an expression with a terminator (`;` or `$`).
\[ S = \text{sum}(N); \]

A ‘return statement’ and ‘break statement’ are also primitives to construct ‘statements.’ As you can see the syntactic definition of ‘if statement’ and ‘for statement’, each of their bodies consists of a single ‘statement.’ Usually, you need several statements in such a body. To solve this contradictory requirement, you may use the ‘compound statement.’ A ‘compound statement’ is a sequence of ‘statement’s enclosed by a left brace ‘{’ and a right brace ‘}’. Thus, you can use multiple statement as if it were a single statement.

```asir
if ( I == 0 ) {
    J = 1;
    K = 2;
    L = 3;
}
```

No terminator symbol is necessary after ‘}’, because ‘{’ statement sequence ‘}’ already forms a statement, and it satisfies the syntactical requirement of the ‘if statement.’

### 4.2.6 return statement

There are two forms of return statement.

```asir
return expression;
```

```asir
return;
```

Both forms are used for exiting from a function. The former returns the value of the expression as a function value. The function value of the latter is not defined.

### 4.2.7 if statement

There are two forms of if statement.

```asir
if ( expression ) statement and if ( expression ) statement
else statement
```

The interpretation of these forms are obvious. However, be careful when another if statement comes at the place for ‘statement’. Let us examine the following example.

```asir
if ( expression1 )
    if ( expression2 ) statement1
else
    statement2
```

One might guess statement2 after else corresponds with the first if ( expression1 ) by its appearance of indentation. But, as a matter of fact, the Asir parser decides that it correspond with the second if ( expression2 ). Ambiguity due to such two kinds of forms of if statement is thus solved by introducing a rule that a statement preceded by an else matches to the nearest preceding if.

Therefore, rearrangement of the above example for improving readability according to the actual interpretation gives the following.

```asir
if ( expression1 ) {
    if ( expression2 ) statement1 else statement2
```
On the other hand, in order to reflect the indentation, it must be written as the following.

```plaintext
if ( expression1 ) {
  if ( expression2 ) statement1
} else
  statement2
```

When `if` is used in the top level, the `if` expression should be terminated with `$` or `;`. If there is no terminator, the next expression will be skipped to be evaluated.

### 4.2.8 loop, break, return, continue

There are three kinds of statements for loops (repetitions): the `while` statement, the `for` statement, and the `do` statement.

- **while statement**
  It has the following form.
  ```plaintext
  while ( expression ) statement
  ```
  This statement specifies that `statement` is repeatedly evaluated as far as the `expression` evaluates to a non-zero value. If the expression 1 is given to the `expression`, it forms an infinite loop.

- **for statement**
  It has the following form.
  ```plaintext
  for ( expr list-1; expr; expr list-2 ) statement
  ```
  This is equivalent to the program
  ```plaintext
  expr list-1 (transformed into a sequence of simple statement)
  while ( expr ) {
    statement
    expr list-2 (transformed into a sequence of simple statement)
  }
  ```

- **do statement**
  ```plaintext
  do {
    statement
  } while ( expression )
  ```
  This statement differs from `while` statement by the location of the termination condition: This statement first execute the `statement` and then check the condition, whereas `while` statement does it in the reverse order.

As means for exiting from loops, there are `break` statement and `return` statement. The `continue` statement allows to move the control to a certain point of the loop.

- **break**
  The `break` statement is used to exit the inner most loop.

- **return**
  The `return` statement is usually used to exit from a function call and it is also effective in a loop.
• **continue**
  The **continue** statement is used to move the control to the end point of the loop body. For example, the last expression list will be evaluated in a **for** statement, and the termination condition will be evaluated in a **while** statement.

### 4.2.9 structure definition

A structure data type is a fixed length array and each component of the array is accessed by its name. Each type of structure is distinguished by its name. A structure data type is declared by **struct** statement. A structure object is generated by a built-in function **newstruct**. Each member of a structure is accessed by an operator `->`. If a member of a structure is again a structure, then the specification by `->` can be nested.

```plaintext
[1] struct rat {num, denom};
0
[2] A = newstruct(rat);
{0, 0}
1
2
[5] A;
{1, 2}
[6] struct_type(A);
1
```

### References

Section 6.7.1 [**newstruct**], page 73, Section 6.7.3 [**struct_type**], page 76

### 4.2.10 various expressions

Major elements to construct expressions are the following:

- **addition**, **subtraction**, **multiplication**, **division**, **exponentiation**
  The exponentiation is denoted by `'^'`. (This differs from C language.) Division denoted by `'/'` is used to operate in a field, for example, `2/3` results in a rational number `2/3`. For integer division and polynomial division, both including remainder operation, built-in functions are provided.

  ```plaintext
  x+1 A^2*B*afo X/3
  ```

- **programming variables with indices**
  An element of a vector, a matrix or a list can be referred to by indexing. Note that the indices begin with number 0. When the referred element is again a vector, a matrix or a list, repeated indexing is also effective.

  ```plaintext
  V[0] M[1][2]
  ```

- **comparison operation**
  There are comparison operations `'='` for equivalence, `!'='` for non-equivalence, `'>'`, `'<','='`, and `'<='` for larger or smaller. The results of these operations are either value 1 for the truth, or 0 for the false.
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- logical expression
  There are two binary logical operations ‘&&’ for logical ‘conjunction’ (and), ‘||’ for logical ‘disjunction’ (or), and one unary logical operation ‘!’ for logical ‘negation’ (not). The results of these operations are either value 1 for the truth, and 0 for the false.

- assignment
  Value assignment of a program variable is usually done by ‘=’. There are special assignments combined with arithmetic operations. (‘+=’, ‘-=’, ‘*=’, ‘/=’, ‘^=’)
  \[
  A = 2 \quad A *= 3 \quad \text{(the same as} \quad A = A*3; \text{The others are alike.)}
  \]

- function call
  A function call is also an expression.

- ‘++’, ‘--’
  These operators are attached to or before a program variable, and denote special operations and values.
  \[
  A++ \quad \text{the expression value is the previous value of} \quad A, \quad \text{and} \quad A = A+1 \\
  A-- \quad \text{the expression value is the previous value of} \quad A, \quad \text{and} \quad A = A-1 \\
  ++A \quad A = A+1, \quad \text{and the value is the one after increment of} \quad A \\
  --A \quad A = A-1, \quad \text{and the value is the one after decrement of} \quad A
  \]

4.2.11 preprocessor

The Asir user language imitates C language. A typical features of C language include macro expansion and file inclusion by the preprocessor cpp. Also, Asir read in user program files through cpp. This enables Asir user to use #include, #define, #if etc. in his programs.

- #include
  Include files are searched within the same directory as the file containing #include so that no arguments are passed to cpp.

- #define
  This can be used just as in C language.

- #if
  This is conveniently used to comment out a large part of a user program that may contain comments by /* and */, because such comments cannot be nested.

the following are the macro definitions in ‘defs.h’.

```plaintext
#define ZERO 0
#define NUM 1
#define POLY 2
#define RAT 3
#define LIST 4
#define VECT 5
#define MAT 6
#define STR 7
#define N_Q 0
#define N_R 1
#define N_A 2
#define N_B 3
#define N_C 4
#define V_IND 0
```
Since we are utilizing the C preprocessor, it cannot properly preprocess expressions with $. For example, even if LIST is defined, LIST in the expression LIST$ is not replaced. Add a blank before $, i.e., write as LIST $ to make the preprocessor replace it properly.

4.2.12 option

If a user defined function is declared with N arguments, then the function is callable with N arguments only.

A function with indefinite number of arguments can be realized by using a list or an array as its argument. Another method is available as follows:

```plaintext
% cat factor

def factor(F)
{
    Mod = getopt(mod);
    ModType = type(Mod);
    if ( ModType == 1 ) /* 'mod' is not specified. */
        return fctr(F);
    else if ( ModType == 0 ) /* 'mod' is a number */
        return modfctr(F,Mod);
}

load("factor");
factor(x^5-1);
[[1,1],[x-1,1],[x^4+x^3+x^2+x+1,1]]
factor(x^5-1|mod=11);
```
In the second call of `factor()`, `|mod=11` is placed after the argument `x^5-1`, which appears in the declaration of `factor()`. This means that the value 11 is assigned to the keyword `mod` when the function is executed. The value can be retrieved by `getopt(mod)`. We call such machinery `option`. If the option for `mod` is not specified, `getopt(mod)` returns an object whose type is `-1`. By this feature, one can describe the behaviour of the function when the option is not specified by `if` statements. After `'|'` one can append any number of options separated by `','`.

```plaintext
[100] xxx(1,2,x^2-1,[1,2,3]|proc=1,index=5);
```

Optimal arguments may be given as a list with the key word `option_list` as `option_list=[["key1",value1],["key2",value2],...]`. It is equivalent to pass the optional arguments as `key1=value1,key2=value2,...`.

```plaintext
[101] dp_gr_main([x^2+y^2-1,x*y-1]|option_list=[{"v",[x,y]},{"order",[[x,5,y,1]]}]);
```

Since `getopt()` returns an option list, the optional argument `option_list=...` is useful when we call functions with optional arguments from a function with optional arguments to pass all the optional parameters.

```plaintext
% cat foo.rr

def foo(F)
{
    OPTS=getopt();
    return factor(F|option_list=OPTS);
}

[3] load("foo.rr")$
[4] foo(x^5-1|mod=11);
[[1,1],[x+6,1],[x+2,1],[x+10,1],[x+7,1],[x+8,1]]
```

### 4.2.13 module

Function names and variables in a library may be encapsulated by `module`. Let us see an example of using `module`.

```plaintext
module stack;

static Sp $
Sp = 0$
static Ssize$
Ssize = 100$
static Stack $
Stack = newvect(Ssize)$
localf push $
localf pop $

def push(A) {
    if (Sp >= Ssize) {print("Warning: Stack overflow\nDiscard the top"); pop();}
    Stack[Sp] = A;
    Sp++;
}

def pop() {
```
local A;
if (Sp <= 0) {print("Stack underflow"); return 0;}
Sp--;
A = Stack[Sp];
return A;
}
endmodule;

def demo() {
    stack.push(1);
    stack.push(2);
    print(stack.pop());
    print(stack.pop());
}

Module is encapsulated by the sentences module module name and endmodule. A variable of a module is declared with the key word static. The static variables cannot be referred nor changed out of the module, but it can be referred and changed in any functions in the module. The static variables must be declared before the definitions of functions, because the one-path parser of asir automatically assume variables as local variables if there is no declaration for them. A global variable which can be referred and changed in or out of the module is declared with the key word extern.

Any function defined in a module must be declared forward with the keyword localf. In the example above, push and pop are declared. This declaration is necessary.

A function functionName defined in a module moduleName can be called by the expression moduleName.functionName(arg1, arg2, ...) out of the module. Inside the module, moduleName, is not necessary. In the example below, the functions push and pop defined in the module stack are called out of the module.
    stack.push(2);
    print( stack.pop() );
2

Any function name defined in a module is local. In other words, the same function name may be used out of the module to define a different function.

The module structure of asir is introduced to develop large libraries. In order to load libraries on demand, the command module_definedp will be useful. The below is an example of demand loading.
    if (!module_definedp("stack")) load("stack.rr") $

It is not necessary to declare local variables in asir. As you see in the example of the stack module, we may declare local variables by the key word local. Once this key word is used, asir requires to declare all the variables. In order to avoid some troubles to develop a large libraries, it is recommended to use local declarations.

When we need to call a function in a module before the module is defined, we must make a prototype declaration as the example below.
    /* Prototype declaration of the module stack */
    module stack;
    localf push $
    localf pop $
endmodule;

def demo() {
    print("----------------");
    stack.push(1);
    print(stack.pop());
    print("----------------");
}

module stack;
    /* The body of the module stack */
endmodule;

In order to call functions defined in the top level from the inside of a module, we use :: as in the example below.
def afo() {
    S = "afo, afo";
    return S;
}
module abc;
localf foo,afo $
def foo() {
    G = ::afo();
    return G;
}
def afo() {
    return "afo, afo in abc";
}
endmodule;
end$

[1200] abc.foo();
afo, afo
[1201] abc.afo();
afo, afo in abc

References
    Section 6.12.1 [module_list], page 89, Section 6.12.2 [module_definedp], page 89, Section 6.12.3 [remove_module], page 89.
Chapter 5: Debugger

5 Debugger

5.1 What is Debugger

A debugger dbx is available for C programs on Sun, VAX etc. In dbx, one can use commands such as setting break-point on a source line, stepwise execution, inspecting a variable’s value etc. Asir provides such a dbx-like debugger. In addition to such commands, we adopted several useful commands from gdb. In order to enter the debug-mode, type debug; at the top level of Asir.

Asir also enters the debug-mode by the following means or in the following situations.

- When it reaches a break point while executing a program.
- When the ‘d’ option is selected at an interruption.
- When it detects errors while executing a program.

In this case, to continue the execution of the program is impossible. But because it reports the statement in the user defined function that caused the error, then enters the debug-mode, user can inspect the values of variables at the error state. This helps to analyze the error and debug the program.

- When built-in function error() is called.

5.2 Debugger commands

Only indispensable commands of dbx are supported in the current version. Generally, the effect of a command is the same as that of dbx. There are, however, slight differences: Commands step and next execute the next statement, but not the next line; therefore, if there are multiple statements in one line, one should issue such commands several times to proceed the next line. The debugger reads in ‘.dbxinit’, which allows the same aliases as is used in dbx.

- step: Executes the next statement; if the next statement contains a function call, then enters the function.
- next: Executes the next statement.
- finish: Enter the debug-mode again after finishing the execution of the current function. This is useful when an unnecessary step has been executed.
- cont: Exits from the debug-mode and continues execution.
- quit: Exits from the debug-mode and continues execution.
- up [n]: Move up the call stack one level. Move up the call stack n levels if n is specified.
- down [n]: Move down the call stack one level. Move down the call stack n levels if n is specified.
- frame [n]: Print the current stack frame with no argument. n specifies the stack frame number to be selected. Here the stack frame number is a number at the top of lines displayed by executing where.
list [startline]
list function
Displays ten lines in a source file from startline, the current line if the startline
is not specified, or from the top line of current target function.

print expr
Displays expr.

func function
Set the target function to function.

stop at sourceline [if cond]
stop in function
Set a break-point at the sourceline-th line of the source file, or at the top of the
target function. Break-points are removed whenever the relevant function is
redefined. When if statements are repeatedly encountered, Asir enters debug-
mode only when the corresponding cond parts are evaluated to a non-zero value.

trace expr at sourceline [if cond]
trace expr in function
These are similar to stop. trace simply displays the value of expr and without
entering the debug-mode.

delete n
Remove the break point specified by a number n, which can be known by the
status command.

status
Displays a list of the break-points.

where
Displays the calling sequence of functions from the top level through the current
level.

alias alias command
Create an alias alias for command

The debugger command print can take almost all expressions as its argument. The ordinary
usage is to print the values of (programming) variables. However, the following usage is
worth to remember.

• overwriting the variable
  One might sometimes wish to continue the execution with several values of variables
modified. For such an purpose, take the following procedure.

    (debug) print A
    A = 2
    (debug) print A=1
    A=1 = 1
    (debug) print A
    A = 1

• function call
  A function call is also an expression, therefore, it can appear at the argument place of
print.

    (debug) print length(List)
    length(List) = 14
In this example, the length of the list assigned to the variable `List` is examined by a function `length()`.

(debug) print ctrl("cputime",1)
ctrl("cputime",1) = 1

This example shows such a usage where measuring CPU time is activated from within the debug-mode, even if one might have forgotten to specify the activation of CPU time measurement.

It is also useful to save intermediate results to files from within the debug-mode by the built-in function `bsave()` when one is forced to quit the computation by any reason.

(debug) print bsave(A,"savefile")
bsave(A,"savefile") = 1

Note that continuation of the parent function will be impossible if an error will occur in the function call from within the debug-mode.

### 5.3 Execution example of debugger

Here, the usage of the Debugger is explained by showing an example for debugging a program which computes the integer factorial by a recursive definition.

```asir
% asir
[0] load("fac")$
[3] debug$
(debug) list factorial
1  def factorial(X) {  
2   if ( !X )  
3     return 1;  
4   else  
5     return X * factorial(X - 1);  
6 }  
7 end$
(debug) stop at 5  \(\text{--- setting a break point}\)
(0) stop at "./fac":5
(debug) quit  \(\text{--- leaving the debug-mode}\)
[4] factorial(6);  \(\text{--- call for factorial(6)}\)
stopped in factorial at line 5 in file "./fac"
5  return X * factorial(X - 1);
(debug) where
factorial(), line 5 in "./fac"  \(\text{--- display the calling sequence}\)
(debug) print X
X = 6
(debug) step  \(\text{--- step execution}\)
(enters function)
stopped in factorial at line 2 in file "./fac"
2  if ( !X )
(debug) where
factorial(), line 2 in "./fac"
factorial(), line 5 in "./fac"
(debug) print X
```
Chapter 5: Debugger

X = 5
(debug) delete 0 <-- delete the break point 0
(debug) cont <-- continue execution
720 <-- result = 6!
[5] quit;

5.4 Sample file of initialization file for Debugger

As is previously mentioned, Asir reads in the file ‘$HOME/.dbxinit’ at its invocation. This file is originally used to define various initializing commands for dbx debugger, but Asir recognizes only alias lines. For example, by the setting

```bash
% cat ~/.dbxinit
alias n next
alias c cont
alias p print
alias s step
alias d delete
alias r run
alias l list
alias q quit
```

one can use short aliases, e.g., p, c etc., for frequently used commands such as print, cont etc. One can create new aliases in the debug-mode during an execution.

```plaintext
lex_hensel(La,[a,b,c],0,[a,b,c],0);
stopped in gennf at line 226 in file "/home/usr3/noro/asir/gr"
226 N = length(V); Len = length(G); dp_ord(O); PS = newvect(Len);
(debug) p V
V = [a,b,c]
(debug) c
...
6 Built-in Function

6.1 Numbers

6.1.1 idiv, irem

\( \text{idiv}(i1,i2) \) :: Integer quotient of \( i1 \) divided by \( i2 \).

\( \text{irem}(i1,i2) \) :: Integer remainder of \( i1 \) divided by \( i2 \).

\[ \text{return} \quad \text{integer} \]

\( i1 \) \( i2 \) \text{ integer}

- Integer quotient and remainder of \( i1 \) divided by \( i2 \).
- \( i2 \) must not be 0.
- If the dividend is negative, the results are obtained by changing the sign of the results for absolute values of the dividend.
- One can use \( i1 \% i2 \) for replacement of \text{irem()} which only differs in the point that the result is always normalized to non-negative values.
- Use \text{sdiv()}, \text{srem()} for polynomial quotient.

\[ \begin{align*}
[0] \text{idiv}(100,7); \\
14 \\
[0] \text{idiv}(-100,7); \\
-14 \\
[1] \text{irem}(100,7); \\
2 \\
[1] \text{irem}(-100,7); \\
-2 \\
\end{align*} \]

References
Section 6.3.8 \[ \text{sdiv sdivm srem sremm sqr sqrm} \], page 49, Section 6.3.10 \[ \% \], page 51.

6.1.2 fac

\( \text{fac}(i) \) :: The factorial of \( i \).

\[ \text{return} \quad \text{integer} \]

\( i \) \text{ integer}

- The factorial of \( i \).
- Returns 0 if the argument \( i \) is negative.

\[ \begin{align*}
[0] \text{fac}(50); \\
30414093201713378043612608166064768844377641568960512000000000000 \\
\end{align*} \]
6.1.3 igcd, igcdcntl

\textbf{igcd}(i1, i2)

:: The integer greatest common divisor of \textit{i1} and \textit{i2}.

\textbf{igcdcntl}([i])

:: Selects an algorithm for integer GCD.

\textbf{return} integer

\textit{i1 i2 i} integer

- Function \textbf{igcd()} returns the integer greatest common divisor of the given two integers.
- An error will result if the argument is not an integer; the result is not valid even if one is returned.
- Use \textbf{gcd()}, \textbf{gcdz()} for polynomial GCD.
- Various method of integer GCD computation are implemented and they can be selected by \textbf{igcdcntl}.

0 Euclid algorithm (default)

1 binary GCD

2 bmod GCD

3 accelerated integer GCD

2, 3 are due to [Weber].

In most cases 3 is the fastest, but there are exceptions.

\begin{verbatim}
[0] A=lrandom(10^4)
[1] B=lrandom(10^4)
[2] C=lrandom(10^4)
[5] cputime(1)
[6] igcd(D,E)
0.6sec + gc : 1.93sec(2.531sec)
[7] igcdcntl(1)
[8] igcd(D,E)
0.27sec(0.2635sec)
[9] igcdcntl(2)
[10] igcd(D,E)
0.19sec(0.1928sec)
[12] igcd(D,E)
0.08sec(0.08023sec)
\end{verbatim}

References

Section 6.3.20 \texttt{gcd gcdz}, page 58.
6.1.4 ilcm

ilcm(i1, i2)
  :: The integer least common multiple of i1 and i2.
return integer
i1 i2 integer
- This function computes the integer least common multiple of i1, i2.
- If one of argument is equal to 0, the return 0.

References
Section 6.1.3 [igcd igcdcntl], page 36, Section 6.1.10 [mt_save mt_load], page 39.

6.1.5 isqrt

isqrt(n) :: The integer square root of n.
return non-negative integer
n non-negative integer

6.1.6 inv

inv(i, m) :: the inverse (reciprocal) of i modulo m.
return integer
i m integer
- This function computes an integer such that \( ia \equiv 1 \mod (m) \).
- The integer i and m must be mutually prime. However, inv() does not check it.
  [71] igcd(1234, 4321);
  1
  [72] inv(1234, 4321);
  3239
  [73] irem(3239*1234, 4321);
  1

References
Section 6.1.3 [igcd igcdcntl], page 36.

6.1.7 prime, lprime

prime(index)
lprime(index)
  :: Returns a prime number.
return integer
index integer
The two functions, \texttt{prime()} and \texttt{lprime()}, returns an element stored in the system table of prime numbers. Here, \texttt{index} is a non-negative integer and be used as an index for the prime tables. The function \texttt{prime()} can return one of 1900 primes up to 16381 indexed so that the smaller one has smaller index. The function \texttt{lprime()} can return one of 999 primes which are 8 digit sized and indexed so that the larger one has the smaller index. The two function always returns 0 for other indices.

For more general function for prime generation, there is a \texttt{PARI} function

\begin{verbatim}
pari(nextprime,number).
[95] prime(0); 2
[96] prime(1228); 9973
[97] lprime(0); 99999989
[98] lprime(999); 0
\end{verbatim}

References
Section 6.1.14 [pari], page 41.

6.1.8 \texttt{random}

\texttt{random([seed])}

\textit{seed} \\
\textit{return} non-negative integer

- Generates a random number which is a non-negative integer less than $2^{32}$.
- If a non zero argument is specified, then after setting it as a random seed, a random number is generated.
- As the default seed is fixed, the sequence of the random numbers is always the same if a seed is not set.
- The algorithm is Mersenne Twister (http://www.math.keio.ac.jp/matsumoto/mt.html) by M. Matsumoto and T. Nishimura. The implementation is done also by themselves.
- The period of the random number sequence is $2^{19937}-1$.
- One can save the state of the random number generator with \texttt{mt_save}. By loading the state file with \texttt{mt_load}, one can trace a single random number sequence across multiple sessions.

References
Section 6.1.9 [lrandom], page 38, Section 6.1.10 [mt_save mt_load], page 39.

6.1.9 \texttt{lrandom}

\texttt{lrandom(bit)}

:: Generates a long random number.

\textit{bit}
return integer

- Generates a non-negative integer of at most bit bits.
- The result is a concatenation of outputs of random.

References
Section 6.1.8 [random], page 38, Section 6.1.10 [mt_save mt_load], page 39.

6.1.10 mt_save, mt_load

mt_save(fname)
:: Saves the state of the random number generator.

mt_load(fname)
:: Loads a saved state of the random number generator.

return 0 or 1

fname string

- One can save the state of the random number generator with mt_save. By loading the state file with mt_load, one can trace a single random number sequence across multiple Asir sessions.

[340] random();
3510405877
[341] mt_save("/tmp/mt_state");
1
[342] random();
4290933890
[343] quit;
% asir
This is Asir, Version 991108.
Copyright (C) FUJITSU LABORATORIES LIMITED.
3 March 1994. All rights reserved.
[340] mt_load("/tmp/mt_state");
1
[341] random();
4290933890

References
Section 6.1.8 [random], page 38, Section 6.1.9 [lrandom], page 38.

6.1.11 nm, dn

nm(rat) :: Numerator of rat.

dn(rat) :: Denominator of rat.

return integer or polynomial

rat rational number or rational expression

- Numerator and denominator of a given rational expression.
For a rational number, they return its numerator and denominator, respectively. For a rational expression whose numerator and denominator may contain rational numbers, they do not separate those rational coefficients to numerators and denominators.

For a rational number, the denominator is always kept positive, and the sign is contained in the numerator.

**Risa/Asir** does not cancel the common divisors unless otherwise explicitly specified by the user. Therefore, \( \text{nm()} \) and \( \text{dn()} \) return the numerator and the denominator as it is, respectively.

\[
[2] \ [\text{nm}(-43/8), \text{dn}(-43/8)];
[\text{-43,8}]
\]
\[
[3] \ \text{dn}((x*z)/(x*y));
\ y\ x
\]
\[
[3] \ \text{dn}((x*z)/(x*y));
\ y
\]

References
Section 6.3.21 [red], page 59.

6.1.12 \textbf{conj, real, imag}

\textbf{real}(\text{comp})
:: Real part of \text{comp}.

\textbf{imag}(\text{comp})
:: Imaginary part of \text{comp}.

\textbf{conj}(\text{comp})
:: Complex conjugate of \text{comp}.

\textit{return} \text{comp}
:: complex number

- Basic operations for complex numbers.
- These functions works also for polynomials with complex coefficients.

\[
[111] \ A=(2+@i)^3;
\ (2+11*@i)
\]
\[
[112] \ [\text{real}(A), \text{imag}(A), \text{conj}(A)];
\ [2,11,(2-11*@i)]
\]

6.1.13 \textbf{eval, deval}

\textbf{eval}(\text{obj}[, \text{prec}])

\textbf{deval}(\text{obj})
:: Evaluate \text{obj} numerically.

\textit{return} \text{number or expression}

\textit{obj} \text{general expression}

\textit{prec} \text{integer}

- Evaluates the value of the functions contained in \text{obj} as far as possible.
• **deval** returns double float. Rational numbers remain unchanged in results from **eval**.

• In **eval** the computation is done by **MPFR** library. In **deval** the computation is done by the C math library.

• **deval** cannot handle complex numbers.

• When **prec** is specified, computation will be performed with a precision of about **prec**-digits. If **prec** is not specified, computation is performed with the precision set currently.

  (See Section 6.1.15 [setbprec setprec], page 42.)

• Currently available numerical functions are listed below.

  sin, cos, tan,
  asin, acos, atan,
  sinh, cosh, tanh, asinh, acosh, atanh,
  exp, log, pow(a,b) (a^b)

• Symbols for special values are as the followings. Note that @i cannot be handled by **deval**.

  @i unit of imaginary number
  @pi the number pi, the ratio of circumference to diameter
  @e Napier’s number (exp(1))

  [118] eval(exp(@pi*@i));
       -1.0000000000000000000000000000000
  [119] eval(2^(1/2));
       1.414213562373095048763788073031
  [120] eval(sin(@pi/3));
       0.86602540378443864674620506632
  [121] eval(sin(@pi/3)-3^(1/2)/2,50);
       -2.78791084448179148471 E-58
  [122] eval(1/2);
       1/2
  [123] deval(sin(1)^2+cos(1)^2);
       1

References

  Section 6.14.1 [ctrl], page 91, Section 6.1.15 [setbprec setprec], page 42.

### 6.1.14 pari

`pari(func, arg, prec)`

:: Call **PARI** function **func**.

- **func** Function name of **PARI**.
- **arg** Arguments of **func**.
- **prec** integer

- This command connects **Asir** to **PARI** system so that several functions of **PARI** can be conveniently used from **Risa/Asir**.
• **PARI** [Batut et al.] is developed at Bordeaux University, and distributed as a free software. Though it has a certain facility to computer algebra, its major target is the operation of numbers (bignum, bigfloat) related to the number theory. It facilitates various function evaluations as well as arithmetic operations at a remarkable speed. It can also be used from other external programs as a library. It provides a language interface named ‘gp’ to its library, which enables a user to use PARI as a calculator which runs on UNIX.

- The last argument (optional) int specifies the precision in digits for bigfloat operation. If the precision is not explicitly specified, operation will be performed with the precision set by setprec().

- Currently available functions of PARI system are as follows. Note these are only a part of functions in PARI system. For details of individual functions, refer to the PARI manual. (Some of them can be seen in the following example.)

```plaintext
abs, adj, arg, bigomega, binary, ceil, centerlift, cf, classno, classno2, conj, content, denom, det, det2, detr, dilog, disc, discf, divisors, eigen, eintg1, erf, eta, floor, frac, galois, galoisconj, gamh, gamma, hclassno, hermite, hess, imag, image, image2, indexrank, indsort, initalg, isfund, isprime, ispsp, isqrt, issqfree, issquare, jacobi, jell, ker, keri, kerint, kerintg1, kerint2, kerr, length, lexsort, lift, lindep, lll, lllg1, lllgen, lllgram, lllgramg1, lllgramgen, lllgramint, lllgramkerim, lllgramkerimgen, lllint, lllkerim, lllkerimgen, lllrat, lingamma, logagm, mat, matrixqz2, matrixqz3, matsize, modreverse, mu, nextprime, norm, norml2, numdiv, numer, omega, order, orddet, phi, pnqn, polred, polred2, primroot, psi, quadgen, quadpoly, real, recip, redcomp, redreal, regulat, reorder, reverse, rhoreal, roots, rootslong, round, sigma, signat, simplify, smalldiscf, smallfact, smallpolred, smallpolred2, smith, smith2, sort, sqa, sqred, sqrt, supplement, trace, trans, trunc, type, unit, vec, wf, wf2, zeta
```

• **Asir** currently uses only a very small subset of PARI.

```plaintext
/* Eigen vectors of a numerical matrix */
[0] pari(eigen,newmat(2,2,[[1,1],[1,2]]));
[-1.61803398874989484819771921990 0.61803398874989484826]
[1 1]
/* Roots of a polynomial */
[1] pari(roots,t^2-2);
[-1.41421356237309504876 1.41421356237309504876]
```

References
Section 6.1.15 [setbprec setprec], page 42.

6.1.15 setbprec, setprec

```plaintext
setbprec([n])
setprec([n])
:: setbprec, setprec set the precision for bigfloat operations to n bits, n digits respectively.

return integer
```
When an argument $n$ is given, these functions set the precision for \texttt{bigfloat} operations to $n$ bits or $n$ digits. The return value is always the previous precision regardless of the existence of an argument.

- \textbf{Bigfloat} operations are done by \texttt{MPFR} library.
- This is effective for computations in \texttt{bigfloat}. Refer to \texttt{ctrl()} for turning on the ‘\texttt{bigfloat} flag.’
- There is no upper limit for precision digits. It sets the precision to some digits around the specified precision. Therefore, it is safe to specify a larger value.

\begin{verbatim}
[1] setprec();
15
[2] setprec(100);
15
[3] setprec(100);
99
[4] setbprec();
332
\end{verbatim}


\subsection*{6.1.16 \texttt{setround}}

\texttt{setround(\texttt{[mode]})}

:: Sets the rounding mode \texttt{mode}.

\texttt{return} \hspace{1cm} \texttt{integer}

\texttt{mode} \hspace{1cm} \texttt{integer}

- When an argument \texttt{mode} is given, these functions set the rounding mode for \texttt{bigfloat} operations to \texttt{mode}. The return value is always the previous rounding mode regardless of the existence of an argument. The meanings of rounding modes are as follows

\begin{verbatim}
0 Round to nearest
1 Round toward 0
2 Round toward +infinity
3 Round toward -infinity
\end{verbatim}

- This is effective for computations in \texttt{bigfloat}. Refer to \texttt{ctrl()} for turning on the ‘\texttt{bigfloat} flag.’

\begin{verbatim}
[1] setprec();
15
[2] setprec(100);
15
[3] setprec(100);
99
[4] setbprec();
332
\end{verbatim}

6.1.17 **setmod**

```plaintext
setmod([p])
:: Sets the ground field to GF(p).

*return* integer
```

- $n$ prime less than $2^{27}$
  - Sets the ground field to GF($p$) and returns the value $p$.
  - A member of a finite field does not have any information about the field and the arithmetic operations over GF($p$) are applied with $p$ set at the time.
  - As for large finite fields, see Chapter 10 [Finite fields], page 170.

```plaintext
[0] A=dp_mod(dp_ptod(2*x,[x]),3,[1]);
(2)*<<1>>
[1] A+A;
addmi : invalid modulus
return to toplevel
[1] setmod(3);
3
[2] A+A;
(1)*<<1>>
```

**References**

- Section 8.10.14 [dp_mod dp_rat], page 142, Section 3.2 [Types of numbers], page 14.

6.1.18 **ntoint32, int32ton**

```plaintext
ntoint32(n)
int32ton(int32)
:: Type-conversion between a non-negative integer and an unsigned 32bit integer.

*return* unsigned 32bit integer or non-negative integer
```

- $n$ non-negative integer less than $2^{32}$
- $int32$ unsigned 32bit integer
  - These functions do conversions between non-negative integers (the type id 1) and unsigned 32bit integers (the type id 10).
  - An unsigned 32bit integer is a fundamental construct of **OpenXM** and one often has to send an integer to a server as an unsigned 32bit integer. These functions are used in such a case.

**References**

- Chapter 7 [Distributed computation], page 102, Section 3.2 [Types of numbers], page 14.
6.2 Bit operations

6.2.1 iand, ior, ixor

\[
\text{iand}(i1, i2) \\
:: \text{bitwise and}
\]

\[
\text{ior}(i1, i2) \\
:: \text{bitwise or}
\]

\[
\text{ixor}(i1, i2) \\
:: \text{bitwise xor}
\]

\[\text{return} \quad \text{integer}\]

\[i1 \quad i2 \quad \text{integer}\]

- The absolute value of the argument is regarded as a bit string.
- The sign of the argument is ignored and a non-negative integer is returned.

\[0 \quad \text{ctrl("hex"}, \text{1);}
0x1
\]

\[1 \quad \text{iand(0xeeeeeeeeee0x2984723234812312312);}
0x462224802020202
\]

\[2 \quad \text{ior(0xa0a0a0a0a0a0a0a0a0a0,0xb0c0b0b0b0b0b0b0b);}
0xabacababababababababababab
\]

\[3 \quad \text{ixor(0xfffffffffff,0x234234234234);}
0x2cbdcbdcbdcb;
\]

References

Section 6.2.2 [ishift], page 45.

6.2.2 ishift

\[
\text{ishift}(i, \text{count}) \\
:: \text{bit shift}
\]

\[\text{return} \quad \text{integer}\]

\[i \quad \text{count} \quad \text{integer}\]

- The absolute value of \(i\) is regarded as a bit string.
- The sign of \(i\) is ignored and a non-negative integer is returned.
- If \(\text{count}\) is positive, then \(i\) is shifted to the right. If \(\text{count}\) is negative, then \(i\) is shifted to the left.

\[0 \quad \text{ctrl("hex"}, \text{1);}
0x1
\]

\[1 \quad \text{ishift(0x1000000,12);}
0x1000
\]

\[2 \quad \text{ishift(0x1000,-12);}
0x1000000
\]

\[3 \quad \text{ixor(0x1248,ishift(1,-16)-1);}
\]

References

Section 6.2.1 [iand ior ixor], page 45.
6.3 operations with polynomials and rational expressions

6.3.1 var

\textbf{var}(\textit{rat}) :: \textit{Main variable (indeterminate) of rat}.

\textbf{return} \quad \text{indeterminate}

\textit{rat} \quad \text{rational expression}

\begin{itemize}
  \item See Section 3.1 [Types in Asir], page 11 for main variable.
  \item Indeterminates (variables) are ordered by default as follows.
    x, y, z, u, v, w, p, q, r, s, t, a, b, c, d, e, f, g, h, i, j, k, l, m, n, o. The other variables will be ordered after the above noted variables so that the first comer will be ordered prior to the followers.
\end{itemize}

\begin{itemize}
  \item \texttt{var(x^2+y^2+a^2)}; \rightarrow x
  \item \texttt{var(a*b*c*d*e)}; \rightarrow a
  \item \texttt{var(3/abc+2*xy/efg)}; \rightarrow abc
\end{itemize}

References
Section 6.3.7 [\texttt{ord}], page 49, Section 6.3.2 [\texttt{vars}], page 46.

6.3.2 vars

\textbf{vars}(\textit{obj}) :: \textit{A list of variables (indeterminates) in an expression \textit{obj}}.

\textbf{return} \quad \text{list}

\textit{obj} \quad \text{arbitrary}

\begin{itemize}
  \item Returns a list of variables (indeterminates) contained in a given expression.
  \item Lists variables according to the variable ordering.
\end{itemize}

\begin{itemize}
  \item \texttt{vars(x^2+y^2+a^2)}; \rightarrow [x,y,a]
  \item \texttt{vars(3/abc+2*xy/efg)}; \rightarrow [abc,xy,efg]
  \item \texttt{vars([x,y,z])}; \rightarrow [x,y,z]
\end{itemize}

References
Section 6.3.1 [\texttt{var}], page 46, Section 6.3.3 [\texttt{uc}], page 47, Section 6.3.7 [\texttt{ord}], page 49.

6.3.3 uc

\textbf{uc()} :: \textit{Create a new indeterminate for an undermined coefficient}.

\textbf{return} \quad \text{indeterminate with its \texttt{vtype} 1.}
At every evaluation of command \texttt{uc()}, a new indeterminate in the sequence of indeterminates \_0, \_1, \_2, \ldots is created successively.

Indeterminates created by \texttt{uc()} cannot be input on the keyboard. By this property, you are free, no matter how many indeterminates you will create dynamically by a program, from collision of created names with indeterminates input from the keyboard or from program files.

Functions, \texttt{rtostr()} and \texttt{strtov()}, are used to create ordinary indeterminates (indeterminates having 0 for their \texttt{vtype}).

Kernel sub-type of indeterminates created by \texttt{uc()} is 1. (vtype(uc())=1)

\begin{verbatim}
[0] A=uc();
_0
[1] B=uc();
_1
[2] (uc()+uc())^2;
_2^2+2*_3*_2+_3^2
[3] (A+B)^2;
_0^2+2*_1*_0+_1^2
\end{verbatim}

6.3.4 \texttt{coef}

\texttt{coef(poly, deg[, var])}

:: The coefficient of a polynomial \textit{poly} at degree \textit{deg} with respect to the variable \textit{var} (main variable if unspecified).

\texttt{return} \hspace{1em} \texttt{polynomial}

\texttt{poly} \hspace{1em} \texttt{polynomial}

\texttt{var} \hspace{1em} \texttt{indeterminate}

\texttt{deg} \hspace{1em} non-negative integer

- The coefficient of a polynomial \textit{poly} at degree \textit{deg} with respect to the variable \textit{var}.

- The default value for var is the main variable, i.e., \texttt{var(poly)}.

- For multi-variate polynomials, access to coefficients depends on the specified indeterminates. For example, taking \texttt{coef} for the main variable is much faster than for other variables.

\begin{verbatim}
[0] A = (x+y+z)^3;
  x^3+(3*y+3*z)*x^2+(3*y^2+6*z*y+3*z^2)*x+y^3+3*z*y^2+3*z^2*y+z^3
[1] coef(A,1,y);
  3*x^2+6*z*x+3*z^2
[2] coef(A,0);
  y^3+3*z*y^2+3*z^2*y+z^3
\end{verbatim}

References

Section 6.3.1 [var], page 46, Section 6.3.5 [deg mindeg], page 48.
6.3.5 deg, mindeg

\[ \text{deg}(\text{poly}, \text{var}) \]
:: The degree of a polynomial \text{poly} with respect to variable \text{var}.

\[ \mindeg(\text{poly}, \text{var}) \]
:: The least exponent of the terms with non-zero coefficients in a polynomial \text{poly} with respect to the variable \text{var}. In this manual, this quantity is sometimes referred to the minimum degree of a polynomial for short.

\text{return} \quad \text{non-negative integer}

\text{poly} \quad \text{polynomial}
\text{var} \quad \text{indeterminate}

- The least exponent of the terms with non-zero coefficients in a polynomial \text{poly} with respect to the variable \text{var}. In this manual, this quantity is sometimes referred to the minimum degree of a polynomial for short.

- Variable \text{var} must be specified.

\[ [0] \ \text{deg}((x+y+z)^{10}, x); \]
10
\[ [1] \ \text{deg}((x+y+z)^{10}, w); \]
0
\[ [75] \ \mindeg(x^2+3*x*y, x); \]
1

6.3.6 nmono

\[ \text{nmono}(\text{rat}) \]
:: Number of monomials in rational expression \text{rat}.

\text{return} \quad \text{non-negative integer}
\text{rat} \quad \text{rational expression}

- Number of monomials with non-zero number coefficients in the full expanded form of the given polynomial.
- For a rational expression, the sum of the numbers of monomials of the numerator and denominator.
- A function form is regarded as a single indeterminate no matter how complex arguments it has.

\[ [0] \ \text{nmono}((x+y)^{10}); \]
11
\[ [1] \ \text{nmono}((x+y)^{10}/(x+z)^{10}); \]
22
\[ [2] \ \text{nmono}(\sin((x+y)^{10})); \]
1

References
Section 6.8.3 [vtype], page 78.
6.3.7 ord

\texttt{ord([\text{varlist}])}

:: It sets the ordering of indeterminates (variables).

\texttt{return list of indeterminates}

\texttt{varlist list of indeterminates}

- When an argument is given, this function rearranges the ordering of variables (indeterminates) so that the indeterminates in the argument \texttt{varlist} precede and the other indeterminates follow in the system’s variable ordering. Regardless of the existence of an argument, it always returns the final variable ordering.

- Note that no change will be made to the variable ordering of internal forms of objects which already exists in the system, no matter what reordering you specify. Therefore, the reordering should be limited to the time just after starting \texttt{Asir}, or to the time when one has decided himself to start a totally new computation which has no relation with the previous results. Note that unexpected results may be obtained from operations between objects which are created under different variable ordering.

\texttt{[0] ord(); [x,y,z,u,v,w,p,q,r,s,t,a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,_x,_y,_z,_u,_v, \_w,\_p,\_q,\_r,\_s,\_t,\_a,\_b,\_c,\_d,\_e,\_f,\_g,\_h,\_i,\_j,\_k,\_l,\_m,\_n,\_o, exp(_x),(_x)^(_y),log(_x),(_x)^(_y-1),cos(_x),sin(_x),tan(_x), (-_x^2+1)^(-1/2),cosh(_x),sinh(_x),tanh(_x), (_x^2+1)^(-1/2),(_x^2-1)^(-1/2) [1] ord([dx,dy,dz,a,b,c]); [dx,dy,dz,a,b,c,x,y,z,u,v,w,p,q,r,s,t,d,e,f,g,h,i,j,k,l,m,n,o,_x,_y, \_z,\_u,\_v,\_w,\_p,\_q,\_r,\_s,\_t,\_a,\_b,\_c,\_d,\_e,\_f,\_g,\_h,\_i,\_j,\_k,\_l,\_m,\_n, \_o,exp(_x),(_x)^(_y),log(_x),(_x)^(_y-1),cos(_x),sin(_x),tan(_x), (-_x^2+1)^(-1/2),cosh(_x),sinh(_x),tanh(_x), (_x^2+1)^(-1/2),(_x^2-1)^(-1/2)]}

6.3.8 sdiv, sdivm, srem, sremm, sqr, sqrm

\texttt{sdiv(poly1,poly2[,v])}
\texttt{sdivm(poly1,poly2,mod[,v])}

:: Quotient of poly1 divided by poly2 provided that the division can be performed within polynomial arithmetic over the rationals.

\texttt{srem(poly1,poly2[,v])}
\texttt{sremm(poly1,poly2,mod[,v])}

:: Remainder of poly1 divided by poly2 provided that the division can be performed within polynomial arithmetic over the rationals.

\texttt{sqr(poly1,poly2[,v])}
\texttt{sqrm(poly1,poly2,mod[,v])}

:: Quotient and remainder of poly1 divided by poly2 provided that the division can be performed within polynomial arithmetic over the rationals.

\texttt{return sdiv(), sdivm(), srem(), sremm() : polynomial sqr(), sqrm() : a list [quotient,remainder]}
Regarding \( poly1 \) as an uni-variate polynomial in the main variable of \( poly2 \), i.e. \( \var(poly2) \) (v if specified), \( \text{sdiv()} \) and \( \text{srem()} \) compute the polynomial quotient and remainder of \( poly1 \) divided by \( poly2 \).

- \( \text{sdivm()} \), \( \text{sremm()} \), \( \text{sqrm()} \) execute the same operation over GF(mod).

- Division operation of polynomials is performed by the following steps: (1) obtain the quotient of leading coefficients; let it be Q; (2) remove the leading term of \( poly1 \) by subtracting, from \( poly1 \), the product of \( Q \) with some powers of main variable and \( poly2 \); obtain a new \( poly1 \); (3) repeat the above step until the degree of \( poly1 \) become smaller than that of \( poly2 \). For fulfillment, by operating in polynomials, of this procedure, the divisions at step (1) in every repetition must be an exact division of polynomials. This is the true meaning of what we say “division can be performed within polynomial arithmetic over the rationals.”

- There are typical cases where the division is possible: leading coefficient of \( poly2 \) is a rational number; \( poly2 \) is a factor of \( poly1 \).

- Use \( \text{sqr()} \) to get both the quotient and remainder at once.

- Use \( \text{idiv()} \), \( \text{irem()} \) for integer quotient.

- For remainder operation on all integer coefficients, use \%.

```
[0] \text{sdiv((x+y+z)^3,x^2+y+a)};
  x+3*y+3*z
[1] \text{srem((x+y+z)^2,x^2+y+a)};
  (2*y+2*z)*x+y^2+(2*z-1)*y+z^2-a
[2] X=(x+y+z)*(x-y-z)^2;
  x^3+(-y-z)*x^2+(-y^2-2*z*y-z^2)*x+y^2+3+3*z*y^2+3*z^2-y^2-z^3
[3] Y=(x+y+z)*x^2+(-y^2-2*z*y-z^2)*x-y^3-3*z*y^2-3*z^2*y-z^3
[4] G=\text{gcd}(X,Y);
  x^2-y^2-2*z*y-z^2
[5] \text{sqr(X,G)};
  [x-y-z,0]
[6] \text{sqr(Y,G)};
  [x+y+z,0]
[7] \text{sdiv(y*x^3+x+1,y*x+1)};
  \text{divsp: cannot happen}
  \text{return to toplevel}
```

References

Section 6.1.1 [idiv irem], page 35, Section 6.3.10 [%], page 51.

6.3.9 \( \text{tdiv} \)

\( \text{tdiv(poly1,poly2)} \)

:: Tests whether \( poly2 \) divides \( poly1 \).
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return Quotient if poly2 divides poly1, 0 otherwise.

poly1 poly2

polynomial

- Tests whether poly2 divides poly1 in polynomial ring.
- One application of this function: Consider the case where a polynomial is certainly an irreducible factor of the other polynomial, but the multiplicity of the factor is unknown. Application of tdiv() to the polynomials repeatedly yields the multiplicity.

\[
Y = (x+y+z)^5(x-y-z)^3;
\]
\[
x^8 + (2y+2z)x^7 + (-2y^2-4z*y-2z^2)x^6 + (-6y^3-18z*y^2-18z^2*y-6z^3)x^5
\]
\[
+ (6y^4+30z*y^3+60z^2*y^3+60z^3*y^2+30z^4*y+6z^5)*x^4
\]
\[
+ (2y^5+12z*y^4+30z^2*y^4+40z^3*y^3+30z^4*y^2+12z^5*y+2z^6)*x^3
\]
\[
+ (-2y^6-14z*y^5-42z^2*y^5-70z^3*y^4-70z^4*y^3-42z^5*y+2z^6)*x^2
\]
\[
- 14z^7y^7-28z^6y^7-28z^5y^7-56z^4y^7-70z^3y^7-70z^2y^7-70z^4y^2
\]
\[
- 56z^5y^2-28z^6y^2-8-z^7y^8-8-z^8y^7-28z^5y^7-70z^4y^7
\]
\[
[11] \text{for}(I=0,F=x+y+z,T=Y; T=tdiv(T,F); I++);
\]
\[
[12] I;
\]
\[
5
\]

References

Section 6.1.1 \[idiv irem\], page 35.

6.3.10 \%

poly \% m :: integer remainder to all integer coefficients of the polynomial.

return integer or polynomial

poly integer or polynomial with integer coefficients

m integer

- Returns a polynomial whose coefficients are remainders of the coefficients of the input polynomial divided by m.
- The resulting coefficients are all normalized to non-negative integers.
- An integer is allowed for poly. This can be used for an alternative for irem() except that the result is normalized to a non-negative integer.
- Coefficients of poly and m must all be integers, though the type checking is not done.

\[
[0] (x+2)^5 \% 3;
\]
\[
x^5 + x^4 + x^3 + 2x^2 + 2x + 2
\]
\[
[1] (x-2)^5 \% 3;
\]
\[
x^5 + 2x^4 + x^3 + 2x^2 + 2x + 1
\]
\[
[2] (-5) \% 4;
\]
\[
3
\]
\[
[3] irem(-5,4);
\]
\[
-1
\]

References

Section 6.3.8 \[sddiv sdvim srem sremm sqr sqrm\], page 49.
6.3.11 subst, psubst

\[ \text{subst}(\text{rat}, \varn, \text{ratn}) \]
\[ \text{psubst}(\text{rat}, \var, \text{rat}) \]

:: Substitute \text{ratn} for \varn in expression \text{rat}. \ (n=1,2,\ldots) Substitution will be done successively from left to right if arguments are repeated.

\text{return} \quad \text{rational expression}

\text{rat} \quad \text{ratn} \quad \text{rational expression}

\varn \quad \text{indeterminate}

- Substitutes rational expressions for specified kernels in a rational expression.
- \text{subst}(r,v1,r1,v2,r2,\ldots) has the same effect as \text{subst(subst(r,v1,r1),v2,r2,\ldots)}.
- Note that repeated substitution is done from left to right successively. You may get different result by changing the specification order.
- Ordinary \text{subst()} performs substitution at all levels of a scalar algebraic expression creeping into arguments of function forms recursively. Function \text{psubst()} regards such a function form as an independent indeterminate, and does not attempt to apply substitution to its arguments. (The name comes after Partial SUBSTITution.)
- Since \text{Asir} does not reduce common divisors of a rational expression automatically, substitution of a rational expression to an expression may cause unexpected increase of computation time. Thus, it is often necessary to write a special function to meet the individual problem so that the denominator and the numerator do not become too large.
- The same applies to substitution by rational numbers.

\begin{align*}
0 & \text{ subst}(x^3-3*y*x^2+3*y^2*x-y^3,y,2); \\
& x^3-6*x^2+12*x-8 \\
1 & \text{ subst}(0, x,-1); \\
& -27 \\
2 & \text{ subst}(x^3-3*y*x^2+3*y^2*x-y^3,y,2,x,-1); \\
& -27 \\
3 & \text{ subst}(x*y^3,x,y,y,x); \\
& x^4 \\
4 & \text{ subst}(x*y^3,y,x,x,y); \\
& y^4 \\
5 & \text{ subst}(x*y^3,x,t,y,x,t,y); \\
& y*x^3 \\
6 & \text{ subst}(x*sin(x),x,t); \\
& \text{sint}(t)*t \\
7 & \text{ psubst}(x*sin(x),x,t); \\
& \text{sin}(x)*t
\end{align*}

6.3.12 diff

\[ \text{diff}(\text{rat}, \varn) \]
\[ \text{diff}(\text{rat}, \text{varlist}) \]

:: Differentiate \text{rat} successively by \varn’s for the first form, or by variables in \text{varlist} for the second form.
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return expression
rat rational expression which contains elementary functions.
varn indeterminate
varlist list of indeterminates

- Differentiate rat successively by var’s for the first form, or by variables in varlist for the second form.
- Differentiation is performed by the specified indeterminates (variables) from left to right. diff(rat,x,y) is the same as diff(diff(rat,x),y).

\[
\begin{align*}
[0] & \quad \text{diff}((x+2y)^2,x); \\
& \quad 2x+4y \\
[1] & \quad \text{diff}((x+2y)^2,x,y); \\
& \quad 4 \\
[2] & \quad \text{diff}(x/sin(log(x)+1),x); \\
& \quad (sin(log(x)+1)-cos(log(x)+1))/(sin(log(x)+1)^2) \\
[3] & \quad \text{diff}(sin(x),[x,x,x,x]); \\
& \quad sin(x)
\end{align*}
\]

6.3.13 ediff

ediff(poly[,varn]*)
ediff(poly,varlist)
:: Differentiate poly successively by Euler operators of var’s for the first form, or by Euler operators of variables in varlist for the second form.

return polynomial
poly polynomial
varn indeterminate
varlist list of indeterminates

- Differentiation is performed by the specified indeterminates (variables) from left to right. ediff(poly,x,y) is the same as ediff(ediff(poly,x),y).

\[
\begin{align*}
[0] & \quad \text{ediff}((x+2*y)^2,x); \\
& \quad 2x^2+4*y*x \\
[1] & \quad \text{ediff}((x+2*y)^2,x,y); \\
& \quad 4*y*x \\
[2] & \quad \text{ediff}(x/sin(log(x)+1),x); \\
& \quad (sin(log(x)+1)-cos(log(x)+1))/(sin(log(x)+1)^2) \\
[3] & \quad \text{ediff}(sin(x),[x,x,x,x]); \\
& \quad sin(x)
\end{align*}
\]

6.3.14 res

res(var,poly1,poly2[,mod])
:: Resultant of poly1 and poly2 with respect to var.

return polynomial
var indeterminate
poly1 poly2 polynomial
mod prime
- Resultant of two polynomials poly1 and poly2 with respect to var.
- Sub-resultant algorithm is used to compute the resultant.
- The computation is done over GF(mod) if mod is specified.

\[ \text{res}(t, (t^3+1)*x+1, (t^3+1)*y+t); \]
\[-x^3-x^2-y^3\]

**6.3.15 fctr, sqfr**

**fctr(poly)**
:: Factorize polynomial poly over the rationals.

**sqfr(poly)**
:: Gets a square-free factorization of polynomial poly.

**return** list
poly polynomial with rational coefficients
- Factorizes polynomial poly over the rationals. fctr() for irreducible factorization; sqfr() for square-free factorization.
- The result is represented by a list, whose elements are a pair represented as
  \[ [\text{num}, 1], [\text{factor}, \text{multiplicity}], \ldots \].
- Products of all factor^multiplicity and num is equal to poly.
- The number num is determined so that \((poly/num)\) is an integral polynomial and its content (GCD of all coefficients) is 1. (See Section 6.3.18 [ptozp], page 57.)

\[ \text{fctr}(x^{10}-1); \]
\[[[1, 1], [x-1, 1], [x+1, 1], [x^4+x^3+x^2+x+1, 1], [x^4-x^3+x^2-x+1, 1]]\]
\[[1] \text{fctr}(x^3+y^3+(z/3)^3-x*y*z); \]
\[[[1/27, 1], [9*x^2+(-9*y-3*z)*x+9*y^2-3*z*y+z^2, 1], [3*x+3*y+z, 1]]\]
\[[2] A=(a+b+c+d)^2; \]
\[a^2+(2*b+2*c+2*d)*a+b^2+(2*c+2*d)*b+c^2+2*d*c+d^2\]
\[[3] \text{fctr}(A); \]
\[[[1, 1], [a+b+c+d+2]]\]
\[[4] A=(x+1)*(x^2-y^2)^2; \]
\[x^5+x^4-2*y^2*x^3-2*y^2*x^2+y^4*x+y^4\]
\[[5] \text{sqfr}(A); \]
\[[[1, 1], [x+1, 1], [-x^2+y^2, 2]]\]
\[[6] \text{fctr}(A); \]
\[[[1, 1], [x+1, 1], [-x-y, 2], [x-y, 2]]\]

**References**
Section 6.3.16 [ufctrhint], page 55.

**6.3.16 ufctrhint**

**ufctrhint(poly, hint)**
:: Factorizes uni-variate polynomial poly over the rational number field when the degrees of its factors are known to be some integer multiples of hint.
return list

poly uni-variate polynomial with rational coefficients

hint non-negative integer

- By any reason, if the degree of all the irreducible factors of poly is known to be some multiples of hint, factors can be computed more efficiently by the knowledge than fctr().
- When hint is 1, ufctrhint() is the same as fctr() for uni-variate polynomials. An typical application where ufctrhint() is effective: Consider the case where poly is a norm (Chapter 9 [Algebraic numbers], page 156) of a certain polynomial over an extension field with its extension degree d, and it is square free; Then, every irreducible factor has a degree that is a multiple of d.

```
[10] A=t^9-15*t^6-87*t^3-125;
t^9-15*t^6-87*t^3-125
0msec
[11] N=res(t,subst(A,t,x-2*t),A);
-x^81+1215*x^78-567405*x^75+139519665*x^72-19360343142*x^69
+1720634125410*x^66-88249977024390*x^63-4856095669551930*x^60
+1999385245240571421*x^57-15579689952590251515*x^54
+15956967531741971462865*x^51
...
+1403955687720353973535526123612661444550659875*x^6
+10122324287343155430042768923500799484375*x^3
+139262743444047310133459021182733314453125
980msec + gc : 250msec
[12] sqfr(N);
[[-1,1],[x^9-405*x^6-63423*x^3-2460375,1],
[x^18-486*x^15+98739*x^12-9316620*x^9+945468531*x^6-12368049246*x^3
+296607516309,1],[x^18-8667*x^12+19842651*x^6+19683,1],
[x^18-324*x^15+44469*x^12-1180980*x^9+427455711*x^6+2793253896*x^3
+31524548679,1],
[x^18+10773*x^12+2784051*x^6+307546875,1]]
20msec
[13] fctr(N);
[[-1,1],[x^9-405*x^6-63423*x^3-2460375,1],
[x^18-486*x^15+98739*x^12-9316620*x^9+945468531*x^6-12368049246*x^3
+296607516309,1],[x^18-8667*x^12+19842651*x^6+19683,1],
[x^18-324*x^15+44469*x^12-1180980*x^9+427455711*x^6+2793253896*x^3
+31524548679,1],
[x^18+10773*x^12+2784051*x^6+307546875,1]]
167.050sec + gc : 1.890sec
[14] ufctrhint(N,9);
[[-1,1],[x^9-405*x^6-63423*x^3-2460375,1],
[x^18-486*x^15+98739*x^12-9316620*x^9+945468531*x^6-12368049246*x^3
+296607516309,1],[x^18-8667*x^12+19842651*x^6+19683,1],
[x^18-324*x^15+44469*x^12-1180980*x^9+427455711*x^6+2793253896*x^3
+31524548679,1],
[x^18+10773*x^12+2784051*x^6+307546875,1]]
```
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119.340sec + gc : 1.300sec

References

Section 6.3.15 [fctr sqfr], page 54.

6.3.17 modfctr

modfctr(poly,mod)
:: Factorizer over small finite fields

return list

poly Polynomial with integer coefficients

mod non-negative integer

- This function factorizes a polynomial poly over the finite prime field of characteristic mod, where mod must be smaller than $2^{29}$.
- The result is represented by a list, whose elements are a pair represented as $[\text{num},1],[\text{factor},\text{multiplicity}],\ldots$.
- Products of all factor$^\text{multiplicity}$ and num is equal to poly.
- To factorize polynomials over large finite fields, use fctr_ff (see Chapter 10 [Finite fields], page 170, Section 10.5.16 [fctr_ff], page 180).

[0] modfctr(x^10+x^2+1,2147483647);
[[1,1],[x+1513477736,1],[x+2055628767,1],[x+91854880,1],
[x+634005911,1],[x+1513477735,1],[x+634005912,1],
[x^4+1759639395*x^2+2045307031,1]]

[1] modfctr(2*x^6+(y^2+z*y)*x^4+2*z*y^3*x^2+(2*z^2*y^2+z^3*y)*x+z^4,3);
[[2,1],[2*x^3+z*y*x+z^2,1],[2*x^3+y^2*x+2*z^2,1]]

References

Section 6.3.15 [fctr sqfr], page 54.

6.3.18 ptozp

ptozp(poly)
:: Converts a polynomial poly with rational coefficients into an integral polynomial such that GCD of all its coefficients is 1.

return polynomial

poly polynomial

- Converts the given polynomial by multiplying some rational number into an integral polynomial such that GCD of all its coefficients is 1.
- In general, operations on polynomials can be performed faster for integer coefficients than for rational number coefficients. Therefore, this function is conveniently used to improve efficiency.
- Function red does not convert rational coefficients of the numerator. You cannot obtain an integral polynomial by direct use of the function nm(). The function nm() returns the numerator of its argument, and a polynomial with rational coefficients is the numerator of itself and will be returned as it is.
• When the option factor is set, the return value is a list \([g,c]\). Here, \(c\) is a rational number, \(g\) is an integral polynomial and \(poly = c*g\) holds.

\[
\begin{align*}
[0] & \text{ptozp}(2*x+5/3); \\
& 6*x+5 \\
[1] & \text{nm}(2*x+5/3); \\
& 2*x+5/3
\end{align*}
\]

References
Section 6.1.11 [nm dn], page 39.

6.3.19 \texttt{prim, cont}

\texttt{prim(poly [,v])} \\
\hspace{1em} ::\ Primitive part of \(poly\).

\texttt{cont(poly [,v])} \\
\hspace{1em} ::\ Content of \(poly\).

\begin{align*}
\text{return } poly
\end{align*}

\(v\) indeterminate

• The primitive part and the content of a polynomial \(poly\) with respect to its main variable (\(v\) if specified).

\[
\begin{align*}
[0] & E=(y-z)*(x+y)*(x-z)*(2*x-y); \\
& (2*y-2*z)*x^3+(y^2-3*z*y+2*z^2)*x^2+(-y^3+z^2*y)*x+z*y^3-z^2*y^2 \\
[1] & \text{prim}(E); \\
& 2*x^3+(y-2*z)*x^2+(-y^2-z*y)*x+z*y^2 \\
[2] & \text{cont}(E); \\
& y-z \\
[3] & \text{prim}(E,z); \\
& (y-z)*x-z*y+z^2
\end{align*}
\]

References
Section 6.3.1 [var], page 46, Section 6.3.7 [ord], page 49.

6.3.20 \texttt{gcd, gcdz}

\texttt{gcd(poly1,poly2 [,mod])} \\
\hspace{1em} ::\ The polynomial greatest common divisor of \(poly1\) and \(poly2\).

\texttt{gcdz(poly1,poly2)} \\
\hspace{1em} ::\ The polynomial greatest common divisor of \(poly1\) and \(poly2\).

\begin{align*}
\text{return} & \quad \text{polynomial} \\
\text{poly1} & \quad \text{polynomial} \\
\text{poly2} & \quad \text{polynomial} \\
\text{mod} & \quad \text{prime}
\end{align*}

• Functions \texttt{gcd()} and \texttt{gcdz()} return the greatest common divisor (GCD) of the given two polynomials.
- Function `gcd()` returns an integral polynomial GCD over the rational number field. The coefficients are normalized such that their GCD is 1. It returns 1 in case that the given polynomials are mutually prime.
- Function `gcdz()` works for arguments of integral polynomials, and returns a polynomial GCD over the integer ring, that is, it returns `gcd()` multiplied by the contents of all coefficients of the two input polynomials.
- `gcd()` computes the GCD over GF(`mod`) if `mod` is specified.
- Polynomial GCD is computed by an improved algorithm based on Extended Zassenhaus algorithm.
- GCD over a finite field is computed by PRS algorithm and it may not be efficient for large inputs and co-prime inputs.

```
[0] gcd(12*(x^2+2*x+1)^2,18*(x^2+(y+1)*x+y)^3);
  x^3+3*x^2+3*x+1
[1] gcdz(12*(x^2+2*x+1)^2,18*(x^2+(y+1)*x+y)^3);
  6*x^3+18*x^2+18*x+6
[2] gcd((x+y)*(x-y)^2,(x+y)^2*(x-y));
  x^2-y^2
[3] gcd((x+y)*(x-y)^2,(x+y)^2*(x-y),2);
  x^3+y*x^2+y^2*x+y^3
```

References

Section 6.1.3 [igcd igcdcntl], page 36.

6.3.21 red

`red(rat)` :: Reduced form of `rat` by canceling common divisors.

```
return rational expression
rat rational expression
```

- **Asir** automatically performs cancellation of common divisors of rational numbers. But, without an explicit command, it does not cancel common polynomial divisors of rational expressions. (Reduction of rational expressions to a common denominator will be always done.) Use command `red()` to perform this cancellation.
- Cancel the common divisors of the numerator and the denominator of a rational expression `rat` by computing their GCD.
- The denominator polynomial of the result is an integral polynomial which has no common divisors in its coefficients, while the numerator may have rational coefficients.
- Since GCD computation is a very hard operation, it is desirable to detect and remove by any means common divisors as far as possible. Furthermore, a call to this function after swelling of the denominator and the numerator shall usually take a very long time. Therefore, often, to some extent, reduction of common divisors is inevitable for operations of rational expressions.

```
[0] (x^3-1)/(x-1);
  (x^3-1)/(x-1)
[1] red((x^3-1)/(x-1));
  x^2+x+1
```
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[2] \( \text{red}\left(\frac{x^3+y^3+z^3-3x*y*z}{x+y+z}\right) \);
\( x^2+(-y-z)*x+y^2-z*y+z^2 \)

[3] \( \text{red}\left(\frac{3x*y+3}{12x^2+21y^3*x}\right) \);
\( \frac{y}{4*x+7*y^3} \)

[4] \( \text{red}\left(\frac{3/4x^2+5/6x}{2y*x+4/3x}\right) \);
\( \frac{9/8*x+5/4}{3*y+2} \)

References

Section 6.1.11 [nm dn], page 39, Section 6.3.20 [gcd gcdz], page 58,
Section 6.3.18 [ptozp], page 57.

6.4 Univariate polynomials

6.4.1 \texttt{umul}, \texttt{umul_ff}, \texttt{usquare}, \texttt{usquare_ff}, \texttt{utmul}, \texttt{utmul_ff}

\begin{verbatim}
\texttt{umul}(p1,p2)
\texttt{umul_ff}(p1,p2)
  :: Fast multiplication of univariate polynomials
\texttt{usquare}(p1)
\texttt{usquare_ff}(p1)
  :: Fast squaring of a univariate polynomial
\texttt{utmul}(p1,p2,d)
\texttt{utmul_ff}(p1,p2,d)
  :: Fast multiplication of univariate polynomials with truncation
\end{verbatim}

\textit{return} univariate polynomial
\begin{verbatim}
p1 p2 univariate polynomial
d non-negative integer
\end{verbatim}

- These functions compute products of univariate polynomials by selecting an appropriate
  algorithm depending on the degrees of inputs.
- \texttt{umul()}, \texttt{usquare()}, \texttt{utmul()} compute products over the integers. Coefficients in \(\text{GF}(p)\)
  are regarded as non-negative integers less than \(p\).
- \texttt{umul_ff()}, \texttt{usquare_ff()}, \texttt{utmul_ff()} compute products over a finite field. However,
  if some of the coefficients of the inputs are integral, the result may be an integral
  polynomial. So if one wants to assure that the result is a polynomial over the finite
  field, apply \texttt{simp_ff()} to the inputs.
- \texttt{umul_ff()}, \texttt{usquare_ff()}, \texttt{utmul_ff()} cannot take polynomials over \(\text{GF}(2^n)\) as their
  inputs.
- \texttt{umul()}, \texttt{umul_ff()} produce \(p1*p2\). \texttt{usquare()}, \texttt{usquare_ff()} produce \(p1^2\).
- \texttt{utmul()}, \texttt{utmul_ff()} produce \(p1^p2 \mod v^{(d+1)}\), where \(v\) is the variable of \(p1, p2\).
- If the degrees of the inputs are less than or equal to the value returned by \texttt{set_upkara()}\n  (\texttt{set_uptkara()} for \texttt{utmul}, \texttt{utmul_ff}), usual pencil and paper method is used. If the
  degrees of the inputs are less than or equall to the value returned by \texttt{set_upfft()},
  Karatsuba algorithm is used. If the degrees of the inputs exceed it, a combination of
FFT and Chinese remainder theorem is used. First of all sufficiently many primes \( m_i \) within 1 machine word are prepared. Then \( p1*p2 \ mod \ m_i \) is computed by FFT for each \( m_i \). Finally they are combined by Chinese remainder theorem. The functions over finite fields use an improvement by V. Shoup [Shoup].

```
[176] load("fff")$
[177] cputime(1)$
0sec(1.407e-05sec)
[178] setmod_ff(2^160-47);
1461501637330902918203684832716283019655932542929
0sec(0.00028sec)
[179] A=randpoly_ff(100,x)$
0sec(0.001422sec)
[180] B=randpoly_ff(100,x)$
0sec(0.00107sec)
[181] for(I=0;I<100;I++)A*B;
7.77sec + gc : 8.38sec(16.15sec)
[182] for(I=0;I<100;I++)umul(A,B);
2.24sec + gc : 1.52sec(3.767sec)
[183] for(I=0;I<100;I++)umul_ff(A,B);
1.42sec + gc : 0.24sec(1.653sec)
[184] for(I=0;I<100;I++)usquare_ff(A);
1.08sec + gc : 0.21sec(1.297sec)
[185] for(I=0;I<100;I++)utmul_ff(A,B,100);
1.2sec + gc : 0.17sec(1.366sec)
[186] deg(utmul_ff(A,B,100),x); 100
```

References
Section 6.4.3 [set_upkara set_uptkara set_upfft], page 61, Section 6.4.2 [kmul ksquare ktmul], page 61.

6.4.2 kmul, ksquare, ktmul

\texttt{kmul}(p1,p2) \\
:: Fast multiplication of univariate polynomials

\texttt{ksquare}(p1) \\
:: Fast squaring of a univariate polynomial

\texttt{ktmul}(p1,p2,d) \\
:: Fast multiplication of univariate polynomials with truncation

\texttt{return} univariate polynomial

\( p1 \) \( p2 \) univariate polynomial

\( d \) non-negative integer

These functions compute products of univariate polynomials by Karatsuba algorithm.

- These functions do not apply FFT for large degree inputs.
- These functions can compute products over GF(2^n).
6.4.3 set_upkara, set_uptkara, set_upfft

**set_upkara([threshold])**
**set_uptkara([threshold])**
**set_upfft([threshold])**

:: Set thresholds in the selection of an algorithm from $N^2$, Karatsuba, FFT algorithms for univariate polynomial multiplication.

*return* value currently set

*threshold* non-negative integer

- These functions set thresholds in the selection of an algorithm from $N^2$, Karatsuba, FFT algorithms for univariate polynomial multiplication.
- Products of univariate polynomials are computed by $N^2$, Karatsuba, FFT algorithms. The algorithm selection is done according to the degrees of input polynomials and the thresholds.
- See the description of each function for details.

References
Section 6.4.2 [kmul ksquare ktmul], page 61, Section 6.4.1 [umul umul_ff usquare usquare_ff utmul utmul_ff], page 59.

6.4.4 utrunc, udecomp, ureverse

**utrunc(p,d)**
**udecomp(p,d)**
**ureverse(p)**

:: Operations on polynomials

*return* univariate polynomial or list of univariate polynomials

*p* univariate polynomial

*d* non-negative integer

- Let $x$ be the variable of $p$. Then $p$ can be decomposed as $p = pl + x^{-(d+1)}p2$, where the degree of $pl$ is less than or equal to $d$. Under the decomposition, utrunc() returns $pl$ and udecomp() returns $[pl, p2]$.

- Let $e$ be the degree of $p$ and $p[i]$ the coefficient of $p$ at degree $i$. Then ureverse() returns $p[e] + p[e-1]x^+...$
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[132] \texttt{utrunc((x+1)^10,5);}
252x^5+210x^4+120x^3+45x^2+10x+1
[133] \texttt{udecomp((x+1)^10,5);}
[252x^5+210x^4+120x^3+45x^2+10x+1, x^4+10x^3+45x^2+120x+210]
[134] \texttt{ureverse(3*x^3+x^2+2*x);}
2x^2+x+3

References
Section 6.4.6 [udiv urem urembymul urembymul_precomp ugcd], page 63.

6.4.5 \texttt{uinv_as_power_series, ureverse_inv_as_power_series}

\texttt{uinv_as_power_series(p,d)}
\texttt{ureverse_inv_as_power_series(p,d)}
\hspace{1em}:: Computes the truncated inverse as a power series.
\texttt{return} \hspace{1em} \texttt{univariate polynomial}
\texttt{p} \hspace{1em} \texttt{univariate polynomial}
\texttt{d} \hspace{1em} \texttt{non-negative integer}

- For a polynomial \( p \) with a non zero constant term, \texttt{uinv_as_power_series(p,d)} computes a polynomial \( r \) whose degree is at most \( d \) such that \( p*r = 1 \mod x^{(d+1)} \), where \( x \) is the variable of \( p \).
- Let \( e \) be the degree of \( p \). \texttt{ureverse_inv_as_power_series(p,d)} computes \texttt{uinv_as_power_series(p1,d)} for \( p1=ureverse(p,e) \).
- The output of \texttt{ureverse_inv_as_power_series()} can be used as the input of \texttt{rembymul_precomp()}.

[123] \texttt{A=(x+1)^5;}
x^5+5x^4+10x^3+10x^2+5x+1
[124] \texttt{uinv_as_power_series(A,5);}
-126x^5+70x^4-35x^3+15x^2-5x+1
[126] \texttt{A*R;}
-126x^10-560x^9-945x^8-720x^7-210x^6+1
[127] \texttt{A=x^10+x^9;}
x^10+x^9
[128] \texttt{R=ureverse_inv_as_power_series(A,5);}
-x^5+x^4-x^3+x^2-x+1
[129] \texttt{ureverse(A)*R;}
-x^6+1

References
Section 6.4.4 [utrunc udecomp ureverse], page 62, Section 6.4.6 [udiv urem urembymul urembymul_precomp ugcd], page 63.

6.4.6 \texttt{udiv, urem, urembymul, urembymul_precomp, ugcd}

\texttt{udiv(p1,p2)}
\texttt{urem(p1,p2)}
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**urembymul**(p1,p2)
**urembymul_precomp**(p1,p2,inv)
**ugcd**(p1,p2)

:: Division and GCD for univariate polynomials.

return univariate polynomial

p1 p2 inv univariate polynomial

- For univariate polynomials p1 and p2, there exist polynomials q and r such that p1=q*p2+r and the degree of r is less than that of p2. Then udiv returns q, urem and urembymul return r. ugcd returns the polynomial GCD of p1 and p2. These functions are specially tuned up for dense univariate polynomials. In urembymul the division by p2 is replaced with the inverse computation of p2 as a power series and two polynomial multiplications. It speeds up the computation when the degrees of inputs are large.

- **urembymul_precomp** is efficient when one repeats divisions by a fixed polynomial. One has to compute the third argument by **ureverse_inv_as_power_series()**.

```
[177] setmod_ff(2^160-47);
1461501637330902918203684832716283019655932542929
[178] A=randpoly_ff(200,x)$
[179] B=randpoly_ff(101,x)$
[180] cputime(1)$
0sec(1.597e-05sec)
[181] srem(A,B)$
0.15sec + gc : 0.15sec(0.3035sec)
[182] urem(A,B)$
0.11sec + gc : 0.12sec(0.2347sec)
[183] urembymul(A,B)$
0.08sec + gc : 0.09sec(0.1651sec)
[184] R=ureverse_inv_as_power_series(B,101)$
0.04sec + gc : 0.03sec(0.063sec)
[185] urembymul_precomp(A,B,R)$
0.03sec(0.02501sec)
```

References

Section 6.4.5 [uinv_as_power_series, ureverse_inv_as_power_series], page 62.

**6.5 Lists**

**6.5.1 car, cdr, cons, append, reverse, length**

**car**(list) :: The first element of the given non-null list list.

**cdr**(list) :: A list obtained by removing the first element of the given non-null list list.

**cons**(obj,list) :: A list obtained by adding an element obj to the top of the given list list.
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append\((\text{list1}, \text{list2})\)
:: A list obtained by adding all elements in the list list2 according to the order as it is to the last element in the list list1.

reverse\((\text{list})\)
:: reversed list of list.

length\((\text{list}\mid\text{vect})\)
:: Number of elements in a list list and a vector vect.

return car\():\) arbitrary, cdr\(), cons\(), append\(), reverse\():\ list, length\():\ non-negative integer

list list1 list2
list
obj arbitrary

- A list is written in Asir as [obj1, obj2, ...]. Here, obj1 is the first element.
- Function car\() outputs the first element of a non-null list. For a null list, the result should be undefined. In the current implementation, however, it outputs a null list. This treatment for a null list may subject to change in future, and users are suggested not to use the tentative treatment for a null list for serious programming.
- Function cdr\() outputs a list obtained by removing the first element from the input non-null list. For a null list, the result should be undefined. In the current implementation, however, it outputs a null list. This treatment for a null list may subject to change in future, and users are suggested not to use the tentative treatment for a null list for serious programming.
- Function cons\() composes a new list from the input list list and an arbitrary object obj by adding obj to the top of list.
- Function append\() composes a new list, which has all elements of list1 in the same ordering followed by all elements of list2 in the same ordering.
- Function reverse\() returns a reversed list of list.
- Function length\() returns a non-negative integer which is the number of elements in the input list list and the input vector vect. Note that function size should be used for counting elements of matrix.
- Lists are read-only objects in Asir. There elements cannot be modified.
- The n-th element in a list can be referred to by applying the function cdr\() n times repeatedly and cdr\() at last. A more convenient way to access to the n-th element is the use of bracket notation, that is, to attach an index \([n]\) like vectors and matrices. The system, however, follow the n pointers to access the desired element. Subsequently, much time is spent for an element located far from the top of the list.
- Function cdr\() does not create a new cell (a memory quantity). Function append\(), as a matter of fact, repeats cons\() for as many as the length of list1 the first argument. Subsequently, append\() consumes much memory space if its first argument is long. Similar argument applies to function reverse\().

```
[0] L = [[1,2,3],4,[5,6]];
[[1,2,3],4,[5,6]]
```
```lisp
[1] car(L);
[1,2,3]
[2] cdr(L);
[4,[5,6]]
[3] cons(x*y,L);
[y*x,[1,2,3],4,[5,6]]
[4] append([a,b,c],[d]);
[a,b,c,d]
[5] reverse([a,b,c,d]);
[d,c,b,a]
[6] length(L);
3
[7] length(ltov(L));
3
[8] L[2][0];
5

6.6 Arrays

6.6.1 newvect, vector, vect

newvect(len[, list])

vector(len[, list])

:: Creates a new vector object with its length len.

vect([elements])

:: Creates a new vector object by elements.

return vector
len non-negative integer
list list

elements elements of the vector

- vect creates a new vector object by its elements.
- vector is an alias of newvect.
- newvect creates a new vector object with its length len and its elements all cleared to value 0. If the second argument, a list, is given, the vector is initialized by the list elements. Elements are used from the first through the last. If the list is short for initializing the full vector, 0’s are filled in the remaining vector elements.

- Elements are indexed from 0 through len-1. Note that the first element has not index 1.

- List and vector are different types in Asir. Lists are conveniently used for representing many data objects whose size varies dynamically as computation proceeds. By its flexible expressive power, it is also conveniently used to describe initial values for other structured objects as you see for vectors. Access for an element of a list is performed by following pointers to next elements. By this, access costs for list elements differ
for each element. In contrast to lists, vector elements can be accessed in a same time, because they are accessed by computing displacements from the top memory location of the vector object.

Note also, in Asir, modification of an element of a vector causes modification of the whole vector itself, while modification of a list element does not cause the modification of the whole list object.

By this, in Asir language, a vector element designator can be a left value of assignment statement, but a list element designator can NOT be a left value of assignment statement.

- No distinction of column vectors and row vectors in Asir. If a matrix is applied to a vector from left, the vector shall be taken as a column vector, and if from right it shall be taken as a row vector.
- The length (or size or dimension) of a vector is given by function size().
- When a vector is passed to a function as its argument (actual parameter), the vector element can be modified in that function.
- A vector is displayed in a similar format as for a list. Note, however, there is a distinction: Elements of a vector are separated simply by a ‘blank space’, while those of a list by a ‘comma.’

```asir
A=newvect(5);
[0 0 0 0 0 ]
A=newvect(5,[1,2,3,4,[5,6]]);
[ 1 2 3 4 [5,6] ]
A[0];
1
A[4];
[5,6]
size(A);
[5]
length(A);
5
vect(1,2,3,4,[5,6]);
[ 1 2 3 4 [5,6] ]
def afo(V) { V[0] = x; }
afo(A);
A;
[ x 2 3 4 [5,6] ]
```

References
Section 6.6.5 [newmat matrix], page 68, Section 6.6.7 [size], page 70, Section 6.6.2 [ltov], page 67, Section 6.6.3 [vtol], page 67.

### 6.6.2 ltov

**ltov(list)** :: Converts a list into a vector.

```
return vector
list list
```
• Converts a list list into a vector of same length. See also newvect().

[3] A=[1,2,3];
[4] lvto(A);
[ 1 2 3 ]

References
Section 6.6.1 [newvect vector vect], page 65, Section 6.6.3 [vtol], page 67.

6.6.3 vtol

tol(vect)
:: Converts a vector into a list.
return list
vect vector
• Converts a vector vect of length n into a list [vect[0],...vect[n-1]].
• A conversion from a list to a vector is done by newvect().

[3] A=newvect(3,[1,2,3]);
[ 1 2 3 ]
[4] vtol(A);
[1,2,3]

References
Section 6.6.1 [newvect vector vect], page 65, Section 6.6.2 [ltov], page 67.

6.6.4 newbytearray

cnewbytearray(len,[listorstring])
:: Creates a new byte array.
return byte array
len non-negative integer
listorstring list or string
• This function generates a byte array. The specification is similar to that of newvect.
• The initial value can be specified by a character string.
• One can access elements of a byte array just as an array.

[182] A=newbytearray(3);
|00 00 00|
[183] A=newbytearray(3,[1,2,3]);
|01 02 03|
[184] A=newbytearray(3,"abc");
|61 62 63|
[185] A[0];
97
123
A; 

References
Section 6.6.1 [newvect vector vect], page 65.

6.6.5 newmat, matrix

newmat(row, col [,[[a, b, ...],[c, d, ...],...]])
matrix(row, col [,[[a, b, ...],[c, d, ...],...]])

:: Creates a new matrix with row rows and col columns.

return matrix

row col non-negative integer

a b c d arbitrary

- matrix is an alias of newmat.
- If the third argument, a list, is given, the newly created matrix is initialized so that each element of the list (again a list) initializes each of the rows of the matrix. Elements are used from the first through the last. If the list is short, 0’s are filled in the remaining matrix elements. If no third argument is given all the elements are cleared to 0.
- The size of a matrix is given by function size().
- Let M be a program variable assigned to a matrix. Then, M[I] denotes a (row) vector which corresponds with the I-th row of the matrix. Note that the vector shares its element with the original matrix. Subsequently, if an element of the vector is modified, then the corresponding matrix element is also modified.
- When a matrix is passed to a function as its argument (actual parameter), the matrix element can be modified within that function.

[0] A = newmat(3,3,[[1,1,1],[x,y],[x^2]]);
  [ 1 1 1 ]
  [ x y 0 ]
  [ x^2 0 0 ]
[1] det(A);
  -y*x^2
[2] size(A);
  [3,3]
  [ x y 0 ]
getarray : Out of range
return to toplevel

References
Section 6.6.1 [newvect vector vect], page 65, Section 6.6.7 [size], page 70,
Section 6.6.8 [det nd_det invmat], page 70.
6.6.6 \texttt{mat, matr, matc}

\texttt{mat(vector[,...])}

\texttt{matr(vector[,...])}

\hspace{1em}:: \text{Creates a new matrix by list of row vectors.}

\texttt{matc(vector[,...])}

\hspace{1em}:: \text{Creates a new matrix by list of column vectors.}

\text{return} \quad \text{matrix}

\text{vector} \quad \text{array or list}

- \texttt{mat} \text{ is an alias of \texttt{matr}.}
- \text{Each vector has same length. Elements are used from the first through the last. If the list is short, 0's are filled in the remaining matrix elements.}

\begin{verbatim}
[0] matr([1,2,3],[4,5,6],[7,8]);
[1  2  3 ]
[ 4  5  6 ]
[ 7  8  0 ]
[1] matc([1,2,3],[4,5,6],[7,8]);
[ 1  4  7 ]
[ 2  5  8 ]
[ 3  6  0 ]
\end{verbatim}

References

Section 6.6.5 \texttt{newmat matrix}, page 68

6.6.7 \texttt{size}

\texttt{size(vect|mat)}

\hspace{1em}:: \text{A list containing the number of elements of the given vector, \texttt{[size of vect]}, or a list containing row size and column size of the given matrix, \texttt{[row size of mat, column size of mat]}.}

\text{return} \quad \text{list}

\text{vect} \quad \text{vector}

\text{mat} \quad \text{matrix}

- \text{Return a list consisting of the dimension of the vector \texttt{vect}, or a list consisting of the row size and column size of the matrix \texttt{matrix}.}
- \text{Use \texttt{length()} for the size of \texttt{list}, and \texttt{nmono()} for the number of monomials with non-zero coefficients in a rational expression.}

\begin{verbatim}
[0] A = newvect(4);
[ 0 0 0 0 ]
[1] size(A);
[4]
[2] length(A);
4
[3] B = newmat(2,3,[[1,2,3],[4,5,6]]);
\end{verbatim}
Chapter 6: Built-in Function  

[ 1 2 3 ]
[ 4 5 6 ]
[4] size(B);
[2,3]

References

Section 6.5.1 [car cdr cons append reverse length], page 64, Section 6.3.6 [nmono], page 48.

6.6.8 det, nd_det, invmat

\texttt{det(mat[,mod])}

\texttt{nd_det(mat[,mod])}

:: Determinant of \texttt{mat}.

\texttt{invmat(mat)}

:: Inverse matrix of \texttt{mat}.

\texttt{return det: expression, invmat: list}

\texttt{mat matrix}

\texttt{mod prime}

- \texttt{det} and \texttt{nd_det} compute the determinant of matrix \texttt{mat}. \texttt{invmat} computes the inverse matrix of matrix \texttt{mat}. \texttt{invmat} returns a list \texttt{[num,den]}, where \texttt{num} is a matrix and \texttt{num/den} represents the inverse matrix.
- The computation is done over GF(\texttt{mod}) if \texttt{mod} is specified.
- The fraction free Gaussian algorithm is employed. For matrices with multi-variate polynomial entries, minor expansion algorithm sometimes is more efficient than the fraction free Gaussian algorithm.
- \texttt{nd_det} can be used for computing the determinant of a matrix with polynomial entries over the rationals or finite fields. The algorithm is an improved version of the fraction free Gaussian algorithm and it computes the determinant faster than \texttt{det}.

\texttt{[91] A=newmat(5,5)$}
\texttt{[92] V=[x,y,z,u,v];}
\texttt{[x,y,z,u,v]}
\texttt{[93] for(I=0;I<5;I++)for(J=0,B=A[I],W=V[I];J<5;J++)B[J]=W^J;}
\texttt{[94] A;}
\texttt{[ 1 x x^2 x^3 x^4 ]}
\texttt{[ 1 y y^2 y^3 y^4 ]}
\texttt{[ 1 z z^2 z^3 z^4 ]}
\texttt{[ 1 u u^2 u^3 u^4 ]}
\texttt{[ 1 v v^2 v^3 v^4 ]}
\texttt{[95] fctr(det(A));}
\texttt{[[1,1],[u-v,1],[-z+v,1],[-z+u,1],[y-v,1],[y+z,1],[-y+z,1],[-x+u,1],}
\texttt{[-x+z,1],[-x+v,1],[-x+y,1]]}
\texttt{[96] A = newmat(3,3)$}
\texttt{[97] for(I=0;I<3;I++)for(J=0,B=A[I],W=V[I];J<3;J++)B[J]=W^J;}
\texttt{[98] A;}
\texttt{[ 1 x x^2 ]}
invmat(A);
[[ -z*y^2+z^2*y z*x^2-z^2*x -y*x^2+y^2*x 
[ y^2-z^2 -x^2+z^2*x z^2-y^2 
[ -y+z x-z -x+y
, (-y+z)*x^2+(y^2-z^2)*x-y^2+2*y^2+z^2*2]
A*B[0];
[ (-y+z)*x^2+(y^2-z^2)*x-y^2+2*y 0 0 ]
[ 0 (-y+z)*x^2+(y^2-z^2)*x-y^2+2*y 0 ]
[ 0 0 (-y+z)*x^2+(y^2-z^2)*x-y^2+2*y ]
map(red,A*B[0]/B[1]);
[ 1 0 0 ]
[ 0 1 0 ]
[ 0 0 1 ]

References
Section 6.6.5 [newmat matrix], page 68.

6.6.9 qsort

qsort(array[, func])
:: Sorts an array array.
return array (The same as the input; Only the elements are exchanged.)
array array
func function for comparison

• This function sorts an array by quick sort.

• If func is not specified, the built-in comparison function is used and the array is sorted in increasing order.

• If a function of two arguments func which returns 0, 1, or -1 is provided, then an ordering is determined so that A<B if func(A,B)=1 holds, and the array is sorted in increasing order with respect to the ordering.

• The returned array is the same as the input. Only the elements are exchanged.

0 qsort(newvect(10,[1,4,6,7,3,2,9,6,0,-1]));
[ -1 0 1 2 3 4 6 7 9 ]
1 def rev(A,B) { return A>B?-1:(A<B?1:0); }
2 qsort(newvect(10,[1,4,6,7,3,2,9,6,0,-1]),rev);
[ 9 7 6 6 4 3 2 1 0 -1 ]

References
Section 6.3.7 [ord], page 49, Section 6.3.2 [vars], page 46.

6.6.10 rowx, rowm, rowa, colx, colm, cola

rowx(matrix,i,j)
:: Exchanges the i-th and j-th rows.

rowm(matrix,i,c)
:: Multiplies the i-th row by c.
rowa(matrix, i, c)
:: Appends c times the j-th row to the j-th row.

colx(matrix, i, j)
:: Exchanges the i-th and j-th columns.

colm(matrix, i, c)
:: Multiplies the i-th column by c.

cola(matrix, i, c)
:: Appends c times the j-th column to the j-th column.

return matrix

i, j integers

c coefficient

- These operations are destructive for the matrix.

A = newmat(3, 3, [[1, 2, 3], [4, 5, 6], [7, 8, 9]]);

1. rowx(A, 1, 2)$

2. A;

3. rowm(A, 2, x);

4. rowa(A, 0, 1, z);  

References
Section 6.6.5 [newmat matrix], page 68

6.7 Structures

6.7.1 newstruct

newstruct(name)
:: Creates a new structure object whose name is name.

return structure

name string

- This function creates an new structure object whose name is name.
- A structure named name should be defined in advance.
Each member of a structure is specified by its name using the operator \( \rightarrow \). If the specified member is also a structure, the specification by \( \rightarrow \) can be nested.

[0] struct list {h,t};
0
[1] A=newstruct(list);
{0,0}
[2] A->t = newstruct(list);
{0,0}
{0,0}
[4] A->h = 1;
1
[5] A->t->h = 2;
2
[6] A->t->t = 3;
3
[7] A;
{1,2,3}

References

Section 6.7.2 [arfreg], page 74, Section 4.2.9 [structure definition], page 25

6.7.2 arfreg

arfreg(name, add, sub, mul, div, pwr, chsgn, comp)
:: Registers a set of fundamental operations for a type of structure.

return 1
name string
add sub mul div pwr chsgn comp
user defined functions

- This function registers a set of fundamental operations for a type of structure whose name is name.
- The specification of each function is as follows.

add(A,B) A+B
sub(A,B) A-B
mul(A,B) A*B
div(A,B) A/B
pwr(A,B) A\(^{B}\)
chsgn(A) -A
comp(A,B)
    1,0,-1 according to the result of a comparison between A and B.

% cat test
struct a {id,body}
def add(A,B)
{
    C = newstruct(a);
    C->id = A->id; C->body = A->body+B->body;
    return C;
}

def sub(A,B)
{
    C = newstruct(a);
    C->id = A->id; C->body = A->body-B->body;
    return C;
}

def mul(A,B)
{
    C = newstruct(a);
    C->id = A->id; C->body = A->body*B->body;
    return C;
}

def div(A,B)
{
    C = newstruct(a);
    C->id = A->id; C->body = A->body/B->body;
    return C;
}

def pwr(A,B)
{
    C = newstruct(a);
    C->id = A->id; C->body = A->body^B;
    return C;
}

def chsgn(A)
{
    C = newstruct(a);
    C->id = A->id; C->body = -A->body;
    return C;
}

def comp(A,B)
{
    if ( A->body > B->body )
        return 1;
    else if ( A->body < B->body )
        return -1;
else
    return 0;
}

arfreg("a",add,sub,mul,div,pwr,chsgn,comp)
end$
% asir
This is Risa/Asir, Version 20000908.
Copyright (C) FUJITSU LABORATORIES LIMITED.
1994-2000. All rights reserved.
[0] load("./test")$
[11] A=newstruct(a);
{0,0}
[12] B=newstruct(a);
{0,0}
3
[14] B->body = 4;
4
{0,12}

References
    Section 6.7.1 [newstruct], page 73, Section 4.2.9 [structure definition], page 25

6.7.3 struct_type

struct_type(name|object)
    :: Get an identity number of the structure of object and name.

return an integer
name string
object a structure

- struct_type() returns an identity number of the structure or -1 (if an error occurs).

[10] struct list {h,t};
0
[11] A=newstruct(list);
{0,0}
[12] struct_type(A);
3
[13] struct_type("list");
3

References
    Section 6.7.1 [newstruct], page 73, Section 4.2.9 [structure definition], page 25
6.8 Types

6.8.1 type

\texttt{type(obj)} :: Returns an integer which identifies the type of the object \texttt{obj} in question.

\texttt{return} integer

\texttt{obj} arbitrary

- Current assignment of integers for object types is listed below.
  0 0
  1 number
  2 polynomial (not number)
  3 rational expression (not polynomial)
  4 list
  5 vector
  6 matrix
  7 string
  8 structure
  9 distributed polynomial
  10 32bit unsigned integer
  11 error object
  12 matrix over GF(2)
  13 MATHCAP object
  14 first order formula
  -1 VOID object

- For further classification of \texttt{number}, use \texttt{ntype()}. For further classification of \texttt{variable}, use \texttt{vtype()}. 

References

Section 6.8.2 [\texttt{ntype}], page 77, Section 6.8.3 [\texttt{vtype}], page 78.

6.8.2 ntype

\texttt{ntype(num)} :: Classifier of type \texttt{num}. Returns a sub-type number, an integer, for \texttt{obj}.

\texttt{return} integer

\texttt{obj} number

- Sub-types for type number are listed below.
0 rational number
1 floating double (double precision floating point number)
2 algebraic number over rational number field
3 arbitrary precision floating point number (bigfloat)
4 complex number
5 element of a finite field
6 element of a large finite prime field
7 element of a finite field of characteristic 2

- When arithmetic operations for numbers are performed, type coercion will be taken if their number sub-types are different so that the object having smaller sub-type number will be transformed to match the other object, except for algebraic numbers.
- A number object created by `newalg(x^2+1)` and the unit of imaginary number `@i` have different number sub-types, and it is treated independently.
- See Chapter 9 [Algebraic numbers], page 156 for algebraic numbers.

```plaintext
[0] [10/37, ntype(10/37)];
[10/37, 0]
[1] [10.0/37.0, ntype(10.0/37.0)];
[0.27027, 1]
[2] [newalg(x^2+1)+1, ntype(newalg(x^2+1)+1)];
[(#0+1), 2]
[3] [eval(sin(@pi/6)), ntype(eval(sin(@pi/6)))];
[0.49999999999999999991, 3]
[4] [@i+1, ntype(@i+1)];
[(1+1*@i), 4]
```

References
Section 6.8.1 [type], page 76.

6.8.3 vtype

```
vtype(var) :: Type of indeterminates var.
return integer
var indeterminate
```

- Classify indeterminates into sub-types by giving an integer value as follows. For details See Section 3.3 [Types of indeterminates], page 16.

```
0 ordinary indeterminate, which can be directly typed in on a keyboard (a,b,x,afo,bfo,...,etc.)
1 Special indeterminate, created by uc() (_0, _1, _2, ... etc.)
2 function form (sin(x), log(a+1), acosh(1), @pi, @e, ... etc.)
3 functor (built-in functor name, user defined functor, functor for the elementary functions) : sin, log, ... etc
```
• Note: An input ‘a();’ will cause an error, but it changes the system database for identifiers. After this error, you will find ‘vtype(a)’ will result 3. (Identifier a is registered as a user defined functor).

• Usually @pi and @e are treated as indeterminates, whereas they are treated as numbers within functions eval() and pari().

References
Section 6.8.1 [type], page 76, Section 6.8.2 [ntype], page 77, Section 6.3.3 [uc], page 47.

6.9 Operations on functions

6.9.1 call

call(name, args)
:: Call the function name with args.

return a return value of name().

name indefinite (function name)

args a list of arguments

• See Section 6.8.3 [vtype], page 78 for function form.

[0] A=igcd;
igcd
[1] call(A,[4,6]);
2
[2] (*A)(4,6);
2

References
Section 6.8.3 [vtype], page 78.

6.9.2 functor, args, funargs

functor(func)
:: Functor of function form func.

args(func)
:: List of arguments of function form func.

funargs(func)
:: cons(functor(func),args(func)).

return functor(): indeterminate, args(), funargs(): list

func function form

• See Section 6.8.3 [vtype], page 78 for function form.

• Extract the functor and the arguments of function form func.
• Assign a program variable, say $F$, to the functor obtained by $\text{functor()}$. Then, you can type $(\ast F)(x)$ (or $(\ast F)(x,y,...)$ depending on the arity,) to input a function form with argument $x$.

```
[0] functor(sin(x));
  sin
[0] args(sin(x));
  [x]
[0] funargs(sin(3*cos(y)));  ;
  [sin,3*cos(y)]
[1] for (L=[sin,cos,tan];L!=[];L=cdr(L)) {A=car(L);
  print(eval((*A)(@pi/3)));}
0.86602540349122136831
0.5000000002
1.7320508058
```

References
Section 6.8.3 [vtype], page 78.

### 6.10 Strings

#### 6.10.1 rtostr

**rtostr(obj)**

 ::= Convert $obj$ into a string.

**return** string

**obj** arbitrary

• Convert an arbitrary object $obj$ into a string.

• This function is convenient to create variables with numbered (or indexed) names by converting integers into strings and appending them to some name strings.

• Use $\text{strtov()}$ for inverse conversion from string to indeterminate.

```
[0] A=afo;
  afo
[1] type(A);
  2
[2] B=rtostr(A);
  afo
[3] type(B);
  7
[4] B+"1";
  afo1
```

References
Section 6.10.2 [strtov], page 80, Section 6.8.1 [type], page 76.
6.10.2 `strtov`

`strtov(str)`

:: Convert a string `str` into an indeterminate.

```
return intedeterminate
```

`str` string which is valid to constitute an indeterminate.

- Convert a string that is valid for an indeterminate into an indeterminate which have `str` as its print name.
- The valid string for an indeterminate is such a string that begins with a small alphabetical letter possibly followed by any string composed of alphabetical letters, digits or a symbol `_`.
- Use the command to create indeterminates dynamically in programs.

```
[0] A="afo";
afo [1] for (I=0;I<3;I++) {B=strtov(A+rtostr(I)); print([B,type(B)]);}
[afo0,2] [afo1,2] [afo2,2]
```

References
---
Section 6.10.1 [rtostr], page 80, Section 6.8.1 [type], page 76, Section 6.3.3 [uc], page 47.

6.10.3 `eval_str`

`eval_str(str)`

:: Evaluates a string `str`.

```
return object
```

`str` string which can be accepted by `Asir` parser

- This function evaluates a string which can be accepted by `Asir` parser and returns the result.
- The input string should represent an expression.
- This functions is the inversion function of `rtostr()`.

```
[0] eval_str("1+2");
3 [1] fctr(eval_str(rtostr((x+y)^10))); [[1,1],[x+y,10]]
```

References
---
Section 6.10.1 [rtostr], page 80

6.10.4 `strtoascii`, `asciitostr`

`strtoascii(str)`

:: Converts a string into a sequence of ASCII codes.
asciitostr(list)
  :: Converts a sequence of ASCII codes into a string.
return strtoascii():list; asciitostr():string
str
list
  list containing positive integers less than 256.
• strtoascii() converts a string into a list of integers which is a representation of the string by the ASCII code.
• asciitostr() is the inverse of asciitostr().
  [0] strtoascii("abcxyz");
  [97,98,99,120,121,122]
  [1] asciitostr(0);
  abcvxyz
  [2] asciitostr([256]);
  asciitostr : argument out of range
  return to toplevel

6.10.5 str_len, str_chr, sub_str

str_len(str)
  :: Returns the length of a string.
str_chr(str,start,c)
  :: Returns the position of the first occurrence of a character in a string.
sub_str(str,start,end)
  :: Returns a substring of a string.
return str_len(), str_chr():integer; sub_str():string
str c
  string
start end
  non-negative integer
• str_len() returns the length of a string.
• str_chr() scans a string str from the start-th character and returns the position of the first occurrence of the first character of a string c. Note that the top of a string is the 0-th character. It returns -1 if the character does not appear.
• sub_str() generates a substring of str containing characters from the start-th one to the end-th one.
  [185] Line="123 456 (x+y)^3";
  123 456 (x+y)^3
  [186] Sp1 = str_chr(Line,0," ");
  3
  [187] D0 = eval_str(sub_str(Line,0,Sp1-1));
  123
  [188] Sp2 = str_chr(Line,Sp1+1," ");
  7
  [189] D1 = eval_str(sub_str(Line,Sp1+1,Sp2-1));
  456
6.11.1 end, quit

```plaintext
end, quit
```

: Close the currently reading file. At the top level, terminate the Asir session.

- These two functions take no arguments. These functions can be called without a ‘()’. Either function close the current input file. This means the termination of the Asir session at the top level.

- An input file will be automatically closed if it is read to its end. However, if no end$ is written at the last of the input file, the control will be returned to the top level and Asir will be waiting for an input without any prompting. Thus, in order to avoid confusion, putting a end$ at the last line of the input file is strongly recommended.

```plaintext
[6] quit;
```

% References

Section 6.11.2 [load], page 83.

6.11.2 load

```plaintext
load("filename")
```

:: Reads a program file filename.

```plaintext
return (1|0)
```

filename file (path) name

- See Chapter 4 [User language Asir], page 18 for practical programming. Since text files are read through cpp, the user can use, as in C programs, `#include` and `#define` in Asir program source codes. The cpp which is installed to the system with a C compiler is used in the unix version. The mcpp http://mcpp.sourceforge.net is used in the Windows version. Note that the length of a line has a limit for an input mcpp. The OpenXM/bin/ox_cpp is used in the unix/OpenXM version (including cfep/asir for MacOS X). This is the cpp distributed with the Portable C compiler http://pcc.ludd.ltu.se.

- It returns 1 if the designated file exists, 0 otherwise.

- If the filename begins with ‘/’, it is understood as an absolute path name; with ‘.’, relative path name from current directory; otherwise, the file is searched first from directories assigned to an environmental variable ASIRLOADPATH, then if the search ends up in failure, the standard library directory (or directories assigned to ASIR_LIBDIR) shall be searched. On Windows, get_rootdir()\lib is searched if ASIR_LIBDIR is not set.
• We recommend to write an end command at the last line of your program. If not, Asir will not give you a prompt after it will have executed load command. (Escape with an interrupt character (Section 2.7 [Interruption], page 8), if you have lost yourself.) Even in such a situation, Asir itself is still ready to read keyboard inputs as usual. It is, however, embarrassing and may cause other errors. Therefore, to put an end$ at the last line is desirable. (Command end; will work as well, but it also returns and displays verbose.)

• On Windows one has to use ‘/’ as the separator of directory names.

References
Section 6.11.1 [end quit], page 82, Section 6.11.3 [which], page 83, Section 6.14.16 [get_rootdir], page 100.

6.11.3 which

which("filename")
:: This returns the path name for the filename which load() will read.

return path name

filename filename (path name) or 0

• This function searches directory trees according to the same procedure as load() will do. Then, returns a string, the path name to the file if the named file exists; 0 unless otherwise.

• For details of searching procedure, refer to the description about load().

• On Windows one has to use ‘/’ as the separator of directory names.

[0] which("gr");
./gb/gr
[1] which("/usr/local/lib/gr");
0
[2] which("/usr/local/lib/asir/gr");
/usr/local/lib/asir/gr

References
Section 6.11.2 [load], page 83.

6.11.4 output

output(["filename"]):: Writes the return values and prompt onto file filename.

return 1

filename filename

• Standard output stream of Asir is redirected to the specified file. While Asir is writing its outputs onto a file, no outputs, except for keyboard inputs and some of error messages, are written onto the standard output. (You cannot see the result on the display.)
To direct the Asir outputs to the standard output, issue the command without argument, i.e., `output()`.

If the specified file already exists, new outputs will be added to the tail of the file. If not, a file is newly created and the outputs will be written onto the file.

When file name is specified without double quotes (""), or when protected file is specified, an error occurs and the system returns to the top level.

If you want to write inputs from the keyboard onto the file as well as Asir outputs, put command `ctrl("echo",1)`, and then redirect the standard output to your desired file.

Contents which are written onto the standard error output, CPU time etc., are not written onto the file.

Reading and writing algebraic expressions which contain neither functional forms nor unknown coefficients (`vtype()` References) are performed more efficiently, with respect to both time and space, by `bload()` and `bsave()`.

On Windows one has to use ‘/’ as the separator of directory names.

```latex
[83] \text{output("afo");}
\text{fctr(x^2-y^2);}
\text{print("afo");}
\text{output();}
1
[87] \text{quit;}
\% cat afo
1
[84] [[1,1],[x+y,1],[x-y,1]]
[85] \text{afo}
0
[86]
```

References
Section 6.14.1 [ctrl], page 91, Section 6.11.5 [bsave bload], page 85.

6.11.5 `bsave`, `bload`

`bsave(obj,"filename")`
:: This function writes `obj` onto `filename` in binary form.

`bload("filename")`
:: This function reads an expression from `filename` in binary form.

`return bsave() : 1, bload() : the expression read`

`obj` arbitrary expression which does not contain neither function forms nor unknown coefficients.

`filename` filename

- Function `bsave()` writes an object onto a file in its internal form (not exact internal form but very similar). Function `bload()` reads the expression from files which is written by `bsave()`. Current implementation support arbitrary expressions, including lists,
arrays (i.e., vectors and matrices), except for function forms and unknown coefficients
(vtype() References.)
• The parser is activated to retrieve expressions written by output(), whereas internal
forms are directly reconstructed by bload() from the bsave()’ed object in the file.
The latter is much more efficient with respect to both time and space.
• It may happen that the variable ordering at reading is changed from that at writing. In
such a case, the variable ordering in the internal expression is automatically rearranged
according to the current variable ordering.
• On Windows one has to use ‘/’ as the separator of directory names.

1. \[ A=(x+y+z+u+v+w)^{20} \]
2. \[ bsave(A,"afo"); \]
3. \[ B = bload("afo"); \]
4. \[ A == B; \]
5. \[ X=(x+y)^{2}; \]
6. \[ X^2+2*y*x+y^2 \]
7. \[ bsave(X,"afo"); \]
8. \[ quit; \]

\[
\text{[0]} \text{ ord([y,x])}\$ \\
\text{[1]} \text{ bload("afo");} \\
\text{y^2+2*x*y+x^2}
\]

\section*{References}
Section 6.11.4 \[\text{output}]\text{, page 84.}

\subsection{6.11.6 bload27}
\texttt{bload27("filename")} \\
\texttt{return expression read} \\
\texttt{filename filename} \\
• In older versions an arbitrary precision integer is represented as an array of 27bit
integers. In the current version it is represented as an array of 32bit integers. By this
incompatibility the bsaved file created by older versions cannot be read in the current
version by \texttt{bload}. \texttt{bload27} is used to read such files.
• On Windows one has to use ‘/’ as the separator of directory names.

\section*{References}
Section 6.11.5 \[\text{bsave bload}]\text{, page 85.}

\subsection{6.11.7 print}
\texttt{print(obj [,nI])} \\
\texttt{:: Displays (or outputs) obj.}
return 0

obj arbitrary

nl flag (arbitrary)

- Displays (or outputs) \textit{obj}.
- It normally adds linefeed code to cause the cursor moving to the next line. If 0 or 2 is given as the second argument, it does not add a linefeed. If the second argument is 0, the output is simply written in the buffer. If the second argument is 2, the output is flushed.
- The return value of this function is 0. If command \texttt{print(rat);} is performed at the top level, first the value of \textit{rat} will be printed, followed by a linefeed, followed by a 0 which is the value of the function and followed by a linefeed and the next prompt. (If the command is terminated by a ‘$’, e.g., \texttt{print(rat)$}, The last 0 will not be printed.)
- Formatted outputs are not currently supported. If one wishes to output multiple objects by a single \texttt{print()} command, use list like \{\textit{obj1}, \ldots\}, which is not so beautiful, but convenient to minimize programming efforts.

\begin{verbatim}
[8] def cat(L) { while ( L != []) { print(car(L),0); L = cdr(L);} print(""); }
[9] cat([xyz,123,"gahaha"])$
xyz123gahaha
\end{verbatim}

6.11.8 access

\texttt{access(file)}
:: testing an existence of \textit{file}.

\texttt{return (110)}

\texttt{file} filename

6.11.9 remove_file

\texttt{remove_file(file)}
:: Delete an file \textit{file}.

\texttt{return 1}

\texttt{file} filename

6.11.10 open_file, close_file, get_line, get_byte, put_byte, purge_ stdin

\texttt{open_file("filename","mode")}
:: Opens \textit{filename} for reading.

\texttt{close_file(num)}
:: Closes the file indicated by a descriptor \textit{num}. 

get_line([num])
:: Reads a line from the file indicated by a descriptor num.

get_byte(num)
:: Reads a byte from the file indicated by a descriptor num.

put_byte(num, c)
:: Writes a byte c to the file indicated by a descriptor num.

purge_stdin()
:: Clears the buffer for the standard input.

return open_file() : integer (fd id); close_file() : 1; get_line() : string; get_byte(), put_byte() : integer

filename file (path) name
mode string
num non-negative integer (file descriptor)

* open_file() opens a file. If mode is not specified, a file is opened for reading. If mode
  is specified, it is used as the mode specification for C standard I/O function fopen().
  For example "w" requests that the file is truncated to zero length or created for writing.
  "a" requests that the file is opened for writing or created if it does not exist. The stream
  pointer is set at the end of the file. If successful, it returns a non-negative integer as the
  file descriptor. Otherwise the system error function is called. Unnecessary files should
  be closed by close_file(). If the special file name unix://stdin or unix://stdout
  or unix://stderr is given, it returns the file descriptor for the standard input or the
  standard output or the standard error stream respectively. The mode argument is
  ignored in this case.

* get_line() reads a line from an opened file and returns the line as a string. If no
  argument is supplied, it reads a line from the standard input.

* get_byte() reads a byte from an opened file and returns the it as an integer.

* put_byte() writes a byte from an opened file and returns the the byte as an integer.

* A get_line() call after reading the end of file returns an integer 0.

* Strings can be converted into internal forms with string manipulation functions such
  as sub_str(), eval_str().

* purge_stdin() clears the buffer for the standard input. When a function receives a
  character string from get_line(), this functions should be called in advance in order
  to avoid an incorrect behavior which is caused by the characters already exists in the
  buffer.

  [185] Id = open_file("test");
  0
  [186] get_line(Id);
  12345

  [187] get_line(Id);
  67890
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[188] get_line(Id);
0
[189] type(00);
0
[190] close_file(Id);
1
[191] open_file("test");
1
[192] get_line(1);
12345

[193] get_byte(1);
54 /* the ASCII code of '6' */
[194] get_line(1);
7890 /* the rest of the last line */
[195] def test() { return get_line(); }
[196] def test1() { purge_stdin(); return get_line(); }
[197] test();
   /* a remaining newline character has been read */
   /* returns immediately */
[198] test1();
123; /* input from a keyboard */
123; /* returned value */

References
Section 6.10.3 [eval_str], page 81, Section 6.10.5 [str_len str_chr sub_str], page 82.

6.12 Operations for modules

6.12.1 module_list

module_list()
:: Get the list of loaded modules.
return The list of loaded modules.

[1040] module_list();
[gr,primdec,bfct,sml,gnuplot,tigers,phc]

References
See Section 4.2.13 [module], page 28.

6.12.2 module_definedp

module_definedp(name)
:: Testing an existense of the module name.
return (10)

name a module name

- If the module name exists, then module_definedp returns 1. otherwise 0.

    [100] module_definedp("gr");
    1

References
Section 6.12.1 [module_list], page 89, See Section 4.2.13 [module], page 28.

6.12.3 remove_module

remove_module(name)
:: Remove the module name.

return (110)
name a module name

- [100] remove_module("gr");
  1

References
See Section 4.2.13 [module], page 28.

6.13 Numerical functions

6.13.1 dacos, dasin, datan, dcos, dsin, dtan

dacos(num)
:: Get the value of Arccos of num.

dasin(num)
:: Get the value of Arcsin of num.

datan(num)
:: Get the value of Arctan of num.

dcos(num)
:: Get the value of cos of num.

dsin(num)
:: Get the value of sin of num.

dtan(num)
:: Get the value of tan of num.

return floating point number

num number

- Compute numerical values of trigonometric functions.
These functions use the standard mathematical library of C language. So results depend on operating systems and a C compilers.

\[ 4 \times \text{d Satan}(1); \]
\[ 3.14159 \]

6.13.2 \text{dabs, dexp, dlog, dsqrt}

\text{dabs}(\text{num})
:: Get the absolute value of \text{num}.

\text{dexp}(\text{num})
:: Get the value of exponent of \text{num}.

\text{dlog}(\text{num})
:: Get the value of logarithm of \text{num}.

\text{dsqrt}(\text{num})
:: Get the value of square root of \text{num}.

\text{return} \quad \text{floating point number}
\text{num} \quad \text{number}

- Compute numerical values of elementary functions.
- These functions use the standard mathematical library of C language. So results depend on operating systems and a C compilers.

\[ 0 \times \text{dexp}(1); \]
\[ 2.71828 \]

6.13.3 \text{ceil, floor, rint, dceil, dfloor, drint}

\text{ceil}(\text{num})
\text{dceil}(\text{num})
:: Get the ceiling integer of \text{num}.

\text{floor}(\text{num})
\text{dfloor}(\text{num})
:: Get the floor integer of \text{num}.

\text{rint}(\text{num})
\text{drint}(\text{num})
:: Get the round integer of \text{num}.

\text{return} \quad \text{integer}
\text{num} \quad \text{number}

\[ 0 \times \text{dceil}(1.1); \]
\[ 1 \]
6.14 Miscellaneous

6.14.1 ctrl

```
ctrl("switch", [obj])
:: Sets the value of switch.
```

```
return   value of switch
switch   switch name
obj      parameter
```

- This function is used to set or to get the values of switches. The switches are used to control an execution of Asir.
- If `obj` is not specified, the value of `switch` is returned.
- If `obj` is specified, the value of `switch` is set to `obj`.
- Switches are specified by strings, namely, enclosed by two double quotes.
- Here are the switches of Asir.

**cputime** If ‘on’, CPU time and GC time is displayed at every top level evaluation of Asir command; if ‘off’, not displayed. See Section 6.14.6 [cputime tstart tstop], page 95. (The switch is also set by command `cputime(1)`, and reset by `cputime(0)`.)

**nez** Selection for EZGCD algorithm. It is set to 1 by default. Ordinary users need not change this setting.

**echo** If ‘on’, inputs from the standard input will be echoed onto the standard output. When executing to load a file, the contents of the file will be written onto the standard output. If ‘off’, the inputs will not be echoed. This command will be useful when used with command `output`.

**bigfloat** If ‘on’, floating operations will be done by PARI system with arbitrary precision floating point operations. Default precision is set to 9 digits. To change the precision, use command `setbprec setprec`. If ‘off’, floating operations will be done by Asir’s own floating operation routines with a fixed precision operations of standard floating double.

**adj** Sets the frequency of garbage collection. A rational number greater than or equal to 1 can be specified. The default value is 3. If a value closer to 1 is specified, larger heap is allocated and as a result, the frequency of garbage collection decreases. See Section 2.4 [Command line options], page 5.

**verbose** If ‘on’ a warning messages is displayed when a function is redefined.

**quiet_mode** If 1 is set, the copyright notice has been displayed at boot time.

**prompt** If the value is 0, then prompt is not output. If the value is 1, then the standard prompt is output. Asir prompt can be customized by giving a C-style format string. Example (for unix asir):

```
ctrl("prompt", \\
033[32m[\%d]:= \033[0m")
```
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hex
If 1 is set, integers are displayed as hexadecimal numbers with prefix 0x.
if -1 is set, hexadecimal numbers are displayed with ‘|’ inserted at every 8 hexadecimal digits.

real_digit
Sets the number of digits used to print a floating double.

double_output
If set to 1, any floating double is printed in the style ddd.ddd.

fortran_output
If ‘on’ polynomials are displayed in FORTRAN style. That is, a power is represented by ‘**’ instead of ‘^’. The default value is ‘off’.

ox_batch
If ‘on’, the OpenXM send buffer is flushed only when the buffer is full. If ‘off’, the buffer is always flushed at each sending of data or command. The default value is ‘off’. See Chapter 7 [Distributed computation], page 102.

ox_check
If ‘on’ the check by mathcap is done before sending data. The default value is ‘on’. See Chapter 7 [Distributed computation], page 102.

ox_exchange_mathcap
If ‘on’ Asir forces the exchange of mathcaps at the communication startup. The default value is ‘on’. See Chapter 7 [Distributed computation], page 102.

debug_window
When 1, a debug window pops up. (The default value is 0.)

fake_ox_reset
When 1, the function ox_reset sends ox_shutdown instead of ox_reset to servers which do not implement ox_reset protocol. It is checked by the optional argument (the fourth argument) containing the string "no_ox_reset" of mathcap if ox_reset is implemented or not. When you use ox_pari, it is recommended to set this flag 1. (The default value is 0.) See Chapter 7 [Distributed computation], page 102.

References
Section 6.14.6 [cputime tstart tstop], page 95, Section 6.11.4 [output], page 84, Section 6.1.14 [pari], page 41, Section 6.1.15 [setbprec setprec], page 42, Section 6.1.13 [eval deval], page 40.

6.14.2 debug

dep
:: Forces to enter into debugging mode.

Function debug is a function with no argument. It can be called without ‘()’.
• In the debug-mode, you are prompted by (debug) and the debugger is ready for commands. Typing in quit (Note! without a semicolon.) brings you to exit the debug-mode.
• See Chapter 5 [Debugger], page 31 for details.
6.14.3 error

\texttt{error(message)}
:: Forces \texttt{Asir} to cause an error and enter debugging mode.

\texttt{message} \hspace{1em} \texttt{string}

- When \texttt{Asir} encounters a serious error such that it finds difficult to continue execution, it, in general, tries to enter debugging mode before it returns to top level. The command \texttt{error()} forces a similar behavior in a user program.
- The argument is a string which will be displayed when \texttt{error()} will be executed.
- You can enter the debug-mode when your program encounters an illegal value for a program variable, if you have written the program so as to call \texttt{error()} upon finding such an error in your program text.

\% cat mod3
\begin{verbatim}
def mod3(A) {
  if ( type(A) >= 2 )
    error("invalid argument");
  else
    return A \% 3;
}
\end{verbatim}

\% asir
[0] load("mod3");
1
[3] mod3(5);
2
[4] mod3(x);
invalid argument
stopped in mod3 at line 3 in file "/mod3"
3 error("invalid argument");
(debug) print A
A = x
(debug) quit
return to toplevel
[4]

References

Section 6.14.2 [debug], page 93.

6.14.4 help

\texttt{help(["function"])}
:: Displays the description of function \texttt{function}. 
return 0

function function name

- If invoked without argument, it displays rough usage of Asir.
- If a function name is given and if there exists a file with the same name in the directory `help` under standard library directory, the file is displayed by a command set to the environmental variable PAGER or else command `more`.
- If the LANG environment variable is set and its value begins with "japan" or "ja_JP", then the file in `help-ja` is displayed. If its value does not begin with "japan" or "ja_JP", then the file in `help-en` is displayed.
- On Windows HTML-style help is available from the menu.

6.14.5 time

time() :: Returns a four element list consisting of total CPU time, GC time, the elapsed time and also total memory quantities requested from the start of current Asir session.

return list

- These are commands regarding CPU time and GC time.
- The GC time is the time regarded to spent by the garbage collector, and the CPU time is the time defined by subtracting the GC time from the total time consumed by command Asir. Their unit is ‘second.’
- Command time() returns total CPU time and GC time measured from the start of current Asir session. It also returns the elapsed time. Time unit is ‘second.’ Moreover, it returns total memory quantities in words (usually 4 bytes) which are requested to the memory manager from the beginning of the current session. The return value is a list and the format is [CPU time, GC time, Memory, Elapsed time].
- You can find the CPU time and GC time for some computation by taking the difference of the figure reported by time() at the beginning and the ending of the computation.
- Since arbitrary precision integers are NOT used for counting the total amount of memory request, the number will eventually happen to become meaningless due to integer overflow.
- When cputime switch is active by ctr1() or by cputime(), the execution time will be displayed after every evaluation of top level statement. In a program, however, in order to know the execution time for a sequence of computations, you have to use time() command, for an example.
- On UNIX, if getrusage() is available, time() reports reliable figures. On Windows NT it also gives reliable CPU time. However, on Windows 95/98, the reported time is nothing but the elapsed time of the real world. Therefore, the time elapsed in the debug-mode and the time of waiting for a reply to interruption prompting are added to the elapsed time.

[72] T0=time();
[2.390885,0.484358,46560,9.157768]
[73] G=hgr(katsura(4),[u4,u3,u2,u1,u0],2)$
[74] T1=time();
\[
\text{CPU}, 6.577163, \text{GC}, 7.221549
\]

References
Section 6.14.6 [cputime tstart tstop], page 95, Section 6.14.8 [currenttime], page 97.

6.14.6 cputime, tstart, tstop

cputime(onoff)
:: Stop displaying cputime if its argument is 0, otherwise start displaying cputime after every top level evaluation of Asir command.

tstart() :: Resets and starts timer for CPU time and GC time.

tstop() :: Stops timer and then displays CPU time GC time elapsed from the last time when timer was started.

return 0

onoff flag (arbitrary)

- Command cputime() with NON-ZERO argument enables Asir to display CPU time and GC time after every evaluation of top level Asir command. The command with argument 0 disables displaying them.
- Command tstart() starts measuring CPU time and GC time without arguments. The parentheses ‘()’ may be omitted.
- Command tstop() stops measuring CPU time and GC time and displays them without arguments. The parentheses ‘()’ may be omitted.
- Command cputime(onoff) has same meaning as ctrl("cputime", onoff).
- Nested use of tstart() and tstop() is not expected. If such an effect is desired, use time().
- On and off states by cputime() have effects only to displaying mode. Time for evaluation of every top level statement is always measured. Therefore, even after a computation has already started, you can let Asir display the timings, whenever you enter the debug-mode and execute cputime(1).

\[
\text{tstart}$
\]

\[
\text{tstop}$
\]

80msec + gc : 40msec

References
6.14.7 timer

timer(interval, expr, val)
  :: Compute an expression under the interval timer.

return result
interval interval (second)
expr expression to be computed
val a value to be returned when the timer is expired

- timer() computes an expression under the interval timer. If the computation finishes within the specified interval, it returns the result of the computation. Otherwise it returns the third argument.
- The third argument should be distinguishable from the result on success.

[0] load("cyclic");
  1
[10] timer(10, dp_gr_main(cyclic(7), [c0,c1,c2,c3,c4,c5,c6],1,1,0),0);
  interval timer expired (VTALRM)
  0
[11]

6.14.8 currenttime

currenttime()
  :: Get current time.

return UNIX time.

- See also time(3) in UNIX manuals.

[0] currenttime();
  1071639228
[1]

6.14.9 sleep

sleep(interval)
  :: Suspend computation for an interval.

return 1
interval interval (micro second)

- See also usleep(3) in UNIX manuals.

[0] sleep(1000);
  1
[1]
6.14.10 heap

heap() :: Heap area size currently in use.
return non-negative integer

- Command heap() returns an integer which is the byte size of current Asir heap area. Heap is a memory area where various data for expressions and user programs of Asir and is managed by the garbage collector. While Asir is running, size of the heap is monotonously non-decreasing against the time elapsed. If it happens to exceed the real memory size, most (real world) time is consumed for swapping between real memory and disk memory.

- For a platform with little real memory, it is recommended to set up Asir configuration tuned for GC functions by -adj option at the activation of Asir. (See Section 2.4 [Command line options], page 5.)

```
% asir -adj 16
[0] load("fctrdatan")$
0
[97] cputime(1)$
0msec
[98] heap();
524288
0msec
[99] fctr(Wang[8])$
3.190sec + gc : 3.420sec
[100] heap();
1118208
0msec
[101] quit;
% asir
[0] load("fctrdatan")$
0
[97] cputime(1)$
0msec
[98] heap();
827392
0msec
[99] fctr(Wang[8])$
3.000sec + gc : 1.180sec
[100] heap();
1626112
0msec
[101] quit;
```

References
Section 2.4 [Command line options], page 5.

6.14.11 version
version()
:: Version identification number of Asir.

return integer

- Command version() returns the version identification number, an integer of Asir in use.

[0] version();
991214

6.14.12 shell

shell(command)
:: Execute shell commands described by a string command.

return integer
command string

Execute shell commands described by a string command by a C function system(). This returns the exit status of shell as its return value.

[0] shell("ls");
alp das katsura ralg suit
alg defs.h kimura ratint test
alpi edet kimura3 robot texput.log
asir.o fee mfee sasa wang
asir_symtab gr mksym shira wang_data
base gr.h mp snf1 wt
bgk help msubst solve
chou hom p sp
const ifplot proot strum
 cyclic is r sugar
0
[1]

6.14.13 map

map(function, arg0, arg1, ...)  
:: Applies a function to each member of a list or an array.

return an object of the same type as arg0.

function the name of a function
arg0 list, vector or matrix
arg1 ... arbitrary (the rest of arguments)

- Returns an object of the same type as arg0. Each member of the returned object is the return value of a function call where the first argument is the member of arg0 corresponding to the member in the returned object and the rest of the argument are arg1, . . . .
- function is a function name itself without ‘"’. 
• A program variable cannot be used as \textit{function}.
• If \textit{arg0} is neither list nor array this function simply returns the value of \texttt{function(arg0,arg1,...)}.

\begin{verbatim}
[82] def afo(X) { return X^3; }
[83] map(afo,[1,2,3]);
[1,8,27]
\end{verbatim}

\subsection*{6.14.14 \texttt{flist}}

\texttt{flist([mname])}

:: Returns the list of function names currently defined.

\begin{description}
\item \texttt{mname} \quad a module name
\item \texttt{return} \quad list of character strings
\end{description}

Returns the list of names of built-in functions and user defined functions currently defined. The return value is a list of character strings. If a module name is specified, only the names of functions defined in the module are returned.

• The names of built-in functions are followed by those of user defined functions.

\begin{verbatim}
[77] flist();
[defpoly,newalg,mainalg,algtorat,rattoalg,getalg,alg,algv,...]
\end{verbatim}

\subsection*{6.14.15 \texttt{delete\_history}}

\texttt{delete\_history([index])}

:: Deletes the history.

\begin{description}
\item \texttt{return} \quad 0
\item \texttt{index} \quad Index of history to be deleted.
\end{description}

• Deletes all the histories without an argument.
• Deletes the history with index \texttt{index} if specified.
• A history is an expression which has been obtained by evaluating an input given for a prompt with an index. It can be taken out by \texttt{@index}, which means that the expression survives garbage collections.
• A large history may do harm in the subsequent memory management and deleting the history by \texttt{delete\_history()}, after saving it in a file by \texttt{bsave()}, is often effective.

\begin{verbatim}
[0] (x+y+z)^100$
[1] @0;
...
[2] delete\_history(0);
[3] @0;
0
\end{verbatim}

\subsection*{6.14.16 \texttt{get\_rootdir}}

\texttt{get\_rootdir()}

:: Gets the name of \texttt{Asir} root directory.
return  string
- On UNIX it returns the value of an environment variable `ASIR_LIBDIR` or `/usr/local/lib/asir` if `ASIR_LIBDIR` is not set.
- On Windows the name of Asir root directory is returned.
- By using relative path names from the value of this function, one can write programs which contain file operations independent of the install directory.

6.14.17 getopt

getopt([key])
:: Returns the value of an option.

return  object
- When a user defined function is called, the number of arguments must be equal to that in the declaration of the function. A function with indefinite number of arguments can be realized by using options (see Section 4.2.12 [option], page 27). The value of a specified option is retrieved by getopt.
- If getopt() is called with no argument, then it returns a list `[[key1,value1], [key2,value2], ...]`. In the list, each key is an option which was specified when the function executing getopt was invoked, and value is the value of the option.
- If an option key is specified upon a function call, getopt return the value of the option. If such an option is not specified, the it returns an object of VOID type whose object identifier is -1. By examining the type of the returned value with type(), one knows whether the option is set or not.
- Options are specified as follows:
  `xxx(A,B,C,D|x=X,y=Y,z=Z)`
  That is, the options are specified by a sequence of key=value seperated by ',', after `|`.

References
Section 4.2.12 [option], page 27, Section 6.8.1 [type], page 76.

6.14.18 getenv

getenv(name)
:: Returns the value of an environment variable.

return  string
- Returns the value of an environment variable name.

    [0] getenv("HOME");
    /home/pcrf/noro
7 Distributed computation

7.1 OpenXM

On Asir distributed computations are done under OpenXM (Open message eXchange protocol for Mathematics), which is a protocol for exchanging mainly mathematical objects between processes. See http://www.math.sci.kobe-u.ac.jp/OpenXM/ for the details of OpenXM. In OpenXM a distributed computation is done as follows:

1. A client requests something to a server.
2. The server does works according to the request.
3. The client requests to send data to the server.
4. The server sends the data to the client and the client gets the data.

The server is a stack machine. That is data objects sent by the client are pushed to the stack of the server. If the server gets a command, then the data are popped form the stack and they are used as arguments of a function call.

In OpenXM, the result of a computation done in the server is simply pushed to the stack and the data is not written to the communication stream without requests from the client.

OpenXM protocol consists of two components: CMO (Common Mathematical Object format) which determines a common format of data representations and SM (StackMachine command) which specifies actions on servers. These are wrapped as OX expressions to indicate the sort of data when they are sent.

To execute a distributed computation by OpenXM, one has to invoke OpenXM servers and to establish communications between the client and the servers. ox_launch(), ox_launch_nox(), ox_launch_generic() are prepared for such purposes. Furthermore the following functions are available.

ox_push_cmo()
It requests a server to push an object to the stack of a server.

ox_pop_cmo()
It request a server to pop an object from the stack of a server.

ox_cmo_rpc()
It requests to execute a function on a server. The result is pushed to the stack of the server.

ox_execute_string()
It requests a server to parse and execute a string by the parser and the evaluator of the server. The result is pushed to the stack of the server.

ox_push_cmd()
It requests a server to execute a command.

ox_get()
It gets an object from a data stream.
7.2 Mathcap

A server or a client does not necessarily implement full specifications of OpenXM. If a program sends data unknown to its peer, an unrecoverable error may occur. To avoid such a case OpenXM provides a scheme not to send data unknown to peers. It is realized by exchanging the list of supported CMO and SM. The list is called mathcap. Mathcap is also defined as a CMO and the elements are 32bit integers or strings. The format of mathcap is as follows.

```
[[version number, server name], SMtaglist, [OXtag, CMOtaglist], [OXtag, CMOtaglist], ...]
```

[OXtag, CMOtaglist] indicates that available object tags for a category of data specified by OXtag. For example `ox_asir` accepts the local object format used by Asir and the mathcap from `ox_asir` reflects the fact.

If "ox_check" switch of ctrl is set to 1, the check by a mathcap is done before data is sent. If "ox_check" switch of ctrl is set to 0, the check is not done. By default it is set to 1.

7.3 Stackmachine commands

The stackmachine commands are provided to request a server to execute various operations. They are automatically sent by built-in functions of Asir, but one often has to send them manually. They are represented by 32bit integers. One can send them by calling ox_push_cmd(). Typical stackmachine commands are as follows. SM.xxx=yyy means that SM.xxx is a mnemonic and that yyy is its value.

**SM_popSerializedLocalObject=258**
An object not necessarily defined as CMO is popped from the stack and is sent to the client. This is available only on `ox_asir`.

**SM_popCMO=262**
A CMO object is popped from the stack and is sent to the client.

**SM_popString=263**
An object is popped from the stack and is sent to the client as a readable string.

**SM_mathcap=264**
The server’s mathcap is pushed to the stack.

**SM_pops=265**
Objects are removed from the stack. The number of object to be removed is specified by the object at the top of the stack.

**SM_setName=266**
A variable name is popped form the stack. Then an object is popped and it is assigned to the variable. This assignment is done by the local language of the server.

**SM_evalName=267**
A variable name is popped from the stack. Then the value of the variable is pushed to the stack.
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**SM** executeStringByLocalParser=268
A string popped from the stack is parsed and evaluated. The result is pushed to the stack.

**SM** executeFunction=269
A function name, the number of arguments and the arguments are popped from the stack. Then the function is executed and the result is pushed to the stack.

**SM** beginBlock=270
It indicates the beginning of a block.

**SM** endBlock=271
It indicates the end of a block.

**SM** shutdown=272
It shuts down communications and terminates servers.

**SM** setMathcap=273
It requests a server to register the data at the top of the stack as the client’s mathcap.

**SM** getsp=275
The number of objects in the current stack is pushed to the stack.

**SM** dupErrors=276
The list of all the error objects in the current stack is pushed to the stack.

**SM** nop=300
Nothing is done.

### 7.4 Debugging

In general, it is difficult to debug distributed computations. `ox_asir` provides several functions for debugging.

#### 7.4.1 Error object

When an error has occurred on an OpenXM server, an error object is pushed to the stack instead of a result of the computation. The error object consists of the serial number of the SM command which caused the error, and an error message.

```sh
[340] ox_launch();
0
[341] ox_rpc(0,"fctr",1.2*x);
0
[342] ox_pop_cmo(0);
error([8,fctrp : invalid argument])
```

#### 7.4.2 Resetting a server

`ox_reset()` resets a process whose identifier is number. After its execution the process is ready for receiving data. This function corresponds to the keyboard interrupt on an usual Asir session. It often happens that a request of a client does not correspond correctly to the
result from a server. It is caused by remaining data on data streams. `ox_reset` is effective for such cases.

### 7.4.3 Pop-up command window for debugging

As a server does not have any standard input device such as a keyboard, it is difficult to debug user programs running on the server. `ox_asir` pops up a small command window to input debug commands when an error has occurred during user program execution or \( \text{ox_rpc}(id,"\text{debug}) \) has been executed. The responses to commands are shown in `xterm` to display standard outputs from the server. To close the small window, input `quit`.

### 7.5 Functions for distributed computation

#### 7.5.1 `ox_launch`, `ox_launch_nox`, `ox_shutdown`

- `ox_launch([host[,dir],command])`
- `ox_launch_nox([host[,dir],command])`
  :: Initialize OpenXM servers.
- `ox_shutdown(id)`
  :: Terminates OpenXM servers.

```latex
\begin{align*}
\text{return} & \quad \text{integer} \\
\text{host} & \quad \text{string or 0} \\
\text{dir command} & \quad \text{string} \\
\text{id} & \quad \text{integer}
\end{align*}
```

- Function `ox_launch()` invokes a process to execute `command` on a host `host` and enables Asir to communicate with that process. If the number of arguments is 3, `ox_launch` in `dir` is invoked on `host`. Then `ox_launch` invokes `command`. If `host` is equal to 0, all the commands are invoked on the same machine as the Asir is running. If no arguments are specified, `host`, `dir` and `command` are regarded as 0, the value of `get_rootdir()` and `ox_asir` in the same directory respectively.

- If `host` is equal to 0, then `dir` can be omitted. In such a case `dir` is regarded as the value of `get_rootdir()`.

- If `command` begins with ‘/’, it is regarded as an absolute pathname. Otherwise it is regarded as a relative pathname from `dir`.

- On UNIX, `ox_launch()` invokes `xterm` to display standard outputs from `command`. If X11 is not available or one wants to invoke servers without `xterm`, use `ox_launch_nox()`, where the outputs of `command` are redirected to `/dev/null`. If the environment variable `DISPLAY` is not set, `ox_launch()` and `ox_launch_nox()` behave identically.

- The returned value is used as the identifier for communication.
The peers communicating with Asir are not necessarily processes running on the same machine. The communication will be successful even if the byte order is different from those of the peer processes, because the byte order for the communication is determined by a negotiation between a client and a server.

The following preparations are necessary. Here, Let A be the host on which Asir is running, and B the host on which the peer process will run.

1. Register the hostname of the host A to the ‘~/.rhosts’ of the host B. That is, you should be allowed to access the host B from A without supplying a password.

2. For cases where connection to X is also used, let Xserver authorize the relevant hosts. Adding the hosts can be done by command xhost.

3. If an environment variable ASIR_RSH is set, the content of this variable is used as a program to invoke remote servers instead of rsh. For example,

   % setenv ASIR_RSH "ssh -f -X -A "

   implies that remote servers are invoked by ‘ssh’ and that X11 forwarding is enabled. See the manual of ‘ssh’ for the detail.

4. Some command’s consume much stack space. You are recommended to set the stack size to about 16MB large in ‘.cshrc’ for safe. To specify the size, put limit stacksize 16m for an example.

   When command opens a window on X, it uses the string specified for display: if the specification is omitted, it uses the value set for the environment variable DISPLAY.

   ox_shutdown() terminates OpenXM servers whose identifier is id.

   When Asir is terminated successfully, all I/O streams are automatically closed, and all the processes invoked are also terminated. However, some remote processes may not terminated when Asir is terminated abnormally. If ever Asir is terminated abnormally, you have to kill all the unterminated process invoked by Asir on every remote host. Check by ps command on the remote hosts to see if such processed are alive.

   ‘xterm’ for displaying the outputs from command is invoked with ‘-name ox_term’ option. Thus, by specifying resources for the resource name ‘ox_term’, only the behaviour of the ‘xterm’ can be customized.

   /* iconify on start */
   ox_xterm*iconic:on
   /* activate the scroll bar */
   ox_xterm*scrollBar:on
   /* 1000 lines can be shown by the scrollbar */
   ox_xterm*saveLines:1000

   ox_launch();
   0
   [219] ox_rpc(0,"fctr",x^10-y^10); 0
   [220] ox_pop_local(0);
   [[1,1],[x^4+y*x^3+y^2*x^2+y^3*x+y^4,1],
   [x^4-y*x^3+y^2*x^2-y^3*x+y^4,1],[x-y,1],[x+y,1]]
   [221] ox_shutdown(0); 0
7.5.2 ox_launch_generic

ox_launch_generic(\textit{host}, \textit{launch}, \textit{server}, \textit{use_unix}, \textit{use_ssh}, \textit{use_x}, \textit{conn_to_serv})

\textbf{::} Initialize OpenXM servers.

\textbf{return} \hspace{1em} \textbf{integer}

\textbf{host} \hspace{1em} \textbf{string or 0}

\textbf{launcher server}

\textbf{string}

\textbf{use_unix} \hspace{1em} \textbf{use_ssh} \hspace{1em} \textbf{use_x} \hspace{1em} \textbf{conn_to_serv}

\textbf{integer}

- \textbf{ox_launch_generic()} invokes a control process \textit{launch} and a server process \textit{server} on \textit{host}. The other arguments are switches for protocol family selection, on/off of the X environment, method of process invocation and selection of connection type.
- If \textit{host} is equal to 0, processes are invoked on the same machine as the \textbf{Asir} is running. In this case UNIX internal protocol is always used.
- If \textit{use_unix} is equal to 1, UNIX internal protocol is used. If \textit{use_unix} is equal to 0, Internet protocol is used.
- If \textit{use_ssh} is equal to 1, ‘\textbf{ssh}’ (Secure Shell) is used to invoke processes. If one does not use ‘\textbf{ssh-agent}’, a password (passphrase) is required. If ‘\textbf{sshd}’ is not running on the target machine, ‘\textbf{rsh}’ is used instead. But it will immediately fail if a password is required.
- If \textit{use_x} is equal to 1, it is assumed that X environment is available. In such a case server is invoked under ‘\textbf{xterm}’ by using the current \textbf{DISPLAY} variable. If \textbf{DISPLAY} is not set, it is invoked without X. Note that the processes will hang up if \textbf{DISPLAY} is incorrectly set.
- If \textit{conn_to_serv} is equal to 1, \textbf{Asir} (client) executes \textbf{bind} and \textbf{listen}, and the invoked processes execute \textbf{connect}. If \textit{conn_to_serv} is equal to 0, \textbf{Asir} (client) the invoked processes execute \textbf{bind} and \textbf{listen}, and the client executes \textbf{connect}.

\begin{verbatim}
[342] LIB=get_rootdir(); /export/home/noro/ca/Kobe/build/OpenXM/lib/asir
[343] ox_launch_generic(0,LIB+"/ox_launch",LIB+"/ox_asir",0,0,0,0);
  1
[344] ox_launch_generic(0,LIB+"/ox_launch",LIB+"/ox_asir",1,0,0,0);
  2
[345] ox_launch_generic(0,LIB+"/ox_launch",LIB+"/ox_asir",1,1,0,0);
  3
[346] ox_launch_generic(0,LIB+"/ox_launch",LIB+"/ox_asir",1,1,1,0);
  4
[347] ox_launch_generic(0,LIB+"/ox_launch",LIB+"/ox_asir",1,1,1,1);
\end{verbatim}


5
[348] ox_launch_generic(0,LIB+"/ox_launch",LIB+"/ox_asir",1,1,0,1);
6

References

Section 7.5.1 [ox_launch ox_launch_nox ox_shutdown], page 105,
Section 7.5.2 [ox_launch_generic], page 107

7.5.3 generate_port, try_bind_listen, try_connect, try_accept, register_server

generate_port([use_unix])
:: Generates a port number.

try_bind_listen(port)
:: Binds and listens on a port.

try_connect(host,port)
:: Connects to a port.

try_accept(socket,port)
:: Accepts a connection request.

register_server(control_socket,control_port,server_socket,server_port)
:: Registers the sockets for which connections are established.

return integer or string for generate_port(), integer for the others

use_unix 0 or 1

host string

port control_port server_port
integer or string

socket control_socket server_socket
integer

• These functions are primitives to establish communications between a client and servers.
• generate_port() generates a port name for communication. If the argument is not
  specified or equal to 0, a port number for Internet domain socket is generated randomly.
  Otherwise a file name for UNIX domain (host-internal protocol) is generated. Note that
  it is not assured that the generated port is not in use.
• try_bind_listen() creates a socket according to the protocol family indicated by the
given port and executes bind and listen. It returns a socket identifier if it is successful.
-1 indicates an error.
• try_connect() tries to connect to a port port on a host host. It returns a socket
  identifier if it is successful. -1 indicates an error.
• try_accept() accepts a connection request to a socket socket. It returns a new socket
  identifier if it is successful. -1 indicates an error. In any case socket is automatically
  closed. port is specified to distinguish the protocol family of socket.
• \texttt{register_server()} registers a pair of a control socket and a server socket. A process identifier indicating the pair is returned. The process identifier is used as an argument of \texttt{ox} functions such as \texttt{ox_push_cmo()}.

• Servers are invoked by using \texttt{shell()}, or manually.

\begin{verbatim}
[340] CPort=generate_port();
39896
[341] SPort=generate_port();
37222
[342] CSocket=try_bind_listen(CPort);
3
[343] SSocket=try_bind_listen(SPort);
5

/*
 ox_launch is invoked here :
% ox_launch "127.1" 0 39716 37043 ox_asir "shio:0"
*/

[344] CSocket=try_accept(CSocket,CPort);
6
[345] SSocket=try_accept(SSocket,SPort);
3
[346] register_server(CSocket,CPort,SSocket,SPort);
0
\end{verbatim}

References

Section 7.5.1 [\texttt{ox_launch} \texttt{ox_launch_nox} \texttt{ox_shutdown}], page 105, Section 7.5.2 [\texttt{ox_launch_generic}], page 107, Section 6.14.12 [\texttt{shell}], page 98, Section 7.5.7 [\texttt{ox_push_cmo} \texttt{ox_push_local}], page 111

7.5.4 ‘\texttt{ox_asir}’

‘\texttt{ox_asir}’ provides almost all the functionalities of \texttt{Asir} as an \texttt{OpenXM} server. ‘\texttt{ox_asir}’ is invoked by \texttt{ox_launch} or \texttt{ox_launch_nox}. If X environment is not available or is not necessary, one can use \texttt{ox_launch_nox}.

\begin{verbatim}
[5] ox_launch();
0

[5] ox_launch_nox("127.0.0.1","/usr/local/lib/asir",
"/usr/local/lib/ox_asir");
0

[7] RemoteLibDir = "/usr/local/lib/asir/"$
[8] Machines = ["sumire","rokkaku","genkotsu","shinpuku"];
[sumire,rokkaku,genkotsu,shinpuku]
[9] Servers = map(ox_launch,Machines,RemoteLibDir,
RemoteLibDir+"ox_asir");
[0,1,2,3]
\end{verbatim}

References

Section 7.5.1 [\texttt{ox_launch} \texttt{ox_launch_nox} \texttt{ox_shutdown}], page 105
### 7.5.5 ox_rpc, ox_cmo_rpc, ox_execute_string

**ox_rpc**

```c
ox_rpc(number,"func",arg0,...)
```

**ox_cmo_rpc**

```c
ox_cmo_rpc(number,"func",arg0,...)
```

**ox_execute_string**

```c
ox_execute_string(number,"command",...)
```

:: Calls a function on an OpenXM server

- **return** 0
- **number** integer (process identifier)
- **func** function name
- **command** string
- **arg0** ... arbitrary (arguments)

- Calls a function on an OpenXM server whose identifier is `number`.
- It returns 0 immediately. It does not wait the termination of the function call.
- `ox_rpc()` can be used when the server is `ox_asir`. Otherwise `ox_cmo_rpc()` should be used.
- The result of the function call is put on the stack of the server. It can be received by `ox_pop_local()` or `ox_pop_cmo()`.
- If the server is not `ox_asir`, only data defined in OpenXM can be sent.
- `ox_execute_string` requests the server to parse and execute `command` by the parser and the evaluator of the server. The result is pushed to the stack.

```c
[234] ox_cmo_rpc(0,"dp_ht",dp_ptod((x+y)^10,[x,y]));
0
[235] ox_pop_cmo(0);
(1)*<<10,0>>
[236] ox_execute_string(0,"12345 % 678;"));
0
[237] ox_pop_cmo(0);
141
```

### References

Section 7.5.8 [ox_pop_cmo ox_pop_local], page 112

### 7.5.6 ox_reset, ox_intr, register_handler

**ox_reset**

```c
ox_reset(number)
```

:: Resets an OpenXM server

**ox_intr**

```c
ox_intr(number)
```

:: Sends SIGINT to an OpenXM server

**register_handler**

```c
register_handler(func)
```

:: Registers a function callable on a keyboard interrupt.

- **return** 1
- **number** integer (process identifier)
**func** functor or 0

- **ox_reset()** resets a process whose identifier is `number`. After its execution the process is ready for receiving data.
- After executing **ox_reset()**, sending/receiving buffers and stream buffers are assured to be empty.
- Even if a process is running, the execution is safely stopped.
- **ox_reset()** may be used prior to a distributed computation. It can be also used to interrupt a distributed computation.
- **ox_intr()** sends SIGINT to a process whose identifier is `number`. The action of a server against SIGINT is not specified in OpenXM. ‘`ox_asir`’ immediately enters the debug mode and pops up an window to input debug commands on X window system.
- **register_handler()** registers a function **func()**. If u is specified on a keyboard interrupt, **func()** is executed before returning the toplevel. If **ox_reset()** calls are included in **func()**, one can automatically reset OpenXM servers on a keyboard interrupt.
- If **func** is equal to 0, the setting is reset.

```plaintext
[10] ox_launch();
0
[11] ox_rpc(0,"fctr",x^100-y^100);
0
[12] ox_reset(0); /* usr1 : return to toplevel by SIGUSR1 */
1 /* is displayed on the xterm. */

[340] Procs=[ox_launch(),ox_launch()];
[0,1]
[341] def reset() { extern Procs; map(ox_reset,Procs);}
[342] map(ox_rpc,Procs,"fctr",x^100-y^100);
[0,0]
[343] register_handler(reset);
1
[344] interrupt ?(q/t/c/d/u/w/?) u
Abort this computation? (y or n) y
Calling the registered exception handler...done.
return to toplevel
```

**References**

Section 7.5.5 [ox_rpc ox_cmo_rpc ox_execute_string], page 110

### 7.5.7 ox_push_cmo, ox_push_local

- **ox_push_cmo**(`number`, `obj`) :: Sends `obj` to a process whose identifier is `number`.
- **ox_push_local**(`number`, `obj`) :: Sends `obj` to a process whose identifier is `number`.
- **return** 0
- **number** integer(process identifier)
- **obj** object
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- `ox_push_cmo` is used to send data to an Open_XM other than `ox_asir` and `ox_plot`.
- `ox_push_local` is used to send data to `ox_asir` and `ox_plot`.
- The call immediately returns unless the stream buffer is full.

References

Section 7.5.5 [ox_rpc ox_cmo_rpc ox_execute_string], page 110,
Section 7.5.8 [ox_pop_cmo ox_pop_local], page 112

7.5.8 ox_pop_cmo, ox_pop_local

```phone

ox_pop_local(number)
:: Receives data from a process whose identifier is number.

return received data
```

- `number` integer(process identifier)
  - Receives data from a process whose identifier is `number`.
  - `ox_pop_cmo` can be used to receive data form an Open_XM server other than `ox_asir` and `ox_plot`.
  - `ox_pop_local` can be used to receive data from `ox_asir`, `ox_plot`.
  - If no data is available, these functions block. To avoid it, send SM_popCMO (262) or SM_popSerializedLocalObject (258). Then check the process status by `ox_select`. Finally call `ox_get` for a ready process.

```
[341] ox_cmo_rpc(0,"fctr",x^2-1);
0
[342] ox_pop_cmo(0);
[[1,1],[x-1,1],[x+1,1]]
[343] ox_cmo_rpc(0,"newvect",3);
0
[344] ox_pop_cmo(0);
error([41,cannot convert to CMO object])
[345] ox_pop_local(0);
[ 0 0 0 ]
```

References

Section 7.5.5 [ox_rpc ox_cmo_rpc ox_execute_string], page 110,
Section 7.5.9 [ox_push_cmd ox_sync], page 112,
Section 7.5.12 [ox_select], page 114,
Section 7.5.10 [ox_get], page 113

7.5.9 ox_push_cmd, ox_sync

```phone

ox_push_cmd(number,command)
:: Sends a command command to a process whose identifier is number.

ox_sync(number)
:: Sends OX_SYNC BALL to a process whose identifier is number.

return 0
```

- `number` integer(process identifier)
command integer(command identifier)

- Sends a command or **OX_SYNC_BALL** to a process whose identifier is *number*.
- Data in **OpenXM** are categorized into three types: **OX_DATA**, **OX_COMMAND**, **OX_SYNC_BALL**. Usually **OX_COMMAND** and **OX_SYNC_BALL** are sent implicitly with high level operations, but these functions are prepared to send these data explicitly.
- **OX_SYNC_BALL** is used on the reseting operation by **ox_reset**. Usually **OX_SYNC_BALL** will be ignored by the peer.

```plaintext
    [3] ox_rpc(0,"fctr",x^100-y^100);
    0
    [4] ox_push_cmd(0,258);
    0
    [5] ox_select([0]);
    [0]
    [6] ox_get(0);
    [[1,1],[x^2+y^2,1],[x^4-y*x^3+y^2*x^2-y^3*x+y^4,1],...]
```

References

Section 7.5.5 [ox_rpc ox_cmo_rpc ox_execute_string], page 110, Section 7.5.6 [ox_reset ox_intr register_handler], page 110

7.5.10 ox_get

**ox_get(number)**

:: Receives data form a process whose identifier is *number*.

**return**

**number** integer(process identifier)

- Receives data form a process whose identifier is *number*.
- One may use this function with **ox_push_cmd**.
- **ox_pop_cmo** and **ox_pop_local** is realized as combinations of **ox_push_cmd** and **ox_get**.

```plaintext
    [11] ox_push_cmo(0,123);
    0
    [12] ox_push_cmd(0,262); /* 262=OX_popCMO */
    0
    [13] ox_get(0);
    123
```

References

Section 7.5.8 [ox_pop_cmo ox_pop_local], page 112, Section 7.5.9 [ox_push_cmd ox_sync], page 112

7.5.11 ox_pops

**ox_pops(number [,nitem])**

:: Removes data form the stack of a process whose identifier is *number*.
return 0

number integer (process identifier)
nitem non-negative integer

- Removes data from the stack of a process whose identifier is number. If nitem is specified, nitem items are removed. If nitem is not specified, 1 item is removed.

```c
[69] for(I=1;I<=10;I++) ox_push_cmo(0,I);
[70] ox_pops(0,4);
[71] ox_pop_cmo(0);
```

References
Section 7.5.8 [ox_pop_cmo ox_pop_local], page 112

7.5.12 ox_select

```c
ox_select(nlist[,timeout])
  :: Returns the list of process identifiers on which data is available.
```

return list

nlist list of integers (process identifier)

timeout number

- Returns the list of process identifiers on which data is available.
- If all the processes in nlist are running, it blocks until one of the processes returns data. If timeout is specified, it waits for only timeout seconds.
- By sending SM_popCMO or SM_popSerializedLocalObject with ox_push_cmd() in advance and by examining the process status with ox_select(), one can avoid a hanging up caused by ox_pop_local() or ox_pop_cmo(). In such a case, data can be received by ox_get().

```c
ox_launch();
0
[220] ox_launch();
1
[221] ox_launch();
2
[222] ox_rpc(2,"fctr",x^500-y^500);
0
[223] ox_rpc(1,"fctr",x^100-y^100);
0
[224] ox_rpc(0,"fctr",x^10-y^10);
0
[225] P=[0,1,2];
[0,1,2]
[226] map(ox_push_cmd,P,258);
[0,0,0]
[227] ox_select(P);
```
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7.5.8 ox_get(0);
[[1,1],[x^4+y*x^3+y^2*x^2+y^3*x+y^4,1],
 [x^4-y*x^3+y^2*x^2-y^3*x+y^4,1],[x-y,1],[x+y,1]]

References
Section 7.5.8 [ox_pop_cmo ox_pop_local], page 112, Section 7.5.9 [ox_push_cmd ox_sync], page 112, Section 7.5.10 [ox_get], page 113

7.5.13 ox_flush

ox_flush(id)
:: Flushes the sending buffer.

return 1

id process identifier

- By default the batch mode is off and the sending buffer is flushed at every sending operation of data and command.
- The batch mode is set by "ox_batch" switch of "ctrl".
- If one wants to send many pieces of small data, ctrl("ox_batch",1) may decrease the overhead of flush operations. Of course, one has to call ox_flush(id) at the end of the sending operations.
- Functions such as ox_pop_cmo and ox_pop_local enter a waiting mode immediately after sending a command. These functions always flush the sending buffer.

[340] ox_launch_nox();
0
[341] cputime(1);
0
7e-05sec + gc : 4.8e-05sec(0.000119sec)
[342] for(I=0;I<10000;I++)ox_push_cmo(0,I);
0.232sec + gc : 0.006821sec(0.6878sec)
[343] ctrl("ox_batch",1);
1
4.5e-05sec(3.302e-05sec)
[344] for(I=0;I<10000;I++)ox_push_cmo(0,I); ox_flush(0);
0.08063sec + gc : 0.06388sec(0.4408sec)
[345] 1
9.6e-05sec(0.01317sec)

References
Section 7.5.8 [ox_pop_cmo ox_pop_local], page 112, Section 6.14.1 [ctrl], page 91

7.5.14 ox_get_serverinfo

ox_get_serverinfo([id])
:: Gets server's mathcap and process id.

return list
id   process identifier

- If id is specified, the mathcap of the process whose identifier is id is returned.
- If id is not specified, the list of [id, Mathcap] is returned, where id is the identifier of a currently active process, and Mathcap is the mathcap of the process. Identifier id is returned.

```c
[344] ox_get_serverinfo();
[[0, [[199909080, Ox_system=ox_sm1.plain, Version=2.991118, HOSTTYPE=FreeBSD],
[262, 263, 264, 265, 266, 268, 269, 272, 273, 275, 276],
[[514], [213076434, 1, 2, 4, 5, 17, 19, 20, 22, 23, 24, 25, 26, 30, 31, 60, 61, 27, 33, 40, 16, 34]]]]
```

References
Section 7.2 [Mathcap], page 103.

### 7.5.15 ifplot, conplot, plot, polarplot, plotover

**ifplot**

```c
ifplot(func [, geometry] [, xrange] [, yrange] [, id] [, name])
:: Displays real zeros of a bi-variate function.
```

**conplot**

```c
conplot(func [, geometry] [, xrange] [, yrange] [, zrange] [, id] [, name])
:: Displays real contour lines of a bi-variate function.
```

**plot**

```c
plot(func [, geometry] [, xrange] [, id] [, name])
:: Displays the graph of a univariate function.
```

**polarplot**

```c
polarplot(func [, geometry] [, thetarange] [, id] [, name])
:: Displays the graph of a curve given in polar form.
```

**plotover**

```c
plotover(func, id, number)
Plots on the existing window real zeros of a bivariate function.
```

**return**

```c
integer
```

**func**

```c
polynomial
```

**geometry xrange yrange zrange**

```c
list
```

**id number**

```c
integer
```

**name**

```c
string
```
Function `ifplot()` draws a graph of real zeros of a bi-variate function. Function `conplot()` plots the contour lines for a same argument. Function `plot()` draws the graph of a univariate function. Function `polarplot()` draws the graph of a curve given in polar form $r=f(\theta)$.

- The plotting functions are realized by an OpenXM server. On UNIX it is `ox_plot` in `Asir` root directory. On Windows `engine` acts as `ox_plot`. Of course, it must be activated by `ox_launch()` `ox_launch_nox()`. If the identifier of an active `ox_plot` is specified as `id`, the server is used for drawing pictures. If `id` is not specified, an available `ox_plot` server is used if it exists. If no `ox_plot` server is available, then `ox_launch_nox()` is automatically executed to invoke `ox_plot`.

- Argument `func` is indispensable. Other arguments are optional. The format of optional arguments and their default values (parenthesized) are listed below.

  - `geometry` Window size is specified by $[x,y]$ in unit `dot`. $[300,300]$ for UNIX version;

  - `xrange yrange` Value ranges of the variables are specified by $[v,v_{\text{min}},v_{\text{max}}]$. $([-2,2]$ for each variable.) If this specification is omitted, the indeterminate having the higher order in `func` is taken for `x` and the one with lower order is taken for `y`. To change this selection, specify explicitly by `xrange` and/or `yrange`. For an uni-variate function, the specification is mandatory.

  - `zrange` This specification applies only to `conplot()`. The format is $[v,v_{\text{min}},v_{\text{max}}[\text{step}]]$. If `step` is specified, the height difference of contours is set to $(v_{\text{max}}-v_{\text{min}})/\text{step}$. ($[-2,2,16]$.)

  - `id` This specifies the number of the remote process by which you wish to draw a graph. (The number for the newest active process.)

  - `name` The name of the window. (Plot.) The created window is titled `name:n/m` which means the $m$-th window of the process with process number $n$. These numbers are used for `plotover()`.

- The maximum number of the windows that can be created on a process is 128.

- Function `plotover()` superposes reals zeros of its argument bi-variate function onto the specified window.

- Enlarged plot can be obtained for rectangular area which is specified, on an already existing window with a graph, by dragging cursor with the left button of mouse from the upper-left corner to lower-right corner and then releasing it. Then, a new window is created whose shape is similar to the specified area and whose size is determined so that the largest side of the new window has the same size of the largest side of the original window. If you wish to cancel the action, drag the cursor to any point above or left of the starting point.

  This facility is effective when `precise` button switch is inactive. If `precise` is selected and active, the area specified by the cursor dragging will be rewritten on the same window. This will be explained later.

- A click of the right button will display the current coordinates of the cursor at the bottom area of the window.
• Place the cursor at any point in the right marker area on a window created by `conplot()`, and drag the cursor with the middle mutton. Then you will find the contour lines changing their colors depending on the movement of the cursor and the corresponding height level displayed on the upper right corner of the window.

• Several operations are available on the window: by button operations for UNIX version, and pull-down menus for Windows version.

  `quit` Destroys (kills) the window. While computing, quit the current computation. If one wants to interrupt the computation, use `ox_reset()`.

  `wide` (toggle)
  Will display, on the same window, a new area enlarged by 10 times as large as the current area for both width-direction and height-direction. The current area will be indicated by a rectangle placed at the center. Area specification by dragging the cursor will create a new window with a plot of the graph in the specified area.

  `precise` (toggle)
  When selected and active, `ox_plot` redraws the specified area more precisely by integer arithmetic. This mode uses bisection method based on Sturm sequence computation to locate real zeros precisely. More precise plotting can be expected by this technique than by the default plotting technique, at the expense of significant increase of computing time. As you see by above explanation, this function is only effective to polynomials with rational coefficients. (Check how they differ for \((x^2+y^2-1)^2\).)

  `formula` Displays the expression for the graph.

  `noaxis` (toggle)
  Erase the coordinates.

• Program `ox_plot` may consume much stack space depending on which machine it is running. You are recommended to set the stack size to about 16MB as large in `.cshrc` for safe. To specify the size, put `limit stacksize 16m` for an example.

• You can customize various resources of a window on X, e.g., coloring, shape of buttons etc. The default setting of resources is shown below. For `plot*form*shapeStyle` you can select among `rectangle`, `oval`, `ellipse`, and `roundedRectangle`.

```plaintext
plot*background:white
plot*form*shapeStyle:rectangle
plot*form*background:white
plot*form*quit*background:white
plot*form*wide*background:white
plot*form*precise*background:white
plot*form=formula*background:white
plot*form=noaxis*background:white
plot*form=xcoord*background:white
plot*form=ycoord*background:white
plot*form=level*background:white
plot*form=xdone*background:white
plot*form=ydone*background:white
```
7.5.16 open_canvas, clear_canvas, draw_obj, draw_string

open_canvas(id [, geometry])
:: Opens a canvas, which is a window for drawing objects.

clear_canvas(id, index)
:: Clears a canvas.

draw_obj(id, index, pointorsegment [, color])
:: Draws a point or a line segment on a canvas.

draw_string(id, index, [x, y], string [, color])
:: Draws a character string on a canvas.

return 0

id index color x y
integer

pointorsegment
integer

string character string

- These functions are supplied by the OpenXM server ‘ox_plot’ (‘engine’ on Windows).
- open_canvas opens a canvas, which is a window for drawing objects. One can specify the size of a canvas in pixel by supplying geometry option [x, y]. The default size is [300,300]. This function pushes an integer value onto the stack of the OpenXM server. The value is used to distinguish the opened canvas and one has to pop and maintain the value by ox_pop_cmo for subsequent calls of draw_obj.
- clear_canvas clears a canvas specified by a server id id and a canvas id index.
- draw_obj draws a point or a line segment on a canvas specified by a server id id and a canvas id index. If pointorsegment is [x, y], it is regarded as a point. If pointorsegment is [x, y, u, v], it is regarded as a line segment which connects [x, y] and [u, v]. If color is specified, color/65536 mod 256, color/256 mod 256, color mod 256 are regarded as the values of Red, Green, Blue (Max. 255) respectively.
- draw_string draws a character string string on a canvas specified by a server id id and a canvas id index. The position of the string is specified by [x, y].

[182] Id=ox_launch_nox(0,"ox_plot");
0
[183] open_canvas(Id);
0
[184] Ind=ox_pop_cmo(Id);
0
[185] draw_obj(Id,Ind,[100,100]);
0
[186] draw_obj(Id,Ind,[200,200],0xffff);
0   
[187] draw_obj(Id,Ind,[10,10,50,50],0xff00ff);
0   
[187] draw_string(Id,Ind,[100,50],"hello",0xffff00);
0   
[189] clear_canvas(Id,Ind);
0

References

Section 7.5.1 [ox_launch ox_launch_nox ox_shutdown], page 105, Section 7.5.6 [ox_reset ox_intr register_handler], page 110, Section 7.5.8 [ox_pop_cmo ox_pop_local], page 112.
Chapter 8: Groebner basis computation

8 Groebner basis computation

8.1 Distributed polynomial

A distributed polynomial is a polynomial with a special internal representation different from the ordinary one.

An ordinary polynomial (having type 2) is internally represented in a format, called recursive representation. In fact, it is represented as an uni-variate polynomial with respect to a fixed variable, called main variable of that polynomial, where the other variables appear in the coefficients which may again polynomials in such variables other than the previous main variable. A polynomial in the coefficients is again represented as an uni-variate polynomial in a certain fixed variable, the main variable. Thus, by this recursive structure of polynomial representation, it is called the ‘recursive representation.’

\[(x + y + z)^2 = 1 \cdot x^2 + (2 \cdot y + (2 \cdot z)) \cdot x + ((2 \cdot z) \cdot y + (1 \cdot z^2))\]

On the other hand, we call a representation the distributed representation of a polynomial, if a polynomial is represented, according to its original meaning, as a sum of monomials, where a monomial is the product of power product of variables and a coefficient. We call a polynomial, represented in such an internal format, a distributed polynomial. (This naming may sounds something strange.)

\[(x + y + z)^2 = 1 \cdot x^2 + 2 \cdot xy + 2 \cdot xz + 1 \cdot y^2 + 2 \cdot yz + 1 \cdot z^2\]

For computation of Groebner basis, efficient operation is expected if polynomials are represented in a distributed representation, because major operations for Groebner basis are performed with respect to monomials. From this view point, we provide the object type distributed polynomial with its object identification number 9, and objects having such a type are available by Asir language.

Here, we provide several definitions for the later description.

term  The power product of variables, i.e., a monomial with coefficient 1. In an Asir session, it is displayed in the form like

\[<<0,1,2,3,4>>\]

and also can be input in such a form. This example shows a term in 5 variables. If we assume the 5 variables as a, b, c, d, and e, the term represents \(b\cdot c^{-2}\cdot d^{-3}\cdot e^{-4}\) in the ordinary expression.

term order  Terms are ordered according to a total order with the following properties.

1. For all \(t \cdot t > 1\).
2. For all \(t, s, u \cdot t > s\) implies \(tu > su\).

Such a total order is called a term ordering. A term ordering is specified by a variable ordering (a list of variables) and a type of term ordering (an integer, a list or a matrix).

monomial  The product of a term and a coefficient. In an Asir session, it is displayed in the form like
2**<0,1,2,3,4>>
and also can be input in such a form.

**head term**
**head monomial**
**head coefficient**

Monomials in a distributed polynomial is sorted by a total order. In such representation, we call the monomial that is maximum with respect to the order the head monomial, and its term and coefficient the head term and the head coefficient respectively.

### 8.2 Reading files

Facilities for computing Groebner bases are `dp_gr_main()`, `dp_gr_mod_main()` and `dp_gr_f_main()`. To call these functions, it is necessary to set several parameters correctly and it is convenient to use a set of interface functions provided in the library file `gr`. The facilities will be ready to use after you load the package by `load()`. The package `gr` is placed in the standard library directory of Asir.

```
[0] load("gr")$
```

### 8.3 Fundamental functions

There are many functions and options defined in the package `gr`. Usually not so many of them are used. Top level functions for Groebner basis computation are the following three functions.

In the following description, `plist`, `vlist`, `order` and `p` stand for a list of polynomials, a list of variables (indeterminates), a type of term ordering and a prime less than $2^{27}$ respectively.

**gr**(plist, vlist, order)
Function that computes Groebner bases over the rationals. The algorithm is Buchberger algorithm with useless pair elimination criteria by Gebauer-Moeller, sugar strategy and trace-lifting by Traverso. For ordinary computation, this function is used.

**hgr**(plist, vlist, order)
After homogenizing the input polynomials a candidate of the `\gr` basis is computed by trace-lifting. Then the candidate is dehomogenized and checked whether it is indeed a Groebner basis of the input. Sugar strategy often causes intermediate coefficient swells. It is empirically known that the combination of homogenization and supresses the swells for such cases.

**gr_mod**(plist, vlist, order, p)
Function that computes Groebner bases over GF(p). The same algorithm as `gr()` is used.
8.4 Controlling Groebner basis computations

One can control a Groebner basis computation by setting various parameters. These parameters can be set and examined by a built-in function `dp_gr_flags()`. Without argument it returns the current settings.

```
[100] dp_gr_flags();

[Demand,0,NoSugar,0,NoCriB,0,NoGC,0,NoMC,0,NoRA,0,NoGCD,0,Top,0,
 ShowMag,1,Print,1,Stat,0,Reverse,0,InterReduce,0,Multiple,0]
```

The return value is a list which contains the names of parameters and their values. The meaning of the parameters are as follows. ‘on’ means that the parameter is not zero.

- **NoSugar** If ‘on’, Buchberger’s normal strategy is used instead of sugar strategy.
- **NoCriB** If ‘on’, criterion B among the Gebauer-Moeller’s criteria is not applied.
- **NoGC** If ‘on’, the check that a Groebner basis candidate is indeed a Groebner basis, is not executed.
- **NoMC** If ‘on’, the check that the resulting polynomials generates the same ideal as the ideal generated by the input, is not executed.
- **NoRA** If ‘on’, the interreduction, which makes the Groebner basis reduced, is not executed.
- **NoGCD** If ‘on’, content removals are not executed during a Groebner basis computation over a rational function field.
- **Top** If ‘on’, Only the head term of the polynomial being reduced is reduced.
- **Reverse** If ‘on’, the selection strategy of reducer in a normal form computation is such that a newer reducer is used first.
- **Print** If ‘on’, various informations during a Groebner basis computation is displayed.
- **PrintShort** If ‘on’ and Print is ‘off’, short information during a Groebner basis computation is displayed.
- **Stat** If ‘on’, a summary of informations is shown after a Groebner basis computation. Note that the summary is always shown if Print is ‘on’.
- **ShowMag** If ‘on’ and Print is ‘on’, the sum of bit length of coefficients of a generated basis element, which we call magnitude, is shown after every normal computation. After completing the computation the maximal value among the sums is shown.
- **Content** If a non-zero rational number, in a normal form computation over the rationals, the integer content of the polynomial being reduced is removed when its magnitude becomes `Content` times larger than a registered value, which is set to the magnitude of the input polynomial. After each content removal the registered value is set to the magnitude of the resulting polynomial. `Content` is equal to 1, the simplification is done after every normal form computation. It is empirically known that it is often efficient to set `Content` to 2 for the case
where large integers appear during the computation. An integer value can be set by the keyword **Multiple** for backward compatibility.

**Demand**

If the value (a character string) is a valid directory name, then generated basis elements are put in the directory and are loaded on demand during normal form computations. Each elements is saved in the binary form and its name coincides with the index internally used in the computation. These binary files are not removed automatically and one should remove them by hand.

If **Print** is ‘on’, the following informations are shown.

```
[93] gr(cyclic(4),[c0,c1,c2,c3],0)$
mod= 99999999, eval = []
(0)(0)<0,2,0,0>>(2,3),nb=2,nab=5,rp=2,sugar=2,mag=4
(0)(0)<0,1,2,0>>(1,2),nb=3,nab=6,rp=2,sugar=3,mag=4
(0)(0)<0,1,1,2>>(0,1),nb=4,nab=7,rp=3,sugar=4,mag=6
(0)(0)<0,0,3,2>>(5,6),nb=5,nab=8,rp=2,sugar=5,mag=4
(0)(0)<0,1,0,4>>(4,6),nb=6,nab=9,rp=3,sugar=5,mag=4
(0)(0)<0,0,2,4>>(6,8),nb=7,nab=10,rp=4,sugar=6,mag=6
....gb done
reduceall
.......
membercheck
(0,0)(0,0)(0,0)(0,0)
gbcheck total 8 pairs
.......
UP=(0,0)SP=(0,0)SPM=(0,0)NF=(0,0)NFM=(0.010002,0)ZNFM=(0.010002,0)
PZ=(0,0)NP=(0,0)MP=(0,0)RA=(0,0)MC=(0,0)GC=(0,0)T=40,B=0 M=8 F=6
D=12 ZR=5 NZR=6 Max_mag=6
[94]
```

In this example **mod** and **eval** indicate moduli used in trace-lifting. **mod** is a prime and **eval** is a list of integers used for evaluation when the ground field is a field of rational functions.

The following information is shown after every normal form computation.

```
(TNF)(TCONT)HT(INDEX),nb=NB,nab=NAB,rp=RP,sugar=S,mag=M
```

Meaning of each component is as follows.

**TNF**

CPU time for normal form computation (second)

**TCONT**

CPU time for content removal(second)

**HT**

Head term of the generated basis element

**INDEX**

Pair of indices which corresponds to the reduced S-polynomial
The summary of the informations shown after a Groebner basis computation is as follows. If a component shows timings and it contains two numbers, they are a pair of time for computation and time for garbage collection.

- **UP**: Time to manipulate the list of critical pairs
- **SP**: Time to compute S-polynomials over the rationals
- **SPM**: Time to compute S-polynomials over a finite field
- **NF**: Time to compute normal forms over the rationals
- **NFM**: Time to compute normal forms over a finite field
- **ZNFM**: Time for zero reductions in NFM
- **PZ**: Time to remove integer contets
- **NP**: Time to compute remainders for coefficients of polynomials with coefficients in the rationals
- **MP**: Time to select pairs from which S-polynomials are computed
- **RA**: Time to interreduce the Groebner basis candidate
- **MC**: Time to check that each input polynomial is a member of the ideal generated by the Groebner basis candidate.
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8.5 Setting term orderings

A term is internally represented as an integer vector whose components are exponents with respect to variables. A variable list specifies the correspondences between variables and components. A type of term ordering specifies a total order for integer vectors. A type of term ordering is represented by an integer, a list of integer or matrices.

There are following three fundamental types.

0 (DegRevLex; total degree reverse lexicographic ordering)  
In general, computation by this ordering shows the fastest speed in most Groebner basis computations. However, for the purpose to solve polynomial equations, this type of ordering is, in general, not so suitable. The Groebner bases obtained by this ordering is used for computing the number of solutions, solving ideal membership problem and seeds for conversion to other Groebner bases under different ordering.

1 (DegLex; total degree lexicographic ordering)  
By this type term ordering, Groebner bases are obtained fairly faster than Lex (lexicographic) ordering, too. Alike the DegRevLex ordering, the result, in general, cannot directly be used for solving polynomial equations. It is used, however, in such a way that a Groebner basis is computed in this ordering after homogenization to obtain the final lexicographic Groebner basis.

2 (Lex; lexicographic ordering)  
Groebner bases computed by this ordering give the most convenient Groebner bases for solving the polynomial equations. The only and serious shortcoming is the enormously long computation time. It is often observed that the number coefficients of the result becomes very very long integers, especially if the ideal is 0-dimensional. For such a case, it is empirically true for many cases that i.e., computation by \texttt{gr()} and/or \texttt{hgr()} may be quite effective.

By combining these fundamental orderingl into a list, one can make various term ordering called elimination orderings.
In this example $O_i$ indicates 0, 1 or 2 and $L_i$ indicates the number of variables subject to the corresponding orderings. This specification means the following.

The variable list is separated into sub lists from left to right where the $i$-th list contains $L_i$ members and it corresponds to the ordering of type $O_i$. The result of a comparison is equal to that for the leftmost different sub components. This type of ordering is called an elimination ordering.

Furthermore one can specify a term ordering by a matrix. Suppose that a real $n \times m$ matrix $M$ has the following properties.

1. For all integer vectors $v$ of length $m$ $Mv = 0$ is equivalent to $v = 0$.
2. For all non-negative integer vectors $v$ the first non-zero component of $Mv$ is non-negative.

Then we can define a term ordering such that, for two vectors $t$, $s$, $t > s$ means that the first non-zero component of $M(t-s)$ is non-negative.

Types of term orderings are used as arguments of functions such as $\text{gr}()$. It is also set internally by $\text{dp\_ord}()$ and is used during executions of various functions.

For concrete definitions of term ordering and more information about Groebner basis, refer to, for example, the book [Becker, Weispfenning].

Note that the variable ordering have strong effects on the computation time as well as the choice of types of term orderings.

$B = [x^{10} - t, x^8 - z, x^{31} - x^6 - x - y]$

$\text{gr}(B, [x, y, z, t], 2);$

$x^2 - 2y^7 + (-41 + t^2 - 13 + t - 1)y^2 + (2t^17 - 12t^16 + 42t^14 + 30t^12 + 30t^11 - 6t^9 + 3t^8 + 30t^7 - 12t^6 + 16t^5 - 55t^4 + 55t^3 + 55t^2 + 55t + 55)t^4$,

$(-12 + t^16 + 72t^15 + 30t^14 + 60t^13 + 60t^12 + 60t^11 + 60t^10 + 60t^9 + 60t^8 + 60t^7 + 60t^6 + 60t^5 + 60t^4 + 60t^3 + 60t^2 + 60t + 60)t^3 + 45t^2 + 45t + 45)t^2$,

$(y + t^2 + z^2 + 2)xy + 7 + (20t^2 + 6 + t^1) + y^2 + (-17 + 6 + t^14 + 21 + t^12 - 15 + t^11) + 8 + 20t^9 + 20t^8 - 35 + t^7 - 126t^6 + 70t^5 + 48t^4 + 3t^3 - 25 + t^9)z^2 y + (6t^16 - 36t^15 + 14t^14 + 11 + 90t^10 - 56t^8 - 120t^7 + 7 + 14t^6 + 6 + 64t^5 + 84t^4 + 2 + t^3 + 16t^2 - 2 + 30t + 7)z^4,

$(t^3 - 1)x - y^6 + (-8 + 20t^3 + 24 + 20t^2 + 8 + 36 + t^7 + 3 + 40 + t^5 + 24t^4 + 4 - 6 + t^3 - 20 + 2t^2 - 6 + t + 1)y + (t^17 + 6 + t^14 + 9 + t^12 + 15 + t^11 - 36 + t^9 - 20 + t^8 - 5 + t^7 + 54t^6 + 15t^5 + 10t + 4 - 36 + t^3 - 11 + t^2 - 5 + 9)t^2,

$-y^8 + 8 + 2 + t^7 + 3 + 16 + t^2 y^2 + (8t + 20t^3 + 48 + 13 - 56 + t^11 - 120 + t^10 + 224 + 8 + 160 + t^7 - 56 + 2 + 322 + 24t^2 + 24 + t^6 - 56)z^4 + y^2 + (t^24 - 8 + t^21 + 20 + t^19 + 28 + 18 + 120 + t^16 - 56 + t^15 + 14 + t^14 + 300 + t^13 + 70 + t^12 - 56 + t^11 - 400 + 10 + 84 + t^2 + 268 + t^7 + 84 + t^6 - 56 + t^5 + 5 - 63 + t^4 - 36 + t^3 + 46 + t^2 - 12 - t + 1)z + 2 + t^5 + 8 + 2 + t + 2 + (-2 + t^11 + 8 + t^8 - 20 + t^6 - 12 + t^5 + 4 + t^3 + 8 + t^2 - 10 + t + 20) + t^4 + 3y + 8 + t^4 + 14 + t^3 + 11 + 8 + t^8 + 8 + t^7 - 32 + t^5 + 5 - 6 + t^4 + 9 + t^2 - 2 + t,

$-x^2 + 3 + (t^7 - 2 + t + 4 + 3 + 2 + t^2) + y + (2 + t^6 + 4 + t^3 + 3 + 2 + t^2) + z^2,

$2 + t^2 + y^2 + 3 + z^2 + y^2 + (-2 + t^5 + 4 + t^2 - 6 + 6 + t^4 + 4 + 4 + t^3 + 3 + 5 + t^2 - 2 + t)z$, $z^3 + 3y + 2 + t^3 + 3y + (t^7 + 2 + t^4 - t^4 - t^2 + 2 + t)z^2$,

$t + 3 + y^2 + 2 + t^2 + 3 + y + (t^6 - 2 + t^3 - 1 + 1)z^4, z^5 - t^4$]
As you see in the above example, the Groebner base under variable ordering \([x,y,z,t]\) has a lot of bases and each base itself is large. Under variable ordering \([t,z,y,x]\), however, \(B\) itself is already the Groebner basis. Roughly speaking, to obtain a Groebner base under the lexicographic ordering is to express the variables on the left (having higher order) in terms of variables on the right (having lower order). In the example, variables \(t\), \(z\), and \(y\) are already expressed by variable \(x\), and the above explanation justifies such a drastic experimental results. In practice, however, optimum ordering for variables may not known beforehand, and some heuristic trial may be inevitable.

### 8.6 Weight

Term orderings introduced in the previous section can be generalized by setting a weight for each variable.

\[
0 \text{ dp}_\text{td}(<1,1,1>);
3
\]

\[
1 \text{ dp}_\text{set_weight}([1,2,3])$
2 \text{ dp}_\text{td}(<1,1,1>);
6
\]

By default, the total degree of a monomial is equal to the sum of all exponents. This means that the weight for each variable is set to 1. In this example, the weights for the first, the second and the third variable are set to 1, 2 and 3 respectively. Therefore the total degree of \(<1,1,1>\) under this weight, which is called the weight of the monomial, is \(1\times1+1\times2+1\times3=6\). By setting weights, different term orderings can be set under a type of term ordering. In some case a polynomial can be made weighted homogeneous by setting an appropriate weight.

A list of weights for all variables is called a weight vector. A weight vector is called a sugar weight vector if its elements are all positive and it is used for computing a weighted total degree of a monomial, because such a weight is used instead of total degree in sugar strategy. On the other hand, a weight vector whose elements are not necessarily positive cannot be set as a sugar weight, but it is useful for generalizing term order. In fact, such a weight vector already appeared in a matrix order. That is, each row of a matrix defining a term order is regarded as a weight vector. A block order is also considered as a refinement of comparison by weight vectors. A block order is also considered as a refinement of comparison by weight vectors. It compares two terms by using a weight vector whose elements corresponding to variables in a block is 1 and 0 otherwise, then it applies a tie breaker.

A weight vector can be set by using \(\text{dp}_\text{set_weight}()\). However it is more preferable if a weight vector can be set together with other parameters such as a type of term ordering and a variable order. This is realized as follows.

\[
B=[x+y+z-6,x*y+y*z+z*x-11,x*y*z-6]$
\
\text{[64]} \text{ dp}_\text{gr_main}(B|v=[x,y,z],\text{sugarweight}=[3,2,1],\text{order}=0);
\text{[65]} \text{ dp}_\text{gr_main}(B|v=[x,y,z],\text{order}=[[1,1,0],[0,1,0],[0,0,1]]);
\text{[66]} \text{ dp}_\text{gr_main}(B|v=[y,z,x],\text{order}=[[1,0,1],[0,1,0],[0,0,1]]);
\text{[67]} \text{ dp}_\text{gr_main}(B|v=[y,z,x],\text{order}=[[x,1,y,2,z,3]]);
\]

\[
[z^3-6*z^2+11*z-6,x+y+z-6,-y^2+(-z+6)*y-z^2+6*z-11]
\]

\[
[x^3-6*x^2+11*x-6,x+y+z-6,-x^2+(-y+6)*x-y^2+6*y-11]
\]

\[
[x+y+z-6,x^3-6*x^2+11*x-6,-x^2+(-y+6)*x-y^2+6*y-11]
\]
In each example, a term ordering is specified as options. In the first example, a variable order, a sugar weight vector and a type of term ordering are specified by options \( v \), \( \text{sugarweight} \) and \( \text{order} \) respectively. In the second example, an option \( \text{order} \) is used to set a matrix ordering. That is, the specified weight vectors are used from left to right for comparing terms. The third example shows a variant of specifying a weight vector, where each component of a weight vector is specified variable by variable, and unspecified components are set to zero. In this example, a term order is not determined only by the specified weight vector. In such a case a tie breaker by the graded reverse lexicographic ordering is set automatically. This type of a term ordering specification can be applied only to builtin functions such as \( \text{dp_gr_main()} \), \( \text{dp_gr_mod_main()} \), not to user defined functions such as \( \text{gr()} \).

### 8.7 Groebner basis computation with rational function coefficients

Such variables that appear within the input polynomials but not appearing in the input variable list are automatically treated as elements in the coefficient field by top level functions, such as \( \text{gr()} \).

\[
\begin{align*}
64 & \quad \text{gr}([a*x+b*y-c,d*x+e*y-f],[x,y],2); \\
& \quad [(-e*a+d*b)*x-f*b+e*c,(-e*a+d*b)*y+f*a-d*c]
\end{align*}
\]

In this example, variables \( a, b, c, \) and \( d \) are treated as elements in the coefficient field. In this case, a Groebner basis is computed on a bi-variate polynomial ring \( F[x,y] \) over rational function field \( F = \mathbb{Q}(a,b,c,d) \). Notice that coefficients are considered as a member in a field. As a consequence, polynomial factors common to the coefficients are removed so that the result, in general, is different from the result that would be obtained when the problem is considered as a computation of Groebner basis over a polynomial ring with rational function coefficients. And note that coefficients of a distributed polynomial are limited to numbers and polynomials because of efficiency.

### 8.8 Change of ordering

When we compute a lex order Groebner basis, it is often efficient to compute it via Groebner basis with respect to another order such as degree reverse lex order, rather than to compute it directory by \( \text{gr()} \) etc. If we know that an input is a Groebner basis with respect to an order, we can apply special methods called change of ordering for a Groebner basis computation with respect to another order, without using Buchberger algorithm. The following two functions are ones for change of ordering such that they convert a Groebner basis \( gbase \) with respect to the variable order \( vlist1 \) and the order type \( \text{order} \) into a lex Groebner basis with respect to the variable order \( vlist2 \).

\[
\text{tlex}(gbase, vlist1, order, vlist2)
\]

This function can be used only when \( gbase \) is an ideal over the rationals. The input \( gbase \) must be a Groebner basis with respect to the variable order \( vlist1 \) and the order type \( \text{order} \). Moreover the ideal generated by \( gbase \) must be zero-dimensional. This computes the lex Groebner basis of \( gbase \) by using the modular change of ordering algorithm. The algorithm first computes the lex Groebner basis over a finite field. Then each element in the lex Groebner basis
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over the rationals is computed with undetermined coefficient method and linear equation solving by Hensel lifting.

tolex_tl(gbase, vlist1, order, vlist2, homo)
This function computes the lex Groebner basis of gbase. The input gbase must be a Groebner basis with respect to the variable order vlist1 and the order type order. Buchberger algorithm with trace lifting is used to compute the lex Groebner basis, however the Groebner basis check and the ideal membership check can be omitted by using several properties derived from the fact that the input is a Groebner basis. So it is more efficient than simple repetition of Buchberger algorithm. If the input is zero-dimensional, this function inserts automatically a computation of Groebner basis with respect to an elimination order, which makes the whole computation more efficient for many cases. If homo is not equal to 0, homogenization is used in each step.

For zero-dimensional systems, there are several functions to compute the minimal polynomial of a polynomial and or a more compact representation for zeros of the system. They are all defined in ‘gr’. Refer to the sections for each functions.

8.9 Weyl algebra

So far we have explained Groebner basis computation in commutative polynomial rings. However Groebner basis can be considered in more general non-commutative rings. Weyl algebra is one of such rings and Risa/Asir implements fundamental operations in Weyl algebra and Groebner basis computation in Weyl algebra.

The n dimensional Weyl algebra over a field K, D=K<x1,...,xn,D1,...,Dn> is a non-commutative algebra which has the following fundamental relations:

\[ x_i \cdot x_j - x_j \cdot x_i = 0, D_i \cdot D_j - D_j \cdot D_i = 0, D_i \cdot x_j - x_j \cdot D_i = 0 \quad (i \neq j), \quad D_i \cdot x_i - x_i \cdot D_i = 1 \]

D is the ring of differential operators whose coefficients are polynomials in K[x1,...,xn] and Di denotes the differentiation with respect to xi. According to the commutation relation, elements of D can be represented as a K-linear combination of monomials x1^i1*...*xn^in*D1^j1*...*Dn^jn. In Risa/Asir, this type of monomial is represented by <<i1,...,in,j1,...,jn>> as in the case of commutative polynomial. That is, elements of D are represented by distributed polynomials. Addition and subtraction can be done by +, -, but multiplication is done by calling dp_weyl_mul() because of the non-commutativity of D.

```
[0] A<<1,2,2,1>>;
(1) *<1,2,2,1>;
[1] B<<2,1,1,2>>;
(1) *<2,1,1,2>;
(1) *<3,3,3,3>;
[3] dp_weyl_mul(A,B);
(1) *<3,3,3,3>+(1) *<3,2,3,2>+(4) *<2,3,2,3>+(4) *<2,2,2,2>+(2) *<1,3,1,3>+(2) *<1,2,1,2>;
```

The following functions are available for Groebner basis computation in Weyl algebra: dp_weyl_gr_main(), dp_weyl_gr_mod_main(), dp_weyl_gr_f_main(), dp_weyl_f4_main(), dp_weyl_f5_main().
dp_weyl_f4_mod_main(). Computation of the global b function is implemented as an application.

8.10 Functions for Groebner basis computation

8.10.1 gr, hgr, gr_mod, dgr

\[
\begin{align*}
gr & (\text{plist}, \text{vlist}, \text{order}) \\
hgr & (\text{plist}, \text{vlist}, \text{order}) \\
gr\_mod & (\text{plist}, \text{vlist}, \text{order}, p) \\
dgr & (\text{plist}, \text{vlist}, \text{order}, \text{procs}) \\
\end{align*}
\]

These functions are defined in `gr` in the standard library directory. They compute a Groebner basis of a polynomial list \( \text{plist} \) with respect to the variable \( \text{vlist} \) and the order type \( \text{order} \). \( \text{gr}() \) and \( \text{hgr}() \) compute a Groebner basis over the rationals and \( \text{gr\_mod} \) computes over \( \text{GF}(p) \).

- Variables not included in \( \text{vlist} \) are regarded as included in the ground field.
- \( \text{gr}() \) uses trace-lifting (an improvement by modular computation) and sugar strategy. \( \text{hgr}() \) uses trace-lifting and a cured sugar strategy by using homogenization.
- \( \text{dgr}() \) executes \( \text{gr}() \), \( \text{dgr}() \) simultaneously on two processes in a child process list \( \text{procs} \) and returns the result obtained first. The results returned from both the processes should be equal, but it is not known in advance which method is faster. Therefore this function is useful to reduce the actual elapsed time.
- The CPU time shown after an execution of \( \text{dgr}() \) indicates that of the master process, and most of the time corresponds to the time for communication.
- When the elements of \( \text{plist} \) are distributed polynomials, the result is also a list of distributed polynomials. In this case, firstly the elements of \( \text{plist} \) is sorted by \text{dp\_sort} and the Groebner basis computation is started. Variables must be given in \( \text{vlist} \) even in this case (these variables are dummy).

\[
\begin{align*}
[0] \text{load}(&"\text{gr}")$
[64] \text{load}(&"\text{cyclic}")$
[74] \text{G}=\text{gr}(\text{cyclic}(5),[c0,c1,c2,c3,c4],2); \\
[112] & [c4^15+122*c4^10-122*c4^5-1,...] \\
[75] \text{GM}=\text{gr\_mod}(\text{cyclic}(5),[c0,c1,c2,c3,c4],2,31991)$ \\
24628*c4^15+29453*c4^10+2538*c4^5+7363 \\
[76] (\text{G}[0]*24628-\text{GM}[0])\%31991; \\
0
\end{align*}
\]
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References
Section 8.10.6 [dp_gr_main dp_gr_mod_main dp_gr_f_main dp_weyl_gr_main dp_weyl_gr_mod_main dp_weyl_gr_f_main], page 136, Section 8.10.10 [dp_ord], page 139.

8.10.2 lex_hensel, lex_tl, tolex, tolex_d, tolex_tl

lex_hensel(plist, vlist1, order, vlist2, homo)
lex_tl(plist, vlist1, order, vlist2, homo)

: Groebner basis computation with respect to a lex order by change of ordering

tolex(plist, vlist1, order, vlist2)
tolex_d(plist, vlist1, order, vlist2, procs)
tolex_tl(plist, vlist1, order, vlist2, homo)

: Groebner basis computation with respect to a lex order by change of ordering, starting from a Groebner basis

return list

plist vlist1 vlist2 procs
list

order number, list or matrix

homo flag

- These functions are defined in ‘gr’ in the standard library directory.
- lex_hensel() and lex_tl() first compute a Groebner basis with respect to the variable order vlist1 and the order type order. Then the Groebner basis is converted into a lex order Groebner basis with respect to the variable order vlist2.
- tolex() and tolex_tl() convert a Groebner basis plist with respect to the variable order vlist1 and the order type order into a lex order Groebner basis with respect to the variable order vlist2. tolex_d() does computations of basis elements in tolex() in parallel on the processes in a child process list procs.
- In lex_hensel() and tolex_hensel() a lex order Groebner basis is computed as follows.(Refer to [Noro,Yokoyama].)
  1. Compute a Groebner basis G0 with respect to vlist1 and order. (Only in lex_hensel().)
  2. Choose a prime which does not divide head coefficients of elements in G0 with respect to vlist1 and order. Then compute a lex order Groebner basis Gp over GF(p) with respect to vlist2.
  3. Compute NF, the set of all the normal forms with respect to G0 of terms appearing in Gp.
  4. For each element f in Gp, replace coefficients and terms in f with undetermined coefficients and the corresponding polynomials in NF respectively, and generate a system of linear equation Lf by equating the coefficients of terms in the replaced polynomial with 0.
  5. Solve Lf by Hensel lifting, starting from the unique mod p solution.
6. If all the linear equations generated from the elements in $G_p$ could be solved, then the set of solutions corresponds to a lex order Groebner basis. Otherwise redo the whole process with another $p$.

- In `lex_t1()` and `tolex_t1()` a lex order Groebner basis is computed as follows. (Refer to [Noro, Yokoyama].)
  1. Compute a Groebner basis $G_0$ with respect to $vlist1$ and `order`. (Only in `lex_t1()`.)
  2. If $G_0$ is not zero-dimensional, choose a prime which does not divide head coefficients of elements in $G_0$ with respect to $vlist1$ and `order`. Then compute a candidate of a lex order Groebner basis via trace lifting with $p$. If it succeeds the candidate is indeed a lex order Groebner basis without any check. Otherwise redo the whole process with another $p$.
  3. If $G_0$ is zero-dimensional, starting from $G_0$, compute a Groebner basis $G_1$ with respect to an elimination order to eliminate variables other than the last variable in $vlist2$. Then compute a lex order Groebner basis starting from $G_1$. These computations are done by trace lifting and the selection of a modulus $p$ is the same as in non zero-dimensional cases.

- Computations with rational function coefficients can be done only by `lex_t1()` and `tolex_t1()`.

- If `homo` is not equal to 0, homogenization is used in Buchberger algorithm.

- The CPU time shown after an execution of `tolex_d()` indicates that of the master process, and it does not include the time in child processes.

```
[78] K=katsura(5)
30msec + gc : 20msec
[79] V=[u5,u4,u3,u2,u1,u0]
0msec
[80] G0=hgr(K,V,2)
91.558sec + gc : 15.583sec
[81] G1=lex_hensel(K,V,0,V,0)
49.049sec + gc : 9.961sec
[82] G2=lex_t1(K,V,0,V,1)
31.186sec + gc : 3.500sec
[83] gb_comp(G0,G1);
1
10msec
[84] gb_comp(G0,G2);
1
```

References

Section 8.10.6 [dp_gr_main dp_gr_mod_main dp_gr_f_main dp_weyl_gr_main dp_weyl_gr_mod_main dp_weyl_gr_f_main], page 136, Section 8.10.10 [dp_ord], page 139, Chapter 7 [Distributed computation], page 102

8.10.3 `lex_hensel_gsl`, `tolex_gsl`, `tolex_gsl_d`
\texttt{lex\_hensel\_gsl(\textit{plist}, \textit{vlist1}, \textit{order}, \textit{vlist2}, \textit{homo})}

:: Computation of an GSL form ideal basis

\texttt{tolex\_gsl(\textit{plist}, \textit{vlist1}, \textit{order}, \textit{vlist2})}

\texttt{tolex\_gsl\_d(\textit{plist}, \textit{vlist1}, \textit{order}, \textit{vlist2}, \textit{procs})}

:: Computation of an GSL form ideal basis stating from a Groebner basis

\texttt{return list \textit{plist} \textit{vlist1} \textit{vlist2} \textit{procs}}

\texttt{list \textit{order} number, list or matrix}

\texttt{homo flag}

- \texttt{lex\_hensel\_gsl()} and \texttt{lex\_hensel()} are variants of \texttt{tolex\_gsl()} and \texttt{tolex()} respectively. The results are Groebner basis or a kind of ideal basis, called GSL form. \texttt{tolex\_gsl\_d()} does basis computations in parallel on child processes specified in \texttt{procs}.

- If the input is zero-dimensional and a lex order Groebner basis has the form \([f_0, x_1-f_1, \ldots, x_n-f_n]\) (\(f_0, \ldots, f_n\) are univariate polynomials of \(x_0\); SL form), then this these functions return a list such as \([[x_1, g_1, d_1], \ldots, [x_n, g_n, d_n], [x_0, f_0, f_0']]]\) (GSL form). In this list \(g_i\) is a univariate polynomial of \(x_0\) such that \(d_i^*f_0^*f_i-g_i\) divides \(f_0\) and the roots of the input ideal is \([x_1=g_1/(d_1^*f_0'), \ldots, x_n=g_n/(d_n^*f_0')]\) for \(x_0\) such that \(f_0(x_0)=0\). If the lex order Groebner basis does not have the above form, these functions return a lex order Groebner basis computed by \texttt{tolex()}.

- Though an ideal basis represented as GSL form is not a Groebner basis we can expect that the coefficients are much smaller than those in a Groebner basis and that the computation is efficient. The CPU time shown after an execution of \texttt{tolex\_gsl\_d()} indicates that of the master process, and it does not include the time in child processes.

\begin{verbatim}
[103] K=katsura(5)
[104] V=[u5,u4,u3,u2,u1,u0]
[105] G0=gr(K,V,0)
[106] GSL=tolex_gsl(G0,V,0,V)
[107] GSL[0];
[u1,8635837421130477667200000000*u0^-31-...]
[108] GSL[1];
[u2,10352277157007342793600000000*u0^-31-...]
[109] GSL[5];
[u0,117710218761930641246400000000*u0^-32-...,
376672700038178051988480000000*u0^-31-...]
\end{verbatim}

\textbf{References}

Section 8.10.2 [\texttt{lex\_hensel lex\_tl tolex tolex\_d tolex\_tl}], page 132, Chapter 7 [\texttt{Distributed computation}], page 102

\subsection*{8.10.4 \texttt{gr\_minipoly}, \texttt{minipoly}}

\texttt{gr\_minipoly(\textit{plist}, \textit{vlist}, \textit{order}, \textit{poly}, \textit{v}, \textit{homo})}

:: Computation of the minimal polynomial of a polynomial modulo an ideal
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minipoly(plist, vlist, order, poly, v)
:: Computation of the minimal polynomial of a polynomial modulo an ideal
return polynomial
plist vlist list
order number, list or matrix
poly polynomial
v indeterminate
homo flag

- \texttt{gr\_minipoly()} begins by computing a Groebner basis. \texttt{minipoly()} regards an input as a Groebner basis with respect to the variable order \texttt{vlist} and the order type \texttt{order}.
- Let \( K \) be a field. If an ideal \( I \) in \( K[X] \) is zero-dimensional, then, for a polynomial \( p \) in \( K[X] \), the kernel of a homomorphism from \( K[v] \) to \( K[X] / I \) which maps \( f(v) \) to \( f(p) \) mod \( I \) is generated by a polynomial. The generator is called the minimal polynomial of \( p \) modulo \( I \).
- \texttt{gr\_minipoly()} and \texttt{minipoly()} computes the minimal polynomial of a polynomial \( p \) and returns it as a polynomial of \( v \).
- The minimal polynomial can be computed as an element of a Groebner basis. But if we are only interested in the minimal polynomial, \texttt{minipoly()} and \texttt{gr\_minipoly()} can compute it more efficiently than methods using Groebner basis computation.
- It is recommended to use a degree reverse lex order as a term order for \texttt{gr\_minipoly()}.

\begin{verbatim}
[117] G=tolox(G0,V,0,V)$
43.818sec + gc : 11.202sec
[118] GSL=tolox_gsl(G0,V,0,V)$
17.123sec + gc : 2.590sec
[119] MP=minipoly(G0,V,0,u0,z)$
4.370sec + gc : 780msec
\end{verbatim}

References
Section 8.10.2 \texttt{[lex\_hensel lex\_tl tolex tolex\_d tolex\_tl]}, page 132.

8.10.5 tolemx, minipolym

tolemx(plist, vlist1, order, vlist2, mod)
:: Groebner basis computation modulo \texttt{mod} by change of ordering.

minipolym(plist, vlist1, order, poly, v, mod)
:: Minimal polynomial computation modulo \texttt{mod} the same method as

return tolemx(): list, minipolym(): polynomial
plist vlist1 vlist2 list
order number, list or matrix
mod prime
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- An input \( \text{plist} \) must be a Groebner basis modulo \( \text{mod} \) with respect to the variable order \( \text{vlist1} \) and the order type \( \text{order} \).
- \text{minipolym()} executes the same computation as in \text{minipoly}.
- \text{tolexm()} computes a lex order Groebner basis modulo \( \text{mod} \) with respect to the variable \( \text{vlist2} \), by using FGLM algorithm.

\[
\text{tolexm}(G0,V,0,V,31991);
\]
\[
[8271*u0^31+10435*u0^30+816*u0^29+26809*u0^28+...,...]
\]
\[
\text{minipolym}(G0,V,0,u0,z,31991);
\]
\[
z^32+11405*z^31+20868*z^30+21602*z^29+...
\]

References

Section 8.10.2 \([\text{lex}_\text{hensel} \text{ lex}_\text{tl} \text{ tolex} \text{ tolex}_\text{d} \text{ tolex}_\text{tl}]\), page 132, Section 8.10.4 \([\text{gr}_\text{minipoly} \text{ minipoly}]\), page 134.

8.10.6 \(\text{dp}_\text{gr}_\text{main}, \text{dp}_\text{gr}_\text{mod}_\text{main}, \text{dp}_\text{gr}_\text{f}_\text{main}, \text{dp}_\text{weyl}_\text{gr}_\text{main}, \text{dp}_\text{weyl}_\text{gr}_\text{mod}_\text{main}, \text{dp}_\text{weyl}_\text{gr}_\text{f}_\text{main}\)

\[
\begin{align*}
\text{dp}_\text{gr}_\text{main}(\text{plist}, \text{vlist}, \text{homo}, \text{modular}, \text{order}) \\
\text{dp}_\text{gr}_\text{mod}_\text{main}(\text{plist}, \text{vlist}, \text{homo}, \text{modular}, \text{order}) \\
\text{dp}_\text{gr}_\text{f}_\text{main}(\text{plist}, \text{vlist}, \text{homo}, \text{order}) \\
\text{dp}_\text{weyl}_\text{gr}_\text{main}(\text{plist}, \text{vlist}, \text{homo}, \text{modular}, \text{order}) \\
\text{dp}_\text{weyl}_\text{gr}_\text{mod}_\text{main}(\text{plist}, \text{vlist}, \text{homo}, \text{modular}, \text{order}) \\
\text{dp}_\text{weyl}_\text{gr}_\text{f}_\text{main}(\text{plist}, \text{vlist}, \text{homo}, \text{order})
\end{align*}
\]

\[
\begin{align*}
\text{return} & \quad \text{list} \\
\text{plist} \quad \text{vlist} & \quad \text{list} \\
\text{order} & \quad \text{number, list or matrix} \\
\text{homo} & \quad \text{flag} \\
\text{modular} & \quad \text{flag or prime}
\end{align*}
\]

- These functions are fundamental built-in functions for Groebner basis computation and \text{gr()}, \text{hgr()} and \text{gr_mod()} are all interfaces to these functions. Functions whose names contain weyl are those for computation in Weyl algebra.
- \text{dp}_\text{gr}_\text{f}_\text{main()} and \text{dp}_\text{weyl}_\text{gr}_\text{f}_\text{main()} are functions for Groebner basis computation over various finite fields. Coefficients of input polynomials must be converted to elements of a finite field currently specified by \text{setmod}_\text{ff}().
- If \text{homo} is not equal to 0, homogenization is applied before entering Buchberger algorithm
- For \text{dp}_\text{gr}_\text{mod}_\text{main()}, \text{modular} means a computation over GF(\text{modular}). For \text{dp}_\text{gr}_\text{main()}, \text{modular} has the following mean.
  1. If \text{modular} is 1, trace lifting is used. Primes for trace lifting are generated by \text{lprime()}, starting from \text{lprime}(0), until the computation succeeds.
  2. If \text{modular} is an integer greater than 1, the integer is regarded as a prime and trace lifting is executed by using the prime. If the computation fails then 0 is returned.
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3. If modular is negative, the above rule is applied for -modular but the Groebner basis check and ideal-membership check are omitted in the last stage of trace lifting.

- \( gr(P,V,O) \), \( hgr(P,V,O) \) and \( gr\_mod(P,V,O,M) \) execute \( dp\_gr\_main(P,V,0,1,O) \), \( dp\_gr\_main(P,V,1,1,O) \) and \( dp\_gr\_mod\_main(P,V,0,M,O) \) respectively.
- Actual computation is controlled by various parameters set by \( dp\_gr\_flags() \), other then by homo and modular.

References
Section 8.10.10 \([dp\_ord]\), page 139, Section 8.10.9 \([dp\_gr\_flags\ dp\_gr\_print]\), page 139, Section 8.10.1 \([gr\ hgr\ gr\_mod]\), page 131, Section 10.5.1 \([setmod\_ff]\), page 172, Section 8.4 \([Controlling Groebner basis computations]\), page 123

8.10.7 \( dp\_f4\_main,\ dp\_f4\_mod\_main,\ dp\_weyl\_f4\_main,\ dp\_weyl\_f4\_mod\_main \)

\( dp\_f4\_main(plist,vlist,order) \)
\( dp\_f4\_mod\_main(plist,vlist,order) \)
\( dp\_weyl\_f4\_main(plist,vlist,order) \)
\( dp\_weyl\_f4\_mod\_main(plist,vlist,order) \)

:: Groebner basis computation by F4 algorithm (built-in functions)

\( return \) list
\( plist\ vlist \) list
\( order \) number, list or matrix
- These functions compute Groebner bases by F4 algorithm.
- F4 is a new generation algorithm for Groebner basis computation invented by J.C. Faugere. The current implementation of \( dp\_f4\_main() \) uses Chinese Remainder theorem and not highly optimized.
- Arguments and actions are the same as those of \( dp\_gr\_main() \), \( dp\_gr\_mod\_main() \), \( dp\_weyl\_gr\_main() \), \( dp\_weyl\_gr\_mod\_main() \), except for lack of the argument for controlling homogenization.

References
Section 8.10.10 \([dp\_ord]\), page 139, Section 8.10.9 \([dp\_gr\_flags\ dp\_gr\_print]\), page 139, Section 8.10.1 \([gr\ hgr\ gr\_mod]\), page 131, Section 8.4 \([Controlling Groebner basis computations]\), page 123

8.10.8 \( nd\_gr,\ nd\_gr\_trace,\ nd\_f4,\ nd\_f4\_trace,\ nd\_weyl\_gr,\ nd\_weyl\_gr\_trace \)

\( nd\_gr(plist,vlist,p,order) \)
\( nd\_gr\_trace(plist,vlist,homo,p,order) \)
\( nd\_f4(plist,vlist,modular,order) \)
\( nd\_f4\_trace(plist,vlist,homo,p,order) \)
\( nd\_weyl\_gr(plist,vlist,p,order) \)
\( nd\_weyl\_gr\_trace(plist,vlist,homo,p,order) \)

:: Groebner basis computation (built-in functions)
return list
plist vlist list
order number, list or matrix
homo flag
modular flag or prime

These functions are new implementations for computing Groebner bases.

- \texttt{nd\_gr} executes Buchberger algorithm over the rationals if \( p \) is 0, and that over \( \text{GF}(p) \) if \( p \) is a prime.
- \texttt{nd\_gr\_trace} executes the trace algorithm over the rationals. If \( p \) is 0 or 1, the trace algorithm is executed until it succeeds by using automatically chosen primes. If \( p \) a positive prime, the trace is computed over \( \text{GF}(p) \). If the trace algorithm fails 0 is returned. If \( p \) is negative, the Groebner basis check and ideal-membership check are omitted. In this case, an automatically chosen prime if \( p \) is 1, otherwise the specified prime is used to compute a Groebner basis candidate. Execution of \texttt{nd\_f4\_trace} is done as follows: For each total degree, an F4-reduction of S-polynomials over a finite field is done, and S-polynomials which give non-zero basis elements are gathered. Then F4-reduction over \( \mathbb{Q} \) is done for the gathered S-polynomials. The obtained polynomial set is a Groebner basis candidate and the same check procedure as in the case of \texttt{nd\_gr\_trace} is done.
- \texttt{nd\_f4} executes F4 algorithm over \( \mathbb{Q} \) if \texttt{modular} is equal to 0, or over a finite field \( \text{GF}(\text{modular}) \) if \texttt{modular} is a prime number of machine size (\(<2^{129}\)). If \texttt{plist} is a list of polynomials, then a Groebner basis of the ideal generated by \texttt{plist} is computed. If \texttt{plist} is a list of lists of polynomials, then each list of polynomials are regarded as an element of a free module over a polynomial ring and a Groebner basis of the sub-module generated by \texttt{plist} in the free module. In the latter case a term order in the free module should be specified. This is specified by \([s,\text{ord}]\). If \( s \) is 0 then it means TOP (Term Over Position). If \( s \) is 1 then it means POT 1 (Position Over Term). \texttt{ord} is a term order in the base polynomial ring.
- \texttt{nd\_weyl\_gr}, \texttt{nd\_weyl\_gr\_trace} are for Weyl algebra computation.
- Functions except for F4 related ones can handle rational coefficient cases.
- In general these functions are more efficient than \texttt{dp\_gr\_main}, \texttt{dp\_gr\_mod\_main}, especially over finite fields.

\begin{verbatim}
[38] load("cyclic")$
[49] C=cyclic(7)$
[50] V=vars(C)$
[51] cputime(1)$
[52] dp\_gr\_mod\_main(C,V,0,31991,0)$
  26.06sec + gc : 0.313sec(26.4sec)
[53] nd\_gr(C,V,31991,0)$
  ndv\_alloc=1477188
  5.737sec + gc : 0.1837sec(5.921sec)
[54] dp\_f4\_mod\_main(C,V,31991,0)$
  3.51sec + gc : 0.7109sec(4.221sec)
[55] nd\_f4(C,V,31991,0)$
  1.906sec + gc : 0.126sec(2.032sec)
\end{verbatim}
8.10.9 dp_gr_flags, dp_gr_print

\texttt{dp\_gr\_flags([\textit{list}])}
\texttt{dp\_gr\_print([\textit{i}])}

and showing informations.

\textbf{return} value currently set

\textbf{list} list

\textbf{i} integer

- \texttt{dp\_gr\_flags()} sets and shows various parameters for Groebner basis computation.
- If no argument is specified the current settings are returned.
- Arguments must be specified as a list such as \texttt{["Print",1,"NoSugar",1,...]}. Names of parameters must be character strings.
- \texttt{dp\_gr\_print()} is used to set and show the value of a parameter \texttt{Print} and \texttt{PrintShort}.

\begin{align*}
i=0 & \quad \text{Print}=0, \text{PrintShort}=0 \\
i=1 & \quad \text{Print}=1, \text{PrintShort}=0 \\
i=2 & \quad \text{Print}=0, \text{PrintShort}=1
\end{align*}

This functions is prepared to get quickly the value when a user defined function calling \texttt{dp\_gr\_main()} etc. uses the value as a flag for showing intermediate informations.

References: Section 8.4 [Controlling Groebner basis computations], page 123

8.10.10 dp_ord

\texttt{dp\_ord([\textit{order}])}

:: Set and show the ordering type.

\textbf{return} ordering type (number, list or matrix)

\textbf{order} number, list or matrix

- If an argument is specified, the function sets the current ordering type to \textit{order}. If no argument is specified, the function returns the ordering type currently set.
- There are two types of functions concerning distributed polynomial, functions which take a ordering type and those which don’t take it. The latter ones use the current setting.
- Functions such as \texttt{gr()}, which need a ordering type as an argument, call \texttt{dp\_ord()} internally during the execution. The setting remains after the execution.

Fundamental arithmetics for distributed polynomial also use the current setting. Therefore, when such arithmetics for distributed polynomials are done, the current setting...
must coincide with the ordering type which was used upon the creation of the polynomials. It is assumed that such polynomials were generated under the same ordering type.

- Type of term ordering must be correctly set by this function when functions other than top level functions are called directly.

```
[19] dp_ord(0)
[20] <<1,2,3>>+<<3,1,1>>;
(1)*<<1,2,3>>+(1)*<<3,1,1>>
[21] dp_ord(2)
[22] <<1,2,3>>+<<3,1,1>>;
(1)*<<3,1,1>>+(1)*<<1,2,3>>
```

References

Section 8.5 [Setting term orderings], page 126

8.10.11 dp_set_weight, dp_set_top_weight, dp_weyl_set_weight

dp_set_weight([weight])
:: Set and show the sugar weight.

dp_set_top_weight([weight])
:: Set and show the top weight.

dp_weyl_set_weight([weight])
:: Set and show the weyl weight.

return a vector

weight a list or vector of integers

- **dp_set_weight** sets the sugar weight=weight. It returns the current sugar weight. A sugar weight is a vector with positive integer components and it represents the weights of variables. It is used for computing the weight of a monomial in a graded ordering. It is recommended to append a component 1 at the end of the weight vector for a homogenizing variable.

- **dp_set_top_weight** sets the top weight=weight. It returns the current top weight. It a top weight is set, the weights of monomials under the top weight are firstly compared. If the the weights are equal then the current term ordering is applied as a tie breaker, but the top weight is not used in the tie breaker.

- **dp_weyl_set_weight** sets the weyl weight=weight. It returns the current weyl weight. If a weyl weight w is set, in the comparison by the term order type 11, a term order with the top weight=(-w,w) and the tie breaker=graded reverse lex is applied.

References

Section 8.6 [Weight], page 128

8.10.12 dp_ptod

dp_ptod(poly,vlist)
:: Converts an ordinary polynomial into a distributed polynomial.
return distributed polynomial
poly polynomial
vlist list

- According to the variable ordering vlist and current type of term ordering, this function converts an ordinary polynomial into a distributed polynomial.
- Indeterminates not included in vlist are regarded to belong to the coefficient field.

[50] dp_ord(0);
1
[51] dp_ptod((x+y+z)^2,[x,y,z]);
(1)*<<2,0,0>>+(2)*<<1,1,0>>+(1)*<<0,2,0>>+(2)*<<1,0,1>>+(2)*<<0,1,1>>
+(1)*<<0,0,2>>
[52] dp_ptod((x+y+z)^2,[x,y]);
(1)*<<2,0,0>>+(2)*<<1,1,0>>+(1)*<<0,2,0>>+(2*z)*<<1,0,1>>+(2*z)*<<0,1,1>>
+(z^2)*<<0,0,0>>

References
Section 8.10.13 [dp_dtop], page 141, Section 8.10.10 [dp_ord], page 139.

8.10.13 dp_dtop

dp_dtop(dpoly,vlist)
:: Converts a distributed polynomial into an ordinary polynomial.

return polynomial
dpoly distributed polynomial
vlist list

- This function converts a distributed polynomial into an ordinary polynomial according to a list of indeterminates vlist.
- vlist is such a list that its length coincides with the number of variables of dpoly.

[53] T=dp_ptod((x+y+z)^2,[x,y]);
(1)*<<2,0,0>>+(2)*<<1,1,0>>+(1)*<<0,2,0>>+(2*z)*<<1,0,1>>+(2*z)*<<0,1,1>>
+(z^2)*<<0,0,0>>
[54] P=dp_dtop(T,[a,b]);
z^2+(2*a+2*b)*z+a^2+2*b*a+b^2

8.10.14 dp_mod, dp_rat

dp_mod(p,mod,subst)
:: Converts a distributed polynomial into one with coefficients in a finite field.

dp_rat(p)
:: Converts a distributed polynomial with coefficients in a finite field into one with coefficients in the rationals.

return distributed polynomial
p distributed polynomial
mod prime

subst list

- \texttt{dp\_nf\_mod()} and \texttt{dp\_true\_nf\_mod()} require distributed polynomials with coefficients in a finite field as arguments. \texttt{dp\_mod()} is used to convert distributed polynomials with rational number coefficients into appropriate ones. Polynomials with coefficients in a finite field cannot be used as inputs of operations with polynomials with rational number coefficients. \texttt{dp\_rat()} is used for such cases.

- The ground finite field must be set in advance by using \texttt{setmod()}.

- \texttt{subst} is such a list as \texttt{[[\text{var}, \text{value}], \ldots \}]. This is valid when the ground field of the input polynomial is a rational function field. \text{var}'s are variables in the ground field and the list means that \text{value} is substituted for \text{var} before converting the coefficients into elements of a finite field.

References

Section 8.10.17 \texttt{[dp\_nf\_mod\_dp\_true\_nf\_mod\_dp\_true\_nf\_mod\_dp\_weyl\_nf\_dp\_weyl\_nf\_mod]}, page 143, Section 6.3.11 \texttt{[subst\_psubst]}, page 52, Section 6.1.17 \texttt{[setmod]}, page 44.

\section*{8.10.15 \texttt{dp\_homo}, \texttt{dp\_dehomo}}

\begin{verbatim}
\textbf{dp\_homo}(dpoly)
:: Homogenize a distributed polynomial

\textbf{dp\_dehomo}(dpoly)
:: Dehomogenize a homogenious distributed polynomial
\end{verbatim}

\textbf{return} distributed polynomial

\textbf{dpoly} distributed polynomial

- \texttt{dp\_homo()} makes a copy of \texttt{dpoly}, extends the length of the exponent vector of each term \text{t} in the copy by 1, and sets the value of the newly appended component to \texttt{d-deg(t)}, where \texttt{d} is the total degree of \texttt{dpoly}.

- \texttt{dp\_dehomo()} make a copy of \texttt{dpoly} and removes the last component of each terms in the copy.

- Appropriate term orderings must be set when the results are used as inputs of some operations.

- These are used internally in \texttt{hgr()} etc.

\begin{verbatim}
[202] X=<<1,2,3>>+3*<<1,2,1>>;
(1)*<<1,2,3>>+(3)*<<1,2,1>>
[203] dp\_homo(X);
(1)*<<1,2,3,0>>+(3)*<<1,2,1,2>>
[204] dp\_dehomo(@);
(1)*<<1,2,3>>+(3)*<<1,2,1>>
\end{verbatim}

References

Section 8.10.1 \texttt{[gr\_hgr\_gr\_mod]}, page 131.
8.10.16 dp_ptozp, dp_prim

dp_ptozp(dpoly)
    :: Converts a distributed polynomial poly with rational coefficients into an
    integral distributed polynomial such that GCD of all its coefficients is 1.

dp_prim(dpoly)
    :: Converts a distributed polynomial poly with rational function coefficients
    into an integral distributed polynomial such that polynomial GCD of all its
    coefficients is 1.

return distributed polynomial
dpoly distributed polynomial

- dp_ptozp() executes the same operation as ptozp() for a distributed polynomial. If
  the coefficients include polynomials, polynomial contents included in the coefficients
  are not removed.
- dp_prim() removes polynomial contents.

[208] X=dp_ptod(3*(x-y)*(y-z)*(z-x),[x]);
(-3*y+3*z)*<<2>>+(3*y^2-3*z^2)*<<1>>+(-3*z*y^2+3*z^2*y)*<<0>>
[209] dp_ptozp(X);
(-y+z)*<<2>>+(y^2-z^2)*<<1>>+(-z*y^2+z^2*y)*<<0>>
[210] dp_prim(X);
(1)*<<2>>+(-y-z)*<<1>>+(z*y)*<<0>>

References
Section 6.3.18 [ptozp], page 57.

8.10.17 dp_nf, dp_nf_mod, dp_true_nf, dp_true_nf_mod

dp_nf(indexlist, dpoly, dpolyarray, fullreduce)
dp_weyl_nf(indexlist, dpoly, dpolyarray, fullreduce)
dp_nf_mod(indexlist, dpoly, dpolyarray, fullreduce, mod)
dp_weyl_nf_mod(indexlist, dpoly, dpolyarray, fullreduce, mod)
    :: Computes the normal form of a distributed polynomial. (The result may be
    multiplied by a constant in the ground field.)

return dp_nf() : distributed polynomial, dp_true_nf() : list
indexlist list
dpoly distributed polynomial
dpolyarray array of distributed polynomial
**fullreduce** flag

**mod** prime

- Computes the normal form of a distributed polynomial.
- Functions whose name contain `weyl` compute normal forms in Weyl algebra. The description below also applies to the functions for Weyl algebra.
- `dp_nf_mod()` and `dp_true_nf_mod()` require distributed polynomials with coefficients in a finite field as arguments.
- The result of `dp_nf()` may be multiplied by a constant in the ground field in order to make the result integral. The same is true for `dp_nf_mod()`, but it returns the true normal form if the ground field is a finite field.
- `dp_true_nf()` and `dp_true_nf_mod()` return such a list as `[nm, dn]`. Here `nm` is a distributed polynomial whose coefficients are integral in the ground field, `dn` is an integral element in the ground field and `nm/dn` is the true normal form.
- `dpolyarray` is a vector whose components are distributed polynomials and `indexlist` is a list of indices which is used for the normal form computation.
- When argument `fullreduce` has non-zero value, all terms are reduced. When it has value 0, only the head term is reduced.
- As for the polynomials specified by `indexlist`, one specified by an index placed at the preceding position has priority to be selected.
- In general, the result of the function may be different depending on `indexlist`. However, the result is unique for Groebner bases.
- These functions are useful when a fixed non-distributed polynomial set is used as a set of reducers to compute normal forms of many polynomials. For single computation `p_nf` and `p_true_nf` are sufficient.

```math
[0] load("gr")$
[64] load("katsura")$
[69] K=katsura(4)$
[70] dp_ord(2)$
[71] V=[u_0,u_1,u_2,u_3,u_4]$
[72] DP1=newvect(length(K),map(dp_ptod,K,V))$
[73] G=gr(K,V,2)$
[74] DP2=newvect(length(G),map(dp_ptod,G,V))$
[75] T=dp_ptod((u_0-u_1+u_2-u_3+u_4)^2,V)$
[76] dp_dtop(dp_nf([0,1,2,3,4],T,DP1,1),V);
  u_4^2+(6*u_3+2*u_2+6*u_1-2)*u_4+9*u_3^2+(6*u_2+18*u_1-6)*u_3+u_2^2
    +(6*u_1-2)*u_2+9*u_1^2-6*u_1+1$
[77] dp_dtop(dp_nf([4,3,2,1,0],T,DP1,1),V);
  -5*u_4^2+(-4*u_3-4*u_2-2*u_1)*u_4+u_3^2-2-3*u_3-3*u_2^2+2*u_1-2)*u_2^2+2-3*u_1+1$
[78] dp_dtop(dp_nf([0,1,2,3,4],T,DP2,1),V);
  -1130879768451657780886122972730785203470970010204714556333530492210
    456775930005716505560062087150928400876150217079820311439477560587583
    488*u_4^15+...
[79] dp_dtop(dp_nf([4,3,2,1,0],T,DP2,1),V);
  -1130879768451657780886122972730785203470970010204714556333530492210
    456775930005716505560062087150928400876150217079820311439477560587583
```
488*u4^15+...
[80] @78==@79;
1

References
Section 8.10.13 [dp_dtop], page 141, Section 8.10.10 [dp_ord], page 139, Section
8.10.14 [dp_mod dp_rat], page 142, Section 8.10.28 [p_nf p_nf_mod p_
true_nf p_true_nf_mod], page 150.

8.10.18 dp_hm, dp_ht, dp_hc, dp_rest

dp_hm(dpoly)
:: Gets the head monomial.

dp_ht(dpoly)
:: Gets the head term.

dp_hc(dpoly)
:: Gets the head coefficient.

dp_rest(dpoly)
:: Gets the remainder of the polynomial where the head monomial is removed.

return dp_hm(), dp_ht(), dp_rest() : distributed polynomial dp_hc() : number or polynomial

dpoly distributed polynomial

- These are used to get various parts of a distributed polynomial.
- The next equations hold for a distributed polynomial $p$.

\[
p = dp_hm(p) + dp_rest(p)
\]

\[
dp_hm(p) = dp_hc(p) \cdot dp_ht(p)
\]

\[
[87] \text{dp_ord(0)}$
\]

\[
[88] X=\text{ptozp}((a46^2+7/10*a46+7/48)*u3^4-50/27*a46^2-35/27*a46-49/216)$
\]

\[
[89] T=\text{dp_ptod}(X, [u3,u4,a46])$
\]

\[
[90] \text{dp_hm(T)};
\]

\[
(2160)<<4,0,2>>$
\]

\[
[91] \text{dp_ht(T)};
\]

\[
(1)<<4,0,2>>$
\]

\[
[92] \text{dp_hc(T)};
\]

\[
2160$
\]

\[
[93] \text{dp_rest(T)};
\]

\[
(1512)<<4,0,1>>+(315)<<4,0,0>>+(-4000)<<0,0,2>>+(-2800)<<0,0,1>>
\]

\[
+(-490)<<0,0,0>>$
\]

8.10.19 dp_td, dp_sugar

dp_td(dpoly)
:: Gets the total degree of the head term.

dp_sugar(dpoly)
:: Gets the sugar of a polynomial.
Function `dp_td()` returns the total degree of the head term, i.e., the sum of all exponent of variables in that term.

Upon creation of a distributed polynomial, an integer called `sugar` is associated. This value is the total degree of the virtually homogenized one of the original polynomial.

The quantity `sugar` is an important guide to determine the selection strategy of critical pairs in Groebner basis computation.

```
[74] dp_ord(0)
[75] X=<<1,2>>+<<0,1>>
[76] Y=<<1,2>>+<<1,0>>
[77] Z=X-Y;
(-1)*<<1,0>>+(1)*<<0,1>>
[78] dp_sugar(T);
3
```

8.10.20 dp_lcm

```
dp_lcm(dpoly1, dpoly2)
:: Returns the least common multiple of the head terms of the given two polynomials.
```

8.10.21 dp_redble

```
dp_redble(dpoly1, dpoly2)
:: Checks whether one head term is divisible by the other head term.
```

References
Section 8.10.28 [p_nf p_nf_mod p_true_nf p_true_nf_mod], page 150.
C;
(1)\times\langle 1,1,1,0,0 \rangle+(1)\times\langle 0,1,1,1,0 \rangle+(1)\times\langle 1,1,0,0,1 \rangle+(1)\times\langle 1,0,0,1,1 \rangle

T;
(3)\times\langle 2,1,0,0,0 \rangle+(3)\times\langle 1,2,0,0,0 \rangle+(1)\times\langle 0,3,0,0,0 \rangle+(6)\times\langle 1,1,1,0,0 \rangle

for ( ; T; T = dp_rest(T)) print(dp_redble(T,C));

References
Section 8.10.26 [dp_red dp_red_mod], page 149.

**8.10.22 dp_subd**

dp_subd(dpoly1, dpoly2)
:: Returns the quotient monomial of the head terms.

return distributed polynomial
dpoly1 dpoly2
distributed polynomial

- Gets dp_ht(dpoly1)/dp_ht(dpoly2). The coefficient of the result is always set to 1.
- Divisibility assumed.

[162] dp_subd(\langle 1,2,3,4,5 \rangle,\langle 1,1,2,3,4 \rangle);
(1)\times\langle 0,1,1,1,1 \rangle

References
Section 8.10.26 [dp_red dp_red_mod], page 149.

**8.10.23 dp_vtoe, dp_etov**

dp_vtoe(vect)
:: Converts an exponent vector into a term.

dp_etov(dpoly)
:: Convert the head term of a distributed polynomial into an exponent vector.

return dp_vtoe : distributed polynomial, dp_etov : vector
vect vector
dpoly distributed polynomial

- dp_vtoe() generates a term whose exponent vector is vect.
- dp_etov() generates a vector which is the exponent vector of the head term of dpoly.

[211] X=\langle 1,2,3 \rangle;
(1)\times\langle 1,2,3 \rangle
[212] V=dp_etov(X);
[ 1 2 3 ]
[214] Y=dp_vtoe(V);
(1)\times\langle 1,2,4 \rangle
8.10.24 dp_mbase

dp_mbase(dp list)
:: Computes the monomial basis

return list of distributed polynomial

dp list list of distributed polynomial

- Assuming that dp list is a list of distributed polynomials which is a Groebner basis with respect to the current ordering type and that the ideal $I$ generated by dp list in $K[X]$ is zero-dimensional, this function computes the monomial basis of a finite dimensional $K$-vector space $K[X]/I$.

- The number of elements in the monomial basis is equal to the $K$-dimension of $K[X]/I$.

```kotlin
[215] K=katsura(5)$
[216] V=[u5,u4,u3,u2,u1,u0]$ 
[217] G0=gr(K,V,0)$
[218] H=map(dp_ptod,G0,V)$
[219] map(dp_ptod,dp_mbase(H),V)$
[u0^5,u4*u0^3,u3*u0^3,u2*u0^3,u1*u0^3,u0^4,u3^2*u0,u2*u3*u0,u1*u3*u0,
 u1*u2*u0,u1^2*u0,u4*u0^2,u3*u0^2,u2*u0^2,u1*u0^2,u0^3,u3^2,u2*u3,u1*u3,
 u1*u2,u1^2,u4*u0,u3*u0,u2*u0,u1*u0,u0^2,u4,u3,u2,u1,u0,1]
```

References
Section 8.10.1 [gr hgr gr_mod], page 131.

8.10.25 dp_mag

dp_mag(p)
:: Computes the sum of bit lengths of coefficients of a distributed polynomial.

return integer

p distributed polynomial

- This function computes the sum of bit lengths of coefficients of a distributed polynomial $p$. If a coefficient is non integral, the sum of bit lengths of the numerator and the denominator is taken.

- This is a measure of the size of a polynomial. Especially for zero-dimensional system coefficient swells are often serious and the returned value is useful to detect such swells.

- If ShowMag and Print for dp_gr_flags() are on, values of dp_mag() for intermediate basis elements are shown.

```kotlin
[221] X=dp_ptod((x+2*y)^10,[x,y])$
[222] dp_mag(X); 115
```

References
Section 8.10.9 [dp_gr_flags dp_gr_print], page 139.
8.10.26 dp_red, dp_red_mod

\[
\text{dp\_red}(\text{dpoly1, dpoly2, dpoly3}) \\
\text{dp\_red\_mod}(\text{dpoly1, dpoly2, dpoly3, mod})
\]
:: Single reduction operation

\[
\text{return list } \\
dpoly1 \text{ dpoly2 dpoly3} \\
\text{distributed polynomial vlist} \\
\text{list mod prime}
\]

- Reduces a distributed polynomial, \(\text{dpoly1 + dpoly2}\), by \(\text{dpoly3}\) for single time.
- An input for \text{dp\_red\_mod()} must be converted into a distributed polynomial with coefficients in a finite field.
- This implies that the divisibility of the head term of \(\text{dpoly2}\) by the head term of \(\text{dpoly3}\) is assumed.
- When integral coefficients, computation is so carefully performed that no rational operations appear in the reduction procedure. It is computed for integers \(a\) and \(b\), and a term \(t\) as: \(a(\text{dpoly1 + dpoly2})-bt\ \text{dpoly3}\).
- The result is a list \([a \text{ dpoly1}, a \text{ dpoly2} - bt \text{ dpoly3}]\).

\[
[157] \text{D}=(3)*\lll<2,1,0,0,0>+\lll<1,2,0,0,0>+(1)*\lll<0,3,0,0,0>>; \\
(3)*\lll<2,1,0,0,0>+\lll<1,2,0,0,0>+(1)*\lll<0,3,0,0,0>> \\
[158] \text{R}=(6)*\lll<1,1,1,0,0>>; \\
(6)*\lll<1,1,1,0,0>> \\
[159] \text{C}=12*\lll<1,1,1,0,0>+(1)*\lll<0,1,1,0,1>+(1)*\lll<1,1,0,1>>; \\
(12)*\lll<1,1,1,0,0>+(1)*\lll<0,1,1,1,0>>+(1)*\lll<1,1,0,1> \\
[160] \text{dp\_red}(D,R,C); \\
[(6)*\lll<2,1,0,0,0>+(6)*\lll<1,2,0,0,0>+(2)*\lll<0,3,0,0,0>>, \\
(-1)*\lll<0,1,1,1,0>+(1)*\lll<1,1,0,1>>]
\]

References

Section 8.10.14 [dp\_mod dp\_rat], page 142.

8.10.27 dp_sp, dp_sp_mod

\[
\text{dp\_sp}(\text{dpoly1, dpoly2}) \\
\text{dp\_sp\_mod}(\text{dpoly1, dpoly2, mod})
\]
:: Computation of an S-polynomial

\[
\text{return distributed polynomial } \\
dpoly1 \text{ dpoly2} \\
\text{distributed polynomial mod prime}
\]

- This function computes the S-polynomial of \(\text{dpoly1}\) and \(\text{dpoly2}\).
- Inputs of \text{dp\_sp\_mod()} must be polynomials with coefficients in a finite field.
The result may be multiplied by a constant in the ground field in order to make the result integral.

```
[227] X=dp_ptod(x^2*y+x*y,[x,y]);
(1)*<<2,1>>+(1)*<<1,1>>
[228] Y=dp_ptod(x*y^2+x*y,[x,y]);
(1)*<<1,2>>+(1)*<<1,1>>
[229] dp_sp(X,Y);
(-1)*<<2,1>>+(1)*<<1,2>>
```

References
Section 8.10.14 [dp_mod dp_rat], page 142.

8.10.28 \( p_{\text{nf}}, p_{\text{nf}\mod}, p_{\text{true\ nf}}, p_{\text{true\ nf}\mod} \)

\[
p_{\text{nf}}(\text{poly, plist, vlist, order})
\]

\[
p_{\text{nf}\mod}(\text{poly, plist, vlist, order, mod})
\]

:: Computes the normal form of the given polynomial. (The result may be multiplied by a constant.)

\[
p_{\text{true\ nf}}(\text{poly, plist, vlist, order})
\]

\[
p_{\text{true\ nf}\mod}(\text{poly, plist, vlist, order, mod})
\]

:: Computes the normal form of the given polynomial. (The result is returned as a form of \([\text{numerator}, \text{denominator}]\))

\[
\text{return} \quad p_{\text{nf}} : \text{polynomial, } p_{\text{true\ nf}} : \text{list}
\]

\[
\text{poly} \quad \text{polynomial}
\]

\[
\text{plist} \quad \text{list}
\]

\[
\text{vlist} \quad \text{list}
\]

\[
\text{order} \quad \text{number, list or matrix}
\]

\[
\text{mod} \quad \text{prime}
\]

- Defined in the package ‘gr’.
- Obtains the normal form of a polynomial by a polynomial list.
- These are interfaces to \(dp_{\text{nf}}(), dp_{\text{true\ nf}}(), dp_{\text{nf}\mod}(), dp_{\text{true\ nf}\mod}()\)
- The polynomial \(\text{poly}\) and the polynomials in \(\text{plist}\) is converted, according to the variable ordering \(\text{vlist}\) and type of term ordering \(\text{otype}\), into their distributed polynomial counterparts and passed to \(dp_{\text{nf}}()\).
- \(dp_{\text{nf}}(), dp_{\text{true\ nf}}(), dp_{\text{nf}\mod}()\) and \(dp_{\text{true\ nf}\mod}()\) is called with value 1 for \text{fullreduce}.
- The result is converted back into an ordinary polynomial.
- As for \(p_{\text{true\ nf}}(), p_{\text{true\ nf}\mod}()\) refer to \(dp_{\text{true\ nf}}()\) and \(dp_{\text{true\ nf}\mod}()\).

```plain
[79] K = katsura(5)$
[80] V = [u5,u4,u3,u2,u1,u0]$  
[81] G = hgr(K,V,2)$
[82] p_nf(K[1],G,V,2);
0
[83] L = p_true_nf(K[1]+1,G,V,2);
```
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[{-1503..., -1503...}]

[84] L[0]/L[1];
1

References
Section 8.10.12 [dp_ptod], page 141, Section 8.10.13 [dp_dtop], page 141, Section 8.10.10 [dp_ord], page 139, Section 8.10.17 [dp_nf dp_nf_mod dp_true_nf dp_true_nf_mod dp_weyl nf dp_weyl nf_mod], page 143.

8.10.29 p_terms

p_terms(poly, vlist, order)
:: Monomials appearing in the given polynomial is collected into a list.
return list
poly polynomial
vlist list
order number, list or matrix

- Defined in the package `gr'.
- This returns a list which contains all non-zero monomials in the given polynomial. The monomials are ordered according to the current type of term ordering and vlist.
- Since polynomials in a Groebner base often have very large coefficients, examining a polynomial as it is may sometimes be difficult to perform. For such a case, this function enables to examine which term is really exists.

\[
\begin{align*}
G &= \text{gr}(\text{katsura}(5), [u5, u4, u3, u2, u1, u0], 2) \\
\text{p_terms}(G[0], [u5, u4, u3, u2, u1, u0], 2); \\
&= [u5, u0^31, u0^30, u0^29, u0^28, u0^27, u0^26, u0^25, u0^24, u0^23, u0^22, \\
u0^21, u0^20, u0^19, u0^18, u0^17, u0^16, u0^15, u0^14, u0^13, u0^12, u0^11, \\
u0^10, u0^9, u0^8, u0^7, u0^6, u0^5, u0^4, u0^3, u0^2, u0, 1]
\end{align*}
\]

8.10.30 gb_comp

gb_comp(plist1, plist2)
:: Checks whether two polynomial lists are equal or not as a set
return 0 or 1
plist1 plist2

- This function checks whether plist1 and plist2 are equal or not as a set.
- For the same input and the same term ordering different functions for Groebner basis computations may produce different outputs as lists. This function compares such lists whether they are equal as a generating set of an ideal.

\[
\begin{align*}
C &= \text{cyclic}(6) \\
V &= \text{cyclic}(6) \\
G &= \text{gr}(C, V, 0) \\
G &= \text{tolex}(G0, V, 0, V) \\
G &= \text{lex_tl}(C, V, 0, V, 0) \\
gb_comp(G, GG); \\
&= 1
\end{align*}
\]
8.10.31 katsura, hkatsura, cyclic, hcyclic

\[ \text{katsura}(n) \]
\[ \text{hkatsura}(n) \]
\[ \text{cyclic}(n) \]
\[ \text{hcyclic}(n) \]

:: Generates a polynomial list of standard benchmark.

\[ n \text{ integer} \]

- Function \text{katsura()} is defined in \text{'katsura'}, and function \text{cyclic()} in \text{'cyclic'}.
- These functions generate a series of polynomial sets, respectively, which are often used for testing and benchmarking: \text{katsura}, \text{cyclic} and their homogenized versions.
- Polynomial set \text{cyclic} is sometimes called by other name: \text{Arnborg}, \text{Lazard}, and \text{Davenport}.

```
[74] load("katsura")$
[79] load("cyclic")$
[89] katsura(5);
[u0+2*u4+2*u3+2*u2+2*u1+2*u5-1, 2*u4*u0-u4+2*u1*u3+u2^2+2*u5*u1,
  2*u3*u0+2*u1*u4-u3+(2*u1+2*u5)*u2, 2*u2*u0+2*u2*u4+(2*u1+2*u5)*u3
-2*u1^2, 2*u1*u0+(2*u3+2*u5)*u4+2*u2*u3+2*u1*u2-u1, u0^2-2*u0+2*u4^2+2*u3^2+2*u2^2+2*u1^2+2*u5^2]
[90] hkatsura(5);
[-t+u0+2*u4+2*u3+2*u2+2*u1+2*u5,
  -u4*t+2*u4*u0+2*u1*u3+u2^2+2*u5*u1,-u3*t+2*u3*u0+2*u1*u4+(2*u1+2*u5)*u2,
  -u2*t+2*u2*u0+2*u2*u4+(2*u1+2*u5)*u3+u1^2,
  -u1*t+2*u1*u0+(2*u3+2*u5)*u4+2*u2*u3+2*u1*u2,
  -u0*t+u0^2+2*u4^2+2*u3^2+2*u2^2+2*u1^2+2*u5^2]
[91] cyclic(6);
[c5*c4*c3*c2*c1*c0-1, (((c4+c5)*c3+c5*c4+c3)*c1+c5+c4+c3)*c0+c5*c4+c3*c2, (((c3+c5)*c2+c5*c4+c3)*c1+c5*c4+c3)*c0+c4*c3+c2*c1+c5*c4+c3, ((c2+c5)*c1+c5*c4)*c0+c3*c2+c1+c4*c3*c2+c5*c4+c3, (c1+c5)*c0+c2*c1+c3*c2+c4*c3+c5+c4, c0+c1+c2+c3+c4+c5]
[92] hcyclic(6);
[-c^6+c5*c4*c3*c2*c1*c0, (((c4+c5)*c3+c5*c4)*c2+c5*c4+c3)*c1+c5+c4+c3*c2, (((c3+c5)*c2+c5*c4)*c1+c5*c4+c3)*c0+c4*c3+c2*c1+c5*c4+c3, ((c2+c5)*c1+c5*c4)*c0+c3*c2+c1+c4*c3*c2+c5*c4+c3, (c1+c5)*c0+c2*c1+c3*c2+c4*c3+c5+c4, c0+c1+c2+c3+c4+c5]
```

References

Section 8.10.13 [dp_dtop], page 141.

8.10.32 primadec, primedec

\[ \text{primadec}(\text{plist, vlist}) \]
primedec(plist, vlist)
:: Computes decompositions of ideals.

return
plist list of polynomials
vlist list of variables

- Function primadec() and primedec are defined in ‘primdec’.
- primadec(), primedec() are the function for primary ideal decomposition and prime
decomposition of the radical over the rationals respectively.
- The arguments are a list of polynomials and a list of variables. These functions accept
ideals with rational function coefficients only.
- primadec returns the list of pair lists consisting a primary component and its associated
prime.
- primedec returns the list of prime components.
- Each component is a Groebner basis and the corresponding term order is indicated by
the global variables PRIMAORD, PRIMEORD respectively.
- primadec implements the primary decompostion algorithm in [Shimoyama,Yokoyama].
- If one only wants to know the prime components of an ideal, then use primedec because
primadec may need additional costs if an input ideal is not radical.

[84] load("primdec")$
[102] primedec((p*q*x-q^2*y^2+p^2*x^2+p^2*x*p*q*y,
(q^3*y^4-2*q^3*y^3+q^3*y^2)*x-q^3*y^4+q^3*y^3,
-q^3*y^4+2*q^3*y^3+(-q^3+p*q^2)*y^2), [p,q,x,y]);
[[y,x], [y,p], [x,q], [q,p], [x-1,q], [y-1,p], [(y-1)*x-y, q*y^2-2*q*y-p+q]]
[103] primadec((x, z*y, w^2*y-z^3, y^3), [x, y, z, w]);
[[x, z*y, y^2, w^2*y-z^3], [z*y, w, x], [[w, x, z*y, z^3, y^3], [w, z, y, x]]]

References
Section 6.3.15 [fctr sqfr], page 54, Section 8.5 [Setting term orderings],
page 126.

8.10.33 primedec_mod

primedec_mod(plist, vlist, ord, mod, strategy)
:: Computes decompositions of ideals over small finite fields.

return
plist list of polynomials
vlist list of variables
ord number, list or matrix
mod positive integer
strategy integer

- Function primedec_mod() is defined in ‘primdec_mod’ and implements the prime de-
composition algorithm in [Yokoyama].
• `primedec_mod()` is the function for prime ideal decomposition of the radical of a polynomial ideal over small finite field, and they return a list of prime ideals, which are associated primes of the input ideal.

• `primedec_mod()` gives the decomposition over GF(mod). The generators of each resulting component consists of integral polynomials.

• Each resulting component is a Groebner basis with respect to a term order specified by `[vlist, ord]`.

• If `strategy` is non zero, then the early termination strategy is tried by computing the intersection of obtained components incrementally. In general, this strategy is useful when the krull dimension of the ideal is high, but it may add some overhead if the dimension is small.

• If you want to see internal information during the computation, execute `dp_gr_print(2)` in advance.

```plaintext
[0] load("primdec_mod")$
[246] PP444=[x^8+x^2+t,y^8+y^2+t,z^8+z^2+t]$
[247] primedec_mod(PP444,[x,y,z,t],0,2,1);

[=[y+z,x+z,z^8+z^2+t], [x+y,y^2+y+z^2+z+1,z^8+z^2+t],
[y+z+1,x+z+1,z^8+z^2+t], [x+z,y^2+y+z^2+z+1,z^8+z^2+t],
[y+z,x^2+x+z^2+1,z^8+z^2+t], [y+z+1,x^2+x+z^2+1,z^8+z^2+t],
[x+z+1,y^2+y+z^2+1,z^8+z^2+t], [y+z+1,x+z+1,z^8+z^2+t],
[x+y+1,y^2+y+z^2+1,z^8+z^2+t], [y+z,x+z+1,z^8+z^2+t]]$
```

References

Section 6.3.17 [modfctr], page 56, Section 8.10.6 [dp_gr_main dp_gr_mod_main dp_gr_f_main dp_weyl_gr_main dp_weyl_gr_mod_main dp_weyl_gr_f_main], page 136, Section 8.5 [Setting term orderings], page 126, Section 8.10.9 [dp_gr_flags dp_gr_print], page 139.

8.10.34 bfunction, bfct, generic_bfct, ann, ann0

bfunction(f)
bfct(f)
generic_bfct(plist, vlist, dvlist, weight):
    :: Computes the global b function of a polynomial or an ideal
ann(f)
ann0(f)    :: Computes the annihilator of a power of polynomial
return polynomial or list
f polynomial
plist list of polynomials
vlist dvlist list of variables

• These functions are defined in ‘bfct’.

- `bfunction(f)` and `bfct(f)` compute the global $b$-function $b(s)$ of a polynomial $f$. $b(s)$ is a polynomial of the minimal degree such that there exists $P(x,s)$ in $D[s]$, which is a polynomial ring over Weyl algebra $D$, and $P(x,s)f^{s+1}=b(s)f^s$ holds.

- `generic_bfct(f,vlist,dvlist,weight)` computes the global $b$-function of a left ideal $I$ in $D$ generated by $plist$, with respect to $weight$. $vlist$ is the list of $x$-variables, $vlist$ is the list of corresponding $D$-variables.

- `bfunction(f)` and `bfct(f)` implement different algorithms and the efficiency depends on inputs.

- `ann(f)` returns the generator set of the annihilator ideal of $f^s$. `ann(f)` returns a list $[a, list]$, where $a$ is the minimal integral root of the global $b$-function of $f$, and $list$ is a list of polynomials obtained by substituting $s$ in $ann(f)$ with $a$.

- See [Saito, Sturmfels, Takayama] for the details.

```plaintext
[0] load("bfct")$
[216] bfunction(x^3+y^3+z^3+x^2*y^2*z^2+x*y*z);
  -9*s^5-63*s^4-173*s^3-233*s^2-154*s-40
[217] fctr(@);
[[[-1,1],[s+2,1],[3*s+4,1],[3*s+5,1],[s+1,2]]
[218] F = [4*x^3*dt+y*z*dt+dx,x*z*dt+4*y^3*dt+dy,
x*y*dt+5*z^4*dt+dz,-x^4-z*y*x-y^4-z^5+t]$
[219] generic_bfct(F,[t,z,y,x],[dt,dz,dy,dx],[1,0,0,0]);
  20000*s^10-70000*s^9+101750*s^8-79375*s^7+35768*s^6-9277*s^5+
  1278*s^4-72*s^3
[220] P=x^3-y^2$
[221] ann(P);
[2*dy*x+3*dx*y^2,-3*dx*x-2*dy*y+6*s]
[222] ann0(P);
[-1,[2*dy*x+3*dx*y^2,-3*dx*x-2*dy*y-6]]
```

References

Section 8.9 [Weyl algebra], page 130.
9 Algebraic numbers

9.1 Representation of algebraic numbers

In Asir algebraic number fields are not defined as independent objects. Instead, individual algebraic numbers are defined by some means. In Asir an algebraic number field is defined virtually as a number field obtained by adjoining a finite number of algebraic numbers to the rational number field.

A new algebraic number is introduced in Asir in such a way where it is defined as a root of an uni-variate polynomial whose coefficients include already defined algebraic numbers as well as rational numbers. We shall call such a newly defined algebraic number a root. Also, we call such an uni-variate polynomial the defining polynomial of that root.

\[ A0 = \text{newalg}(x^2+1); \]
\[ A1 = \text{newalg}(x^3+A0*x+A0); \]
\[ \text{[type}(A0),\text{ntype}(A0)]; \]
\[ 1,2 \]

In this example, the algebraic number assigned to \( A0 \) is defined as a root of a polynomial \( x^2+1 \); that of \( A1 \) is defined as a root of a polynomial \( x^3+A0*x+A0 \), which you see contains the previously defined root \( A0 \) in its coefficients.

The argument to \text{newalg}(), i.e., the defining polynomial, must satisfy the following conditions.

1. A defining polynomial must be an uni-variate polynomial.
2. A defining polynomial is used to simplify expressions containing that algebraic number.
   The procedure of such simplification is performed by an internal routine similar to the built-in function \text{srem}(), where the defining polynomial is used for the second argument, the divisor. By this reason, the leading coefficient of the defining polynomial must be a rational number (must not be an algebraic number.)
3. Every coefficients of a defining polynomial must be a ‘(multi-variate) polynomial’ in already defined root’s. Here, ‘(multi-variate) polynomial’ means a mathematical concept, not the object type ‘polynomial’ in Asir.
4. A defining polynomial must be irreducible over the field that is obtained by adjoining all root’s contained in its coefficients to the rational number field.

Only the first two conditions (1 and 2) are checked by function \text{newalg}(). Among all, it should be emphasized that no check is done for the irreducibility at all. The reason is that the irreducibility test requires enormously much computation time. You are trusted whether to check it at your own risk.

Once a root has been defined by \text{newalg}() function, it is given the type identifier for a number, and furthermore, the sub-type identifier for an algebraic number. (See Section 6.8.1 [type], page 76, Section 6.8.2 [ntype], page 77.) Also, any rational combination of rational numbers and root’s is an algebraic number.

\[ N = \frac{(A0^2+A1)}{(A1^2-A0-1)}; \]
\[ ((#1+#0^2)/(#1^2-#0-1)) \]
As you see it in the example, a root is displayed as \#n. But, you cannot input that root in its immediate output form. You have to refer to a root by a program variable assigned to the root, or to get it by \texttt{alg(n)} function, or by several other indirect means. A strange use of \texttt{newalg()}, with a same argument polynomial (except for the name of its main variable), will yield the old root instead of a new root though it is apparently inefficient.

\begin{verbatim}
[90] alg(0);
(#0)
[91] newalg(t^2+1);
(#0)
\end{verbatim}

The defining polynomial of a root can be obtained by \texttt{defpoly()} function.

\begin{verbatim}
[96] defpoly(A0);
t#0^2+1
[97] defpoly(A1);
t#1^3+t#0*t#1+t#0
\end{verbatim}

Here, you see a strange expression, t#0 and t#1. They are a specially indeterminates generated and maintained by Asir internally. Indeterminate t#0 corresponds to root #0, and t#0 to #1. These indeterminates also cannot be input directly by a user in their immediate forms. You can get them by several ways: by \texttt{var()} function, or \texttt{algv(n)} function.

\begin{verbatim}
[98] var(@);
t#1
[99] algv(0);
t#0
\end{verbatim}

\section{Operations over algebraic numbers}

In the previous section, we explained about the representation of algebraic numbers and their defining method. Here, we describe operations on algebraic numbers. Only a few functions are built-in, and almost all functions are provided as user defined functions. The file containing their definitions is `sp', and it is placed under the same directory as `gr' is placed, i.e., the standard library directory of Asir.

\begin{verbatim}
[0] load("gr")$
[1] load("sp")$
\end{verbatim}

Or if you always need them, it is more convenient to include the \texttt{load} commands in `\texttt{\$HOME/.asirrc}'.

Like the other numbers, algebraic numbers can get arithmetic operations applied. Simplification, however, by defining polynomials are not automatically performed. It is left to users to simplify their expressions. A fatal error shall result if the denominator expression will be simplified to 0. Therefore, be careful enough when you will create and deal with algebraic numbers which may denominators in their expressions.

Use \texttt{simpalg()} function for simplification of algebraic numbers by defining polynomials.

\begin{verbatim}
[49] T=A0^2+1;
(#0^2+1)
\end{verbatim}
Function \texttt{simpalg()} simplifies algebraic numbers which have the same structures as rational expressions in their appearances.

\begin{verbatim}
[39] A0=newalg(x^2+1);
(#0)
[40] T=(A0^2+A0+1)/(A0+3);
((#0^2+#0+1)/(#0+3))
[41] simpalg(T);
(3/10*#0+1/10)
[42] T=1/(A0^2+1);
((1)/(#0^2+1))
[43] simpalg(T);
div : division by 0
stopped in invalgp at line 258 in file "/usr/local/lib/asir/sp"
258 return 1/A;
(debug)
\end{verbatim}

This example shows an error caused by zero division in the course of program execution of \texttt{simpalg()}, which attempted to simplify an algebraic number expression of which the denominator is 0.

Function \texttt{simpalg()} also can take a polynomial as its argument and simplifies algebraic numbers in its coefficients.

\begin{verbatim}
[43] simpalg(1/A0*x+1/(A0+1));
(-#0)*x+(-1/2*#0+1/2)
\end{verbatim}

Thus, you can operate in polynomials which contain algebraic numbers as you do usually in ordinary polynomials, except for proper simplification by \texttt{simpalg()}. You may sometimes feel needs to convert \texttt{root}'s into indeterminates, especially when you are working for norm computation in algorithms for algebraic factorization. Function \texttt{algptorat()} is used for such cases.

\begin{verbatim}
[83] A0=newalg(x^2+1);
(#0)
[84] A1=newalg(x^3+A0*x+A0);
(#1)
[85] T=(2*A0+A1*A0+A1^2)*x+(1+A1)/(2+A0);
(#1^2+#0*#1+2*#0)*x+((#1+1)/(#0+2))
[86] S=algptorat(T);
(((t#0+2)*t#1^2+(t#0^2+2*t#0)*t#1+2*t#0)*x+(t#1+1)/t#0+2)
[87] algptorat(coef(T,1));
t#1^2+t#0*t#1+2*t#0
\end{verbatim}

As you see by the example, function \texttt{algptorat()} converts \texttt{root}'s, \texttt{#n}, in polynomials and numbers into its associated indeterminates, \texttt{t#n}. As was already mentioned those indeterminates cannot be directly input in their immediate form. The restriction is adopted to avoid the confusion that might happen if the user could input such internally generateable indeterminates.

The associated indeterminate to a \texttt{root} is reversely converted into the \texttt{root} by \texttt{rattoalgp()} function.
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[88] rattoalgp(S,[alg(0)]);
(((#0+2)/(#0+2))*t#1^2+((#0^2+2*#0)/(#0+2))*t#1 +((2*#0^2+4*#0)/(#0+2))*t#1+(1)/(#0+2))*t#1+(1)/(#0+2))

[89] rattoalgp(S,[alg(0),alg(1)]);
(((#0^3+6*#0^2+12*#0+8)*#1^2+(#0^4+6*#0^3+12*#0^2+8*#0)*#1 +2*#0^4+12*#0^3+24*#0^2+16*#0)/(#0^3+6*#0^2+12*#0+8))*x
+(((#0+2)*#1+1)/(#0^2+4*#0+4))

[90] rattoalgp(S,[alg(1),alg(0)]);
(((#0+2)*#1^2+(#0^2+2*#0)*#1+2*#0^2+4*#0)/(#0+2))*x
+((#1+1)/(#0+2))

[91] simpalg(89);
(1)*<<1,0>>+(1)*<<0,1>>

[92] simpalg(90);
(-1)*<<0,0>>

Function \texttt{rattoalgp()} takes as the second argument a list consisting of \texttt{root}'s that you want to convert, and converts them successively from the left. This example shows that apparent difference of the results due to the order of such conversion will vanish by simplification yielding the same result. Functions \texttt{algptorat()} and \texttt{rattoalgp()} can be conveniently used for your own simplification.

9.3 Representation of algebraic numbers by distributed polynomials

Simplification of algebraic numbers containing \texttt{root} is not done automatically and should be done by users. There is another representation of algebraic numbers, for which the results of fundamental operations are automatically simplified. This representations are designed so that operations are efficiently performed especially when the field is a successive extension and it can be used as a ground field for Groebner basis related functions. Internally an algebraic number of this type is defined as an object called \texttt{DAlg}. A \texttt{DAlg} is represented as a fraction. The denominator is an integer and the numerator is a distributed polynomial with integral coefficients.

\texttt{DAlg} is generated as an element of an algebraic number field set by \texttt{set_field()}. There are two methods to generate a \texttt{DAlg}. \texttt{algtodalg()} converts an algebraic number containing \texttt{root} to \texttt{DAlg}. \texttt{dptodalg()} directly converts a distributed polynomial to \texttt{DAlg}.

\texttt{DAlg} is generated as an element of an algebraic number field set by \texttt{set_field()}. There are two methods to generate a \texttt{DAlg}. \texttt{algtodalg()} converts an algebraic number containing \texttt{root} to \texttt{DAlg}. \texttt{dptodalg()} directly converts a distributed polynomial to \texttt{DAlg}.

\texttt{[0] A=newalg(x^2+1);
(#0)
[1] B=newalg(x^3+A*x+A);
(#1)
[2] set_field(B,A));
0
[3] C=algtodalg(A+B);
((1)*<<1,0>>+(1)*<<0,1>>)
[4] C^5;
((-11)*<<2,1>>+(5)*<<2,0>>+(10)*<<1,1>>+(9)*<<1,0>>+(11)*<<0,1>> +(-1)*<<0,0>>)
[5] 1/C;
((2)*<<2,1>>+(-1)*<<2,0>>+(1)*<<1,1>>+(2)*<<1,0>>+(-3)*<<0,1>>)
In this example \( \mathbb{Q}(\alpha, \beta) \) \((\alpha^2 + 1 = 0, \beta^3 + \alpha \beta + \beta = 0)\) is set as the current ground field, and \((\alpha + \beta)^5\) and \(1/(\alpha + \beta)\) are simplified in the field. The numerators of the results are printed as distributed polynomials.

### 9.4 Operations for uni-variate polynomials over an algebraic number field

In the file `sp` are provided functions for uni-variate polynomial factorization and uni-variate polynomial GCD computation over algebraic numbers, and furthermore, as an application of them, functions to compute splitting fields of univariate polynomials.

#### 9.4.1 GCD

Greatest common divisors (GCD) over algebraic number fields are computed by `cr_gcd()` function. This function computes GCD by using modular computation and Chinese remainder theorem and it works for the case where the ground field is a multiple extension.

\[
[63] \text{A=newalg}(t^9-15*t^6-87*t^3-125); \\
(\#0) \\
[64] \text{B=newalg}(75*s^2+(10*A^7-175*A^4-470*A)*s+3*A^8-45*A^5-261*A^2); \\
(\#1) \\
[65] \text{P1=75*x^2+(150*B+10*A^7-175*A^4-395*A)*x} \\
+(75*B^2+(10*A^7-175*A^4-395*A)*B+13*A^8-220*A^5-581*A^2)*x \\
[66] \text{P2=x^2+A*x+A^2} \\
[67] \text{cr_gcd}(\text{P1}, \text{P2}); \\
27*x+((\#0^6-19*\#0^3-65)*\#1-\#0^7+19*\#0^4+38*\#0)
\]

#### 9.4.2 Square-free factorization and Factorization

For square-free factorization (of uni-variate polynomials over algebraic number fields), we employ the most fundamental algorithm which begins first to compute GCD of a polynomial and its derivative. The function to do this factorization is `asq()`.

\[
[116] \text{A=newalg}(x^2+x+1); \\
(\#4) \\
[117] \text{T=simpalg}((x+A+1)*(x^2-2*A-3)^2*(x^3-x-A)^2); \\
x^11+(-4*\#4-4)*x^10+(-10*\#4-4)*x^9+(16*\#4+20)*x^8 \\
+(24*\#4-6)*x^7+(-29*\#4-31)*x^6+(-15*\#4+28)*x^5+(-15*\#4+28)*x^4 \\
+(24*\#4-6)*x^3+(-29*\#4-31)*x^2+(-2*\#4-7)*x+(-3*\#4+8) \\
[118] \text{asq}(\text{T}); \\
[[x^5+(-2*\#4-4)*x^3+(-4)*x^2+2*\#4+3]*x+(\#4-2), \#1^1, 1]]
\]

Like factorization over the rational number field, the result is presented, commonly to both square-free factorization and factorization, as a list whose elements are pairs (list of two elements) in the form `[factor, multiplicity]` without the constant multiple part.

Here, it should be noticed that the products of all factors of the result may DIFFER from the input polynomial by a constant. The reason is that the factors are normalized so that they have integral leading coefficients for the sake of readability.

This incongruity may happen to square-free factorization and factorization commonly.
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The algorithm employed for factorization over algebraic number fields is an improvement of the norm method by Trager. It is especially very effective to factorize a polynomial over a field obtained by adjoining some of its root’s to the base field.

\[ \text{af}(T, [A]); \]
\[ [[x^3-x+(-#4),2], [x^2+(-2*#4-3),2], [x+(#4+1),1]] \]

The function takes two arguments: The second argument is a list of root’s. Factorization is performed over a field obtained by adjoining the root’s to the rational number field. It is important to keep in mind that the ordering of the root’s must obey a restriction: Last defined should come first. The automatic re-ordering is not done. It should be done by yourself.

The efficiency of factorization via norm depends on the efficiency of the norm computation and univariate factorization over the rationals. Especially the latter often causes combinatorial explosion and the computation will stick in such a case.

\[ Bnewline(x^2-2*A-3); \]
\[ (#5) \]
\[ \text{af}(T, [B,A]); \]
\[ [[x+(#5),2], [x^3-x+(-#4),2], [x+(-#5),2], [x+(#4+1),1]] \]

9.4.3 Splitting fields

This operation may be somewhat unusual and for specific interest. (Galois Group for example.) Procedurally, however, it is easy to obtain the splitting field of a polynomial by repeated application of algebraic factorization described in the previous section. The function is \( \text{sp}() \).

\[ \text{sp}(x^5-2); \]
\[ [[x+(-#1),2*x+(-#0^3*#1^3+#0^4*#1^2+2*#1+2*#0), 2*x+(-#0^4*#1^2), 2*x+(-#0^3*#1^3), x+(-#0)], \]
\[ [(#1), t#0 t#0 t#0 t#0 t#0 t#0 t#0 t#0 t#0 t#0 t#0 t#0 t#0 t#0 t#0 t#0 t#0]] \]

Function \( \text{sp}() \) takes only one argument. The result is a list of two elements: The first element is a list of linear factors, and the second one is a list whose elements are pairs (list of two elements) in the form \([\text{root}, \text{algptorat(\text{defining polynomial})}]\). The second element, a list of pairs of form \([\text{root}, \text{algptorat(\text{defining polynomial})}]\), corresponds to the root’s which are adjoined to eventually obtain the splitting field. They are listed in the reverse order of adjoining. Each of the defining polynomials in the list is, of course, guaranteed to be irreducible over the field obtained by adjoining all root’s defined before it.

The first element of the result, a list of linear factors, contains all irreducible factors of the input polynomial over the field obtained by adjoining all root’s in the second element of the result. Because such field is the splitting field of the input polynomial, factors in the result are all linear as the consequence.

Similarly to function \( \text{af}() \), the product of all resulting factors may yield a polynomial which differs by a constant.

9.5 Summary of functions for algebraic numbers
9.5.1 newalg

\texttt{newalg(defpoly)}

\begin{itemize}
  \item Creates a new \texttt{root}.
\end{itemize}

\texttt{return} \quad \texttt{algebraic number (root)}

\texttt{defpoly} \quad \texttt{polynomial}

\begin{itemize}
  \item Creates a new \texttt{root} (algebraic number) with its defining polynomial \texttt{defpoly}.
  \item For constraints on \texttt{defpoly}, See Section 9.1 \texttt{[Representation of algebraic numbers]}, page 156.
\end{itemize}

[0] \texttt{A0=newalg(x^2-2)}; \quad (#0)

Reference

Section 9.5.2 \texttt{[defpoly]}, page 162

9.5.2 defpoly

\texttt{defpoly(alg)}

\begin{itemize}
  \item Returns the defining polynomial of \texttt{root alg}.
\end{itemize}

\texttt{return} \quad \texttt{polynomial}

\texttt{alg} \quad \texttt{algebraic number (root)}

\begin{itemize}
  \item Returns the defining polynomial of \texttt{root alg}.
  \item If the argument \texttt{alg}, a \texttt{root}, is \texttt{#n}, then the main variable of its defining polynomial is \texttt{t#n}.
\end{itemize}

[1] \texttt{defpoly(A0)}; \quad \texttt{t#0^2-2}

Reference

Section 9.5.1 \texttt{[newalg]}, page 162, Section 9.5.3 \texttt{[alg]}, page 162, Section 9.5.4 \texttt{[algv]}, page 163

9.5.3 alg

\texttt{alg(i)} \quad \begin{itemize}
  \item Returns \texttt{#i}, a \texttt{root}.
  \item Because \texttt{#i} cannot be input directly, this function provides an alternative way: input \texttt{alg(i)}.
\end{itemize}

[2] \texttt{x+#0}; \quad \texttt{syntax error}

[3] \texttt{alg(0)}; \quad (#0)

Reference

Section 9.5.1 \texttt{[newalg]}, page 162, Section 9.5.4 \texttt{[algv]}, page 163
9.5.4 \texttt{algv}

\texttt{algv}(i) \quad \text{:: Returns the associated indeterminate with \texttt{alg}(i).}

\textit{return} \quad \text{polynomial}

\textit{i} \quad \text{integer}

\begin{itemize}
  \item Returns an indeterminate $t#i$
  \item Since indeterminate $t#i$ cannot be input directly, it is input by \texttt{algv}(i).
\end{itemize}

\begin{verbatim}
[4] var(defpoly(A0));
t#0
[5] t#0;
syntax error
0
[6] algv(0);
t#0
\end{verbatim}

Reference
Section 9.5.1 [\texttt{newalg}], page 162, Section 9.5.2 [\texttt{defpoly}], page 162, Section 9.5.3 [\texttt{alg}], page 162

9.5.5 \texttt{simpalg}

\texttt{simpalg}(\textit{rat}) \quad \text{:: Simplifies algebraic numbers in a rational expression.}

\textit{return} \quad \text{rational expression}

\textit{rat} \quad \text{rational expression}

\begin{itemize}
  \item Defined in the file `sp'.
  \item Simplifies algebraic numbers contained in numbers, polynomials, and rational expressions by the defining polynomials of root's contained in them.
  \item If the argument is a number having the denominator, it is rationalized and the result is a polynomial in root's.
  \item If the argument is a polynomial, each coefficient is simplified.
  \item If the argument is a rational expression, its denominator and numerator are simplified as a polynomial.
\end{itemize}

\begin{verbatim}
[7] simpalg((1+A0)/(1-A0));
simpalg undefined
return to toplevel
[7] load("sp")$
[46] simpalg((1+A0)/(1-A0));
(-2*#0-3)
[47] simpalg((2-A0)/(2+A0)*x^2-1/(3+A0));
(-2*#0+3)*x^2+1/7*#0-3/7
[48] simpalg((x+1/(A0-1))/(x-1/(A0+1)));
(x+(#0+1))/(x+(-#0+1))
\end{verbatim}
9.5.6 \texttt{algptorat}\\
\texttt{algptorat}(\texttt{poly}) \quad :: \text{ Substitutes the associated indeterminate for every root}\\
\texttt{return} \quad \texttt{polynomial}\\
\texttt{poly} \quad \texttt{polynomial}\\
\begin{itemize}
  \item Defined in the file ‘sp’.
  \item Substitutes the associated indeterminate \texttt{t\#n} for every root \texttt{\#n} in a polynomial.
\end{itemize}\\
\begin{verbatim}
[49] algptorat((-2*alg(0)+3)*x^2+(1/7*alg(0)-3/7));
(-2*t#0+3)*x^2+1/7*t#0-3/7
\end{verbatim}\\
\textbf{Reference} \\
Section 9.5.2 [\texttt{defpoly}], page 162, Section 9.5.4 [\texttt{algv}], page 163\\

9.5.7 \texttt{rattoalgp}\\
\texttt{rattoalgp}(\texttt{poly}, \texttt{alglist}) \quad :: \text{ Substitutes a root for the associated indeterminate with the root.}\\
\texttt{return} \quad \texttt{polynomial}\\
\texttt{poly} \quad \texttt{polynomial}\\
\texttt{alglist} \quad \texttt{list}\\
\begin{itemize}
  \item Defined in the file ‘sp’.
  \item The second argument is a list of root’s. Function \texttt{rattoalgp()} substitutes a root for the associated indeterminate of the root.
\end{itemize}\\
\begin{verbatim}
[51] rattoalgp((-2*algv(0)+3)*x^2+(1/7*algv(0)-3/7),[alg(0)]);
(-2*#0+3)*x^2+(1/7*#0-3/7)
\end{verbatim}\\
\textbf{Reference} \\
Section 9.5.3 [\texttt{alg}], page 162, Section 9.5.4 [\texttt{algv}], page 163\\

9.5.8 \texttt{cr\_gcd}\_a\\
\texttt{cr\_gcd}(\texttt{poly1}, \texttt{poly2}) \quad :: \text{ GCD of two uni-variate polynomials over an algebraic number field.}\\
\texttt{return} \quad \texttt{polynomial}\\
\texttt{poly1} \quad \texttt{poly2} \quad \texttt{polynomial}\\
\begin{itemize}
  \item Defined in the file ‘sp’.
  \item Finds the GCD of two uni-variate polynomials.
\end{itemize}\\
\begin{verbatim}
[76] X=x^6+3*x^5+6*x^4+x^3-3*x^2+12*x+16$
[77] Y=x^6+6*x^5+24*x^4+8*x^3-48*x^2+384*x+1024$
[78] A=newalg(X);
(#0)
[79] cr\_gcd(X,subst(Y,x,x+A));
x+(-#0)
\end{verbatim}
9.5.9 \texttt{sp\_norm}

\texttt{sp\_norm(alg, var, poly, alglist)}
\begin{verbatim}
:: Norm computation over an algebraic number field.
\end{verbatim}

\begin{itemize}
  \item Defined in the file `sp'.
  \item Computes the norm of \texttt{poly} with respect to \texttt{alg}. Namely, if we write $K = \mathbb{Q}(\texttt{alglist} \setminus \{\texttt{alg}\})$, The function returns a product of all conjugates of \texttt{poly}, where the conjugate of polynomial \texttt{poly} is a polynomial in which the algebraic number \texttt{alg} is substituted for its conjugate over $K$.
  \item The result is a polynomial over $K$.
  \item The method of computation depends on the input. Currently direct computation of resultant and Chinese remainder theorem are used but the selection is not necessarily optimal. By setting the global variable \texttt{USE\_RES} to 1, the builtin function \texttt{res()} is always used.
\end{itemize}

\begin{verbatim}
[0] load("sp")$
[39] A0=newalg(x^2+1)$
[40] A1=newalg(x^2+A0)$
[41] sp\_norm(A1,x,x^3+A0*x+A1,[A1,A0]);
   x^6+(2*#0)*x^4+(#0^2)*x^2+(#0)
[42] sp\_norm(A0,x,@@,[A0]);
   x^12+2*x^8+5*x^4+1
\end{verbatim}

9.5.10 \texttt{asq, af, af\_noalg}

\texttt{asq(poly)} :: Square-free factorization of polynomial \texttt{poly} over an algebraic number field.

\texttt{af(poly, alglist)}

\texttt{af\_noalg(poly, defpolylist)} :: Factorization of polynomial \texttt{poly} over an algebraic number field.

\begin{verbatim}
return list
poly polynomial
alglist root list
\end{verbatim}
**defpolylist** root list of pairs of an indeterminate and a polynomial

- Both defined in the file ‘sp’.
- If the inputs contain no root’s, these functions run fast since they invoke functions over the integers. In contrast to this, if the inputs contain root’s, they sometimes take a long time, since cr gcda() is invoked.
- Function af() requires the specification of base field, i.e., list of root’s for its second argument.
- In the second argument alglist, root defined last must come first.
- In af(F,AL), AL denotes a list of roots and it represents an algebraic number field. In AL=[A_n,...,A_1] each A_k should be defined as a root of a defining polynomial whose coefficients are in Q(A(k+1),...,A_n).

```
[1] A1 = newalg(x^2+1);
[2] A2 = newalg(x^2+A1);
[3] A3 = newalg(x^2+A2*x+A1);
[[x^2+(#1)*x+(#0),1]]
```

To call sp_noalg, one should replace each algebraic number ai in poly with an indeterminate vi. defpolylist is a list [[vn,dn(vn,...,v1)],...,[v1,d(v1)]. In this expression di(vi,...,v1) is a defining polynomial of ai represented as a multivariate polynomial.

```
[1] af_noalg(x^2+a2*x+a1,[[a2,a2^2+a1],[a1,a1^2+1]]);
[[x^2+a2*x+a1,1]]
```

- The result is a list, as a result of usual factorization, whose elements is of the form [factor, multiplicity]. In the result of af_noalg, algebraic numbers in factor are replaced by the indeterminates according to defpolylist.
- The product of all factors with multiplicities counted may differ from the input polynomial by a constant.

```
[98] A = newalg(t^2-2);
(#0)
[99] asq(-x^4+6*x^3+(2*alg(0)-9)*x^2+(-6*alg(0))*x-2);
[[-x^2+3*x+(#0),2]]
[100] af(-x^2+3*x+alg(0),[alg(0)]);
[[x+(#0-1),1],[-x+(#0+2),1]]
[101] af_noalg(-x^2+3*x+a,[[a,x^2-2]]);
[[x+a-1,1],[-x+a+2,1]]
```

**Reference**

Section 9.5.8 [cr gcda], page 164, Section 6.3.15 [fctr sqfr], page 54

**9.5.11 sp, sp_noalg**

sp(poly)
sp_noalg(poly)
:: Finds the splitting field of polynomial poly and splits.

return list
poly polynomial
• Defined in the file ‘sp’.
• Finds the splitting field of \( poly \), an uni-variate polynomial over with rational coefficients, and splits it into its linear factors over the field.
• The result consists of a two element list: The first element is the list of all linear factors of \( poly \); the second element is a list which represents the successive extension of the field. In the result of \texttt{sp_noalg} all the algebraic numbers are replaced by the special indeterminate associated with it, that is \( t_i \) for \( i \). By this operation the result of \texttt{sp_noalg} is a list containing only integral polynomials.
• The splitting field is represented as a list of pairs of form \([\text{root, algptorat(defpoly(root))}]\). In more detail, the list is interpreted as a representation of successive extension obtained by adjoining \texttt{root}'s to the rational number field. Adjoining is performed from the right \texttt{root} to the left.
• \texttt{sp()} invokes \texttt{sp_norm()} internally. Computation of norm is done by several methods according to the situation but the algorithm selection is not always optimal and a simple resultant computation is often superior to the other methods. By setting the global variable \texttt{USE_RES} to 1, the builtin function \texttt{res()} is always used.

```pascal
L=sp(x^9-54);
[[x+(-#2),-54*x+(#1^6*#2^4),54*x+(#1^6*#2^4+54*#2),
54*x+(-#1^8*#2^2),-54*x+(#1^5*#2^5),54*x+(#1^5*#2^5+#1^8*#2^2),
-54*x+(-#1^7*#2^3-54*#1),54*x+(-#1^7*#2^3),x+(-#1)],
[(#2),t#2^6+t#1^3*t#2^3+t#1^6],[(#1),t#1^9-54]]
```

```pascal
for(I=0;I<9;I++)M*=L[0][I];
```

```pascal
M=simpalg(M);
```

```pascal
-x^9+54
```

Reference
Section 9.5.10 \[asq af af_noalg\], page 165, Section 9.5.2 \[defpoly\], page 162, Section 9.5.6 \[algptorat\], page 164, Section 9.5.9 \[sp_norm\], page 165.

9.5.12 \texttt{set_field}

\texttt{set_field(rootlist)}

:: Set an algebraic number field as the current ground field.

\texttt{return 0}

\texttt{rootlist} A list of root

• \texttt{set_field()} sets an algebraic number field generated by \texttt{root} in \texttt{rootlist} over \( \mathbb{Q} \).
• You don’t care about the order of \texttt{root} in \texttt{rootlist}, because \texttt{root} are automatically ordered internally.

```pascal
A=newalg(x^2+1);
(#0)
B=newalg(x^3+A);
(#1)
C=newalg(x^4+B);
(#1)
```
9.5.13 \texttt{algtodalg, dalgtoalg, dptodalg, dalgtodp}

\texttt{algtodalg(alg)}
:: Converts an algebraic number \texttt{alg} to a \texttt{DAlg}.

\texttt{dalgtoalg(dalg)}
:: Converts a \texttt{DAlg} \texttt{dalg} to an algebraic number.

\texttt{dptodalg(dp)}
:: Converts an algebraic number \texttt{alg} to a \texttt{DAlg}.

\texttt{dalgtodp(dalg)}
:: Converts a \texttt{DAlg} \texttt{dalg} to an algebraic number.

\textit{return} An algebraic number, a \texttt{DAlg} or a list [distributed polynomial,denominator]

\texttt{alg} an algebraic number containing root

\texttt{dp} a distributed polynomial over \textit{Q}

- These functions are converters between \texttt{DAlg} and an algebraic number containing \texttt{root}, or a distributed polynomial.
- A ground field to which a \texttt{DAlg} belongs must be set by \texttt{set_field()} in advance.
- \texttt{dalgtodp()} returns a list containing the numerator (a distributed polynomial) and the denominator (an integer).
- \texttt{algtodalg()}, \texttt{dptodalg()} return the simplified result.

[0] A=newalg(x^2+1);
(#0)

[1] B=newalg(x^3+A*x+A);
(#1)

[2] set_field([B,A]);
0

[3] C=algtodalg((A+B)^10);
((408)*<<2,1>>+(103)*<<2,0>>+(-36)*<<1,1>>+(-446)*<<1,0>>
+(-332)*<<0,1>>+(-218)*<<0,0>>)

[4] dalgtoalg(C);
((408*#0+103)*#1^2+(-36*#0-446)*#1-332*#0-218)

[5] D=dptodalg((<<10,10>>/10+2*<<5,5>>+1/3*<<0,0>>));
((-9)*<<2,1>>+(-57)*<<2,0>>+(-63)*<<1,1>>+(-12)*<<1,0>>
+(-60)*<<0,1>>+(1)*<<0,0>>)/30

[6] dalgtodp(D);
[(-9)*<<2,1>>+(-57)*<<2,0>>+(-63)*<<1,1>>+(-12)*<<1,0>>
+(-60)*<<0,1>>+(1)*<<0,0>>,30]
10 Finite fields

10.1 Representation of finite fields

On Asir GF(p), GF(2\(^n\)), GF(p\(^n\)) can be defined, where GF(p) is a finite prime field of characteristic p, GF(2\(^n\)) is a finite field of characteristic 2 and GF(p\(^n\)) is a finite extension of GF(p). These are all defined by `setmod_ff()`.

```plaintext
[0] P=pari(nextprime,2^50);
1125899906842679
[1] setmod_ff(P);
1125899906842679
[2] field_type_ff();
1
[3] load("fff");
1
[4] F=defpoly_mod2(50);
x^50+x^4+x^3+x^2+1
[5] setmod_ff(F);
x^50+x^4+x^3+x^2+1
[6] field_type_ff();
2
[7] setmod_ff(x^3+x+1,1125899906842679);
[1*x^3+1*x+1,1125899906842679]
[8] field_type_ff();
3
[9] setmod_ff(3,5);
[3,x^5+2*x+1,1]
[10] field_type_ff();
4
```

If \( p \) is a positive integer, `setmod_ff(p)` sets GF(p) as the current base field. If \( f \) is a univariate polynomial of degree \( n \), `setmod_ff(f)` sets GF(2\(^n\)) as the current base field. GF(2\(^n\)) is represented as an algebraic extension of GF(2) with the defining polynomial \( f \) mod 2. Furthermore, finite extensions of prime finite fields can be defined. See Section 3.2 [Types of numbers], page 14. In all cases the primality check of the argument is not done and the caller is responsible for it.

Correctly speaking there is no actual object corresponding to a 'base field'. Setting a base field means that operations on elements of finite fields are done according to the arithmetics of the base field. Thus, if operands of an arithmetic operation are both rational numbers, then the result is also a rational number. However, if one of the operands is in a finite field, then the other is automatically regarded as in the same finite field and the operation is done in the finite field.

A non zero element of a finite field belongs to the number and has object identifier 1. Its number identifier is 6 if the finite field is GF(p), 7 if it is GF(2\(^n\)).

There are several methods to input an element of a finite field. An element of GF(p) can be input by `simp_ff()`.
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```
[0] P=pari(nextprime,2^50);
1125899906842679
[1] setmod_ff(P);
1125899906842679
[2] A=simp_ff(2^100);
3025
[3] ntype(0);
6
```

In the case of GF(2^\(n\)) the following methods are available.

```
[0] setmod_ff(x^50+x^4+x^3+x^2+1);
x^50+x^4+x^3+x^2+1
[1] A=0;
(0)
[2] ptogf2n(x^50+1);
(0^50+1)
[3] simp_ff(0);
(0^4+0^3+0^2)
[4] ntogf2n(2^10-1);
(0^9+0^8+0^7+0^6+0^5+0^4+0^3+0^2+0^1)
```

Elements of finite fields are numbers and one can apply field arithmetics to them. \(0\) is a generator of GF(2^\(n\)) over GF(2). See Section 3.2 [Types of numbers], page 14.

### 10.2 Univariate polynomials on finite fields

In ‘fff’ square-free factorization, DDF (distinct degree factorization), irreducible factorization and primality check are implemented for univariate polynomials over finite fields.

Factorizers return lists of [factor, multiplicity]. The factor part is monic and the information on the leading coefficient of the input polynomial is abandoned.

The algorithm used in square-free factorization is the most primitive one.

The irreducible factorization proceeds as follows.

1. DDF
2. Nullspace computation by Berlekamp algorithm
3. Root finding of minimal polynomials of bases of the nullspace
4. Separation of irreducible factors by the roots

### 10.3 Polynomials on small finite fields

A multivariate polynomial over small finite field set by setmod_ff(p,n) can be factorized by using a builtin function sffctr(). modfctr() also factorizes a polynomial over a finite prime field. Internally, modfctr() creates a sufficiently large field extension of the specified ground field, and it calls sffctr(), then it constructs irreducible factors over the ground field from the factors returned by sffctr().
10.4 Elliptic curves on finite fields

Several fundamental operations on elliptic curves over finite fields are provided as built-in functions.

An elliptic curve is specified by a vector \([a \ b]\) of length 2, where \(a, b\) are elements of finite fields. If the current base field is a prime field, then \([a \ b]\) represents \(y^2=x^3+ax+b\). If the current base field is a finite field of characteristic 2, then \([a \ b]\) represents \(y^2+xy=x^3+ax^2+b\).

Points on an elliptic curve together with the point at infinity forms an additive group. The addition, the subtraction and the additive inverse operation are provided as `ecm_add_ff()`, `ecm_sub_ff()` and `ecm_chsgn_ff()` respectively. Here the representation of points are as follows.

- 0 denotes the point at infinity.
- The other points are represented by vectors \([x \ y \ z]\) of length 3 with non-zero \(z\).

\([x \ y \ z]\) represents a projective coordinate and it corresponds to \([x/z \ y/z]\) in the affine coordinate. To apply the above operations to a point \([x \ y]\), \([x \ y \ 1]\) should be used instead as an argument. The result of an operation is also represented by the projective coordinate. As the third coordinate is not always equal to 1, one has to divide the first and the second coordinate by the third one to obtain the affine coordinate.

10.5 Functions for Finite fields

10.5.1 `setmod_ff`

```
setmod_ff([p|defpoly2])
setmod_ff([defpoly,p])
setmod_ff([p,n])
```

:: Sets/Gets the current base fields.

return number or polynomial

\(p\) prime

\(defpoly2\) univariate polynomial irreducible over GF(2)

\(defpoly\) univariate polynomial irreducible over GF(\(p\))

\(n\) the extension degree

- If the argument is a non-negative integer \(p\), GF\((p)\) is set as the current base field.
- If the argument is a polynomial \(defpoly2\), GF\((2^\text{deg}(defpoly2 \mod 2)) = GF(2)[t]/(defpoly2(t) \mod 2)\) is set as the current base field.
- If the arguments are a polynomial \(defpoly\) and a prime \(p\), GF\((p^\text{deg}(defpoly)) = GF(p)[t]/(defpoly(t))\) is set as the current base field.
- If the arguments are a prime \(p\) and an extension degree \(n\), GF\((p^n)\) is set as the current base field. \(p^n\) must be less than \(2^5\ 29\) and if \(p\) is greater than or equal to \(2^14\), then \(n\) must be equal to 1.
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- If no argument is specified, the modulus indicating the current base field is returned. If the current base field is GF(\(p\)), \(p\) is returned. If it is GF(\(2^\text{\textit{n}}\)), the defining polynomial is returned. If it is GF(\(p^{\text{\textit{n}}}\)) defined by \texttt{setmod_ff(\textit{defpoly}, \textit{p}, \textit{defpolyp}, \textit{p})} is returned. If it is GF(\(p^{\text{\textit{n}}}\)) defined by \texttt{setmod_ff(\textit{p}, \textit{n})}, \([\textit{p}, \textit{defpoly}, \textit{prim\_elem}]\) is returned. Here, \textit{defpoly} is the defining polynomial of the \textit{n}-th extension, and \textit{prim\_elem} is the generator of the multiplicative group of GF(\(p^{\text{\textit{n}}}\)).

- Any irreducible univariate polynomial over GF(2) is available to set GF(\(2^\text{\textit{n}}\)). However, the use of \texttt{defpoly\_mod2()} is recommended for efficiency.

\begin{verbatim}
[174] defpoly\_mod2(100);
x^100+x^15+1
[175] setmod\_ff(@@);
x^100+x^15+1
[176] setmod\_ff();
x^100+x^15+1
[177] setmod\_ff(x^4+x+1,547);
[1*x^4+1*x+1,547]
[178] setmod\_ff(2,5);
[2,x^5+x^2+1,x]
\end{verbatim}

References
Section 10.5.14 [defpoly\_mod2], page 179

10.5.2 field\_type\_ff

\texttt{field\_type\_ff()}

\texttt{:: Type of the current base field.}

\texttt{return integer}

- Returns the type of the current base field.
- If no field is set, 0 is returned. If GF(\(p\)) is set, 1 is returned. If GF(\(2^\text{\textit{n}}\)) is set, 2 is returned.

\begin{verbatim}
[0] field\_type\_ff();
0
[1] setmod\_ff(3);
3
[2] field\_type\_ff();
1
[3] setmod\_ff(x^2+x+1);
x^2+x+1
[4] field\_type\_ff();
2
\end{verbatim}

References
Section 10.5.1 [setmod\_ff], page 172

10.5.3 field\_order\_ff

\texttt{field\_order\_ff()}

\texttt{:: Order of the current base field.}
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return integer
- Returns the order of the current base field.
- \( q \) is returned if the current base field is \( GF(q) \).

[0] field_order_ff();
field_order_ff : current_ff is not set
return to toplevel
[0] setmod_ff(3);
3
[1] field_order_ff();
3
[2] setmod_ff(x^2+x+1);
x^2+x+1
[3] field_order_ff();
4

References
Section 10.5.1 \[setmod_ff\], page 172

10.5.4 characteristic_ff

characteristic_ff()
:: Characteristic of the current base field.
return integer
- Returns the characteristic of the current base field.
- \( p \) is returned if \( GF(p) \), where \( p \) is a prime, is set. \( 2 \) is returned if \( GF(2^n) \) is set.

[0] characteristic_ff();
characteristic_ff : current_ff is not set
return to toplevel
[0] setmod_ff(3);
3
[1] characteristic_ff();
3
[2] setmod_ff(x^2+x+1);
x^2+x+1
[3] characteristic_ff();
2

References
Section 10.5.1 \[setmod_ff\], page 172

10.5.5 extdeg_ff

extdeg_ff()
:: Extension degree of the current base field over the prime field.
return integer
- Returns the extension degree of the current base field over the prime field.
- \( 1 \) is returned if \( GF(p) \), where \( p \) is a prime, is set. \( n \) is returned if \( GF(2^n) \) is set.
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10.5.6 simp_ff

simp_ff(obj)
  :: Converts numbers or coefficients of polynomials into elements in finite fields.

return number or polynomial

obj number or polynomial

- Converts numbers or coefficients of polynomials into elements in finite fields.
- It is used to convert integers or integral polynomials into elements of finite fields or polynomials over finite fields.
- An element of a finite field may not have the reduced representation. In such case an application of simp_ff ensures that the output has the reduced representation. If a small finite field is set as a ground field, an integer is projected the finite prime field, then it is embedded into the ground field. ptosfp() can be used for direct projection to the ground field.

[0] simp_ff((x+1)^10);
x^10+10*x^9+45*x^8+120*x^7+210*x^6+252*x^5+210*x^4+120*x^3+45*x^2+10*x+1
[1] simp_ff((x+1)^10);
1*x^10+1*x^9+1*x+1
[3] ntype(coef(@@,10));
6
[4] setmod_ff(2,3);
[2,x^3+x+1,x]
[5] simp_ff(1);
@_0
[6] simp_ff(2);
0
[7] ptosfp(2);
@_1

References
  Section 10.5.1 [setmod_ff], page 172
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References
Section 10.5.1 [setmod_ff], page 172, Section 10.5.8 [lmptop], page 176, Section 10.5.10 [gf2nton], page 177, Section 10.5.13 [ptosfp sfptop], page 179

10.5.7 random_ff

random_ff()
:: Random generation of an element of a finite field.

return element of a finite field

• Generates an element of the current base field randomly.
• The same random generator as in random(), lrandom() is used.

[0] random_ff();
random_ff : current_ff is not set
return to toplevel
[0] setmod_ff(pari(nextprime,2^40));
1099511627791
[1] random_ff();
561856154357
[2] random_ff();
45141628299

References
Section 10.5.1 [setmod_ff], page 172, Section 6.1.8 [random], page 38, Section 6.1.9 [lrandom], page 38

10.5.8 lmptop

lmptop(obj)
:: Converts the coefficients of a polynomial over GF(p) into integers.

return integral polynomial

obj polynomial over GF(p)

• Converts the coefficients of a polynomial over GF(p) into integers.
• An element of GF(p) is represented by a non-negative integer r less than p. Each coefficient of a polynomial is converted into an integer object whose value is r.

[0] setmod_ff(pari(nextprime,2^40));
1099511627791
[1] F=simp_ff((x-1)^10);
1*x^10+1099511627781*x^9+45*x^8+1099511627671*x^7+210*x^6
+1099511627539*x^5+210*x^4+1099511627671*x^3+45*x^2+1099511627781*x+1
[2] setmod_ff(547);
547
[3] F=simp_ff((x-1)^10);
1*x^10+537*x^9+45*x^8+427*x^7+210*x^6+295*x^5+210*x^4+427*x^3
+45*x^2+537*x+1
[4] lmptop(F);
x^10+537*x^9+45*x^8+427*x^7+210*x^6+295*x^5+210*x^4+427*x^3
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\[ +45x^2+537x+1 \]

[5] \text{lmptop(coef(F,1));}
537
[6] \text{ntype(@@);}
0

References

Section 10.5.6 \texttt{simp_ff}, page 175

10.5.9 \texttt{ntogf2n}

\texttt{ntogf2n(m)}

:: Converts a non-negative integer into an element of GF(2\textsuperscript{r}).

\texttt{return element of GF(2\textsuperscript{r})}

\texttt{m} non-negative integer

- Let \( m \) be a non-negative integer. \( m \) has the binary representation \( m=m_0m_12^1+...+mk2^k \). This function returns an element of GF(2\textsuperscript{r}) = GF(2)[t]/(g(t)), \( m_0m_1t+...+mk^k \mod g(t) \).

- Apply \texttt{simp_ff()} to reduce the result.

10.5.10 \texttt{gf2nton}

\texttt{gf2nton(m)}

:: Converts an element of GF(2\textsuperscript{r}) into a non-negative integer.

\texttt{return non-negative integer}

\texttt{m} element of GF(2\textsuperscript{r})

- The inverse of \texttt{gf2nton}.

References

Section 10.5.10 \texttt{gf2nton}, page 177

10.5.10 \texttt{gf2nton}

\texttt{gf2nton(m)}

:: Converts an element of GF(2\textsuperscript{r}) into a non-negative integer.

\texttt{return non-negative integer}

\texttt{m} element of GF(2\textsuperscript{r})

- The inverse of \texttt{gf2nton}.

References

Section 10.5.10 \texttt{gf2nton}, page 177
10.5.11 \texttt{ptogf2n}

\texttt{ptogf2n}(\texttt{poly})

:: Converts a univariate polynomial into an element of GF(2\(^n\)).

\texttt{return} \quad \text{element of GF(2\(^n\))}

\texttt{poly} \quad \text{univariate polynomial}

- Generates an element of GF(2\(^n\)) represented by \texttt{poly}. The coefficients are reduced modulo 2. The output is equal to the result by substituting \@ for the variable of \texttt{poly}.

\begin{verbatim}
[1] setmod_ff(x^30+x+1);
x^30+x+1
[2] ptogf2n(x^100);
(\@^100)
\end{verbatim}

References

Section 10.5.12 [\texttt{gf2ntop}], page 178

10.5.12 \texttt{gf2ntop}

\texttt{gf2ntop}(\texttt{m}[,\texttt{v}])

:: Converts an element of GF(2\(^n\)) into a polynomial.

\texttt{return} \quad \text{univariate polynomial}

\texttt{m} \quad \text{an element of GF(2\(^n\))}

\texttt{v} \quad \text{indeterminate}

- Returns a polynomial representing \texttt{m}.
- If \texttt{v} is used as the variable of the output. If \texttt{v} is not specified, the variable of the argument of the latest \texttt{ptogf2n()} call. The default variable is \texttt{x}.

\begin{verbatim}
[1] setmod_ff(x^30+x+1);
x^30+x+1
[2] N=simp_ff(gf2ntop(2^100));
(\@^13+\@^12+\@^11+\@^10)
[5] gf2ntop(N);
[207] gf2ntop(N);
x^13+x^12+x^11+x^10
[208] gf2ntop(N,t);
t^13+t^12+t^11+t^10
\end{verbatim}

References

Section 10.5.11 [\texttt{ptogf2n}], page 178
10.5.13 ptosfp, sfptop

\begin{verbatim}
ptosfp(p) :: Transformation to/from a small finite field

return polynomial

p polynomial

- ptosfp() converts coefficients of a polynomial to elements in a small finite field GF(p^n) set as a ground field. If a coefficient is already an element of the field, no conversion is done. If a coefficient is a positive integer, then its residue modulo p^n is expanded as p-adic integer, then p is substituted by x, finally the polynomial is converted to its corresponding logarithmic representation with respect to the primitive element. For example, GF(3^5) is represented as F(3)[x]/(x^5+2*x+1), and each element of the field is represented as \( \mathbb{Q}_k \) by its exponent k with respect to the primitive element x. 23 = 2*3^2+3+2 is represented as \( 2*3^2+3+2 \) and it is equivalent to \( x^{17} \) modulo \( x^5+2*3+1 \). Therefore an integer 23 is converted to \( \mathbb{Q}_{17} \).

- sfptop() is the inverse of ptosfp().
\end{verbatim}

References
Section 10.5.1 [setmod_ff], page 172, Section 10.5.6 [simp_ff], page 175

10.5.14 defpoly_mod2

defpoly_mod2(d) :: Generates an irreducible univariate polynomial over GF(2).

return univariate polynomial

d positive integer

- Defined in ‘fff’.
- An irreducible univariate polynomial of degree d is returned.
- If an irreducible trinomial \( x^d+x^m+1 \) exists, then the one with the smallest m is returned. Otherwise, an irreducible pentanomial \( x^d+x^m1+x^m2+x^m3+1 \) (m1>m2>m3) is returned. m1, m2 and m3 are determined as follows: Fix m1 as small as possible. Then fix m2 as small as possible. Then fix m3 as small as possible.
References

Section 10.5.1 [setmod_ff], page 172

10.5.15 sffctr

\textbf{sffctr}(\textit{poly})

\hspace{1em}:: \text{Irreducible factorization over a small finite field.}

\textit{return} \hspace{1em} list

\textit{poly} \hspace{1em} polynomial over a finite field

- Factorize \textit{poly} into irreducible factors over a small finite field currently set.
- The result is a list $[[f_1,m_1],[f_2,m_2],...], \text{where } f_i \text{ is a monic irreducible factor and } m_i \text{ is its multiplicity.}$

\begin{verbatim}
[0] setmod_ff(2,10);
[2,x^10+x^3+1,x]
[1] sffctr((z*y^3+z*y)*x^3+(y^5+y^3+z*y^2+z)*x^2+z^11*y*x+z^10*y^3+z^11);
[[[@_0,1],[@_0*z*y*x+@_0*y^3+@_0*z,1],[(@_0*y+@_0)*x+@_0*z^5,2]]
\end{verbatim}

References

Section 10.5.1 [setmod_ff], page 172, Section 6.3.17 [modfctr], page 56

10.5.16 fctr_ff

\textbf{fctr_ff}(\textit{poly})

\hspace{1em}:: \text{Irreducible univariate factorization over a finite field.}

\textit{return} \hspace{1em} list

\textit{poly} \hspace{1em} univariate polynomial over a finite field

- Defined in `fff'.
- Factorize \textit{poly} into irreducible factors over the current base field.
- The result is a list $[[f_1,m_1],[f_2,m_2],...], \text{where } f_i \text{ is a monic irreducible factor and } m_i \text{ is its multiplicity.}$
- The leading coefficient of \textit{poly} is abandoned.

\begin{verbatim}
[178] setmod_ff(2^64-95);
18446744073709551521
[179] fctr_ff(x^5+x+1);
[[1*x+14123390394564558010,1],[1*x+6782485570826905238,1],
[1*x+15987612182027639793,1],[1*x^2+1*x+1,1]]
\end{verbatim}

References

Section 10.5.1 [setmod_ff], page 172

10.5.17 irredcheck_ff

\textbf{irredcheck_ff}(\textit{poly})

\hspace{1em}:: \text{Primality check of a univariate polynomial over a finite field.}

\textit{return} \hspace{1em} 0|1
poly  univariate polynomial over a finite field
- Defined in ‘fff’.
- Returns 1 if poly is irreducible over the current base field. Returns 0 otherwise.

```plaintext
[178] setmod_ff(2^64-95);
18446744073709551521
[179] ] F=x^10+random_ff();
x^10+14687973587364016969
[180] irredcheck_ff(F);
1
```

References
Section 10.5.1 [setmod_ff], page 172

10.5.18 randpoly_ff

```plaintext
randpoly_ff(d,v)
:: Generation of a random univariate polynomial over a finite field.
```

**return** polynomial

d  positive integer

v  indeterminate

- Defined in ‘fff’.
- Generates a polynomial of v such that the degree is less than d and the coefficients are in the current base field. The coefficients are generated by random_ff().

```plaintext
[178] setmod_ff(2^64-95);
18446744073709551521
[179] ] F=x^10+random_ff();
[180] randpoly_ff(3,x);
17135261454578964298*x^2+4766826699653615429*x+18317369440429479651
[181] ] randpoly_ff(3,x);
7565988813172050604*x^2+7430075767279665339*x+46996629886224873544
[182] ] randpoly_ff(3,x);
10247781277095450395*x^2+10243690944992524936*x+4063829049268845492
```

References
Section 10.5.1 [setmod_ff], page 172, Section 10.5.7 [random_ff], page 176

10.5.19 ecm_add_ff, ecm_sub_ff, ecm_chsgn_ff

```plaintext
ecm_add_ff(p1,p2,ec)
ecm_sub_ff(p1,p2,ec)
ecm_chsgn_ff(p1)
:: Addition, Subtraction and additive inverse for points on an elliptic curve.
```

**return** vector or 0

p1 p2  vector of length 3 or 0

ec  vector of length 2
Let $p_1, p_2$ be points on the elliptic curve represented by $ec$ over the current base field. $ecm\_add\_ff(p_1,p_2,ec)$, $ecm\_sub\_ff(p_1,p_2,ec)$ and $ecm\_chsgn\_ff(p_1)$ returns $p_1+p_2$, $p_1-p_2$ and $-p_1$ respectively.

If the current base field is a prime field of odd order, then $ec$ represents $y^2=x^3+ec[0]x+ec[1]$. If the characteristic of the current base field is 2, then $ec$ represents $y^2+xy=x^3+ec[0]x^2+ec[1]$.

The point at infinity is represented by 0.

If an argument denoting a point is a vector of length 3, then it is the projective coordinate. In such a case the third coordinate must not be 0.

If the result is a vector of length 3, then the third coordinate is not equal to 0 but not necessarily 1. To get the result by the affine coordinate, the first and the second coordinates should be divided by the third coordinate.

The check whether the arguments are on the curve is omitted.

References
Section 10.5.1 [setmod_ff], page 172
Appendix A Appendix

A.1 Details of syntax

<expression>:
    '('<expression>')'
    <expression> <binary operator> <expression>
    '+' <expression>
    '-' <expression>
    <left value>
    <left value> <assignment operator> <expression>
    <left value> '++'
    <left value> '--'
    '++' <left value>
    '--' <left value>
    '?' <expression> ':' <expression>
    <function> '(' <expr list> ')' 
    <function> '(' <expr list> '|' <option list> ')
    <string>
    <exponent vector>
    <atom>
    <list>

(See Section 4.2.10 [various expressions], page 25.)

<left value>:
    <program variable> ['['<expression>']']*

<binary operator>:
    '+' '-' '*' '/' '%' '^' (exponentiation)
    '==' '!=' '<' '>' '<=' '>=' '&&' '||'

<assignment operator>:
    '=' '+=' '-=' '*=' '/=' '%=' '^='

<expr list>:
    <empty>
    <expression> ['','<expression>]*

<option>:
    Character sequence beginning with an alphabetical letter '=' <expr>

<option list>:
    <option>
    <option> ['','<option>]*

.list>:
    ['<expr list> ']

<program variable>:
    Sequence of alphabetical letters or numeric digits or _
    that begins with a capital alphabetical letter
    (X,Y,Japan etc.)
Appendix A: Appendix

See Section 4.2.2 [variables and indeterminates], page 20.

<function>:
   Sequence of alphabetical letters or numeric digits or _
   that begins with a small alphabetical letter
   (fctrc,gcd etc.)

<atom>:
   <indeterminate>
   <number>

<indeterminate>:
   Sequence of alphabetical letters or numeric digits or _
   that begin with a small alphabetical letter
   (a,bCD,c1_2 etc.)

(See Section 4.2.2 [variables and indeterminates], page 20.)

<number>:
   <rational number>
   <floating point number>
   <algebraic number>
   <complex number>

(See Section 3.2 [Types of numbers], page 14.)

<rational number>:
   0, 1, -2, 3/4

-floating point number>:
   0.0, 1.2e10

<algebraic number>:
   newalg(x^2+1), alg(0)^2+1

(See Chapter 9 [Algebraic numbers], page 156.)

<complex number>:
   1+i, 2.3*i

<string>:
   character sequence enclosed by two `"`'s.

<exponent vector>:
   `<' expr list `'>'

(See Chapter 8 [Groebner basis computation], page 121.)

<statement>:
   <expression> <terminator>
   <compound statement>
   'break' <terminator>
   'continue' <terminator>
   'return' <terminator>
   'return' <expression> <terminator>
   'if' '(' <expr list> ')' <statement>
   'if' '(' <expr list> ')' <statement> 'else' <statement>
   'for' '(' <expr list> ';' <expr list> ';' <expr list> ')' <statement>
   'do' <statement> 'while' '(' <expr list> ')' <terminator>
   'while' '(' <expr list> ')' <statement>
   'def' <function> '(' <expr list> ')' '{' <variable declaration> <stat list> '}'
A.2 Files of user defined functions

There are several files of user defined functions under the standard library directory. (`/usr/local/lib/asir' by default.) Here, we explain some of them.

`fff' Univariate factorizer over large finite fields (See Chapter 10 [Finite fields], page 170.)

`gr' Groebner basis package. (See Chapter 8 [Groebner basis computation], page 121.)

`sp' Operations over algebraic numbers and factorization, Splitting fields. (See Chapter 9 [Algebraic numbers], page 156.)

`alpi'
`bgk'
`cyclic'
`katsura'
`kimura' Example polynomial sets for benchmarks of Groebner basis computation. (See Section 8.10.31 [katsura hkatsura cyclic hcyclic], page 152.)

`defs.h' Macro definitions. (See Section 4.2.11 [preprocessor], page 26.)

`fctrtest' Test program of factorization of integral polynomials. It includes `factor.tst' of REDUCE and several examples for large multiplicity factors. If this file is `load()'ed, computation will begin immediately. You may use it as a first test whether Asir at you hand runs correctly.

`fctrdata' This contains example polynomials for factorization. It includes polynomials used in `fctrtest'. Polynomials contained in vector `Alg[]' is for the algebraic factorization af(). (See Section 9.5.10 [asq af af noalg], page 165.)

```asir
cputime(1);
x^9-15*x^6-87*x^3-125
```
Appendix A: Appendix

0msec
[177] a0(A0g[5],[newalg(A0g[5]));
[[1,1],[75*x^2+(10*_0^7-175*_0^4-470*_0^1)*x
+(3*_0^5-8-45*_0^5-261*_0^1),1],
[75*x^2+(-10*_0^7+175*_0^4+395*_0^1)*x
+(3*_0^5-8-45*_0^5-261*_0^1),1],
[25*x^2+(25*_0)*x+(*_0^8-15*_0^5-87*_0^2),1],
[x^2+(*_0)*x+(*_0^2),1],
[x+(-*_0),1]]
3.600sec + gc : 1.040sec

‘ifplot’ Examples for plotting. (See Section 7.5.15 [ifplot conplot plot polarplot plotover], page 116.) Vector IS[] contains several famous algebraic curves. Variables H, D, C, S contains something like the suits (Heart, Diamond, Club, and Spade) of cards.

‘num’ Examples of simple operations on numbers.

‘mat’ Examples of simple operations on matrices.

‘ratint’ Indefinite integration of rational functions. For this, files ‘sp’ and ‘gr’ is necessary. A function ratint() is defined. Its returns a rather complex result.

[0] load("gr")$
[45] load("sp")$
[84] load("ratint")$
[102] ratint(x^6/(x^5+x+1),x);
[1/2*x^2,
[(#2)*log(-140*x+(-2737*#2^2+552*#2-131)),
161*t#2^3-23*t#2^2+15*t#2-1],
[(#1)*log(-5*x+(-21*#1-4)),21*t#1^2+3*t#1+1)]

In this example, indefinite integral of the rational function $x^6/(x^5+x+1)$ is computed. The result is a list which comprises two elements: The first element is the rational part of the integral; The second part is the logarithmic part of the integral. The logarithmic part is again a list which comprises finite number of elements, each of which is of form $[\text{root}\ast\log(\text{poly}),\text{defpoly}]$. This pair should be interpreted to sum up the expression $\text{root}\ast\log(\text{poly})$ through all $\text{root}$'s $\text{root}$'s of the $\text{defpoly}$. Here, $\text{poly}$ contains $\text{root}$, and substitution for $\text{root}$ is equally applied to $\text{poly}$. The logarithmic part in total is obtained by applying such interpretation to all element pairs in the second element of the result and then summing them up all.

‘primdec’ Primary ideal decomposition of polynomial ideals and prime composition of radicals over the rationals (see Section 8.10.32 [primadec primedec], page 153).

‘primdec_mod’ Prime decomposition of radicals of polynomial ideals over finite fields (see Section 8.10.33 [primedec_mod], page 154).

‘bfct’ Computation of b-function. (see Section 8.10.34 [bfunction bfct generic_bfct ann ann0], page 155).
A.3 Input interfaces

A command line editing facility and a history substitution facility are built-in for DOS, Windows version of Asir. UNIX versions of Asir do not have such built-in facilities. Instead, the following input interfaces are prepared. This are also available from our ftp server. As for our ftp server See Section 1.3 [How to get Risa/Asir], page 2.

On Windows, `asirgui.exe' has a copy and paste functionality different from Windows convention. Press the left button of the mouse and drag the mouse cursor on a text, then the text is selected and is highlighted. When the button is released, highlighted text returns to the normal state and it is saved in the copy buffer. If the right button is pressed, the text in the copy buffer is inserted at the current text cursor position. Note that the existing text is read-only and one cannot modify it.

A.3.1 fep

Fep is a general purpose front end processor. The author is K. Utashiro (SRA Inc.). Under fep, emacs- or vi-like command line editing and csh-like history substitution are available for UNIX commands, including `asir'.

``
% fep asir
...
[0] fctr(x^5-1);
[[1,1],[x-1,1],[x^4+x^3+x^2+x+1,1]]
[1] !! /* !+Return */
fctr(x^5-1); /* The last input appears. */
...
/* Edit+Return */
fctr(x^5+1);
[[1,1],[x+1,1],[x^4-x^3+x^2-x+1,1]]
``

Fep is a free software and the source is available. However machines or operating systems on which the original one can run are limited. The modified version by us running on several unsupported environments is available from our ftp server.

A.3.2 asir.el

`asir.el' is a GNU Emacs interface for Asir. The author is Koji Miyajima (YVE25250@pcvan.or.jp). In `asir.el', completion of file names and command names is realized other than the ordinary editing functions which are available on Emacs.

`asir.el' is distributed on PC-VAN. The version where several changes have been made according to the current version of Asir is available via ftp.

The way of setting up and the usage can be found at the top of `asir.el'.

A.4 Library interfaces

It is possible to link an Asir library to use the functionalities of Asir from other programs. The necessary libraries are included in the OpenXM distribution (http://www.math.kobe-u.ac.jp/OpenXM). At present only the OpenXM interfaces are available. Here we assume that OpenXM is already installed. In the following $OPENXM_HOME denotes the OpenXM
Appendix A: Appendix

root directory. All the library files are placed in `$OpenXM_HOME/lib`. There are three kinds of libraries as follows.

- `libasir.a`
  It does not contain the functionalities related to PARI and X11. Only `libasir-gc.a` is necessary for linking.

- `libasir_pari.a`
  It does not contain the functionalities related to X11. `libasir-gc.a`, `libpari.a` are necessary for linking.

- `libasir_pari_X.a`
  All the functionalities are included. `libasir-gc.a`, `libpari.a` and libraries related to X11 are necessary for linking.

- `int asir_ox_init(int byteorder)`
  It initializes the library. `byteorder` specifies the format of binary CMO data on the memory. If `byteorder` is 0, the byteorder native to the machine is used. If `byteorder` is 1, the network byteorder is used. It returns 0 if the initialization is successful, -1 otherwise.

- `void asir_ox_push_cmo(void *cmo)`

- `int asir_ox_push_cmo(int cmd)`
  It executes a stack machine command `cmd`.

- `int asir_ox_pop_cmo(void *cmo, int limit)`
  It pops an Asir object at the top of the stack and it converts the object into CMO data. If the size of the CMO data is not greater than `limit`, then the data is written in `cmo` and the size is returned. Otherwise -1 is returned. The size of the array pointed by `cmo` must be at least `limit`. In order to know the size of converted CMO data in advance `asir_ox_pop_cmo` is called.

- `void asir_ox_execute_string(char *str)`
  It evaluates `str` as a string written in the Asir user language. The result is pushed onto the stack.

A program calling the above functions should include `$OpenXM_HOME/include/asir/ox.h`.

In this file all the definitions of OpenXM tags and commands. The following example (`$OpenXM_HOME/doc/oxlib/test3.c`) illustrates the usage of the above functions.

```c
#include <asir/ox.h>
#include <signal.h>

template(int argc, char **argv)
{
  char buf[BUFSIZ+1];
  int c;
  unsigned char sendbuf[BUFSIZ+10];
  unsigned char *result;
  unsigned char h[3];
```
int len, i, j;
static int result_len = 0;
char *kwd, *bdy;
unsigned int cmd;

signal(SIGINT, SIG_IGN);
asir_ox_init(1); /* 1: network byte order; 0: native byte order */
result_len = BUFSIZ;
result = (void *)malloc(BUFSIZ);
while ( 1 ) {
    printf("Input>"); fflush(stdout);
fgets(buf, BUFSIZ, stdin);
    for ( i = 0; buf[i] && isspace(buf[i]); i++ );
    if ( !buf[i] )
        continue;
    kwd = buf+i;
    for ( ; buf[i] && !isspace(buf[i]); i++ );
    buf[i] = 0;
    bdy = buf+i+1;
    if ( !strcmp(kwd, "asir") ) {
        sprintf(sendbuf, "%s;', bdy);
asir_ox_execute_string(sendbuf);
    } else if ( !strcmp(kwd, "push") ) {
        h[0] = 0;
        h[2] = 0;
        j = 0;
        while ( 1 ) {
            for ( ; (c = *bdy) && isspace(c); bdy++ );
            if ( !c )
                break;
            else if ( h[0] ) {
                h[1] = c;
                sendbuf[j++] = strtol(h, 0, 16);
                h[0] = 0;
            } else
                h[0] = c;
            bdy++;
        }
        if ( h[0] )
            fprintf(stderr, "Number of characters is odd.\n");
        else {
            sendbuf[j] = 0;
asir_ox_push_cmo(sendbuf);
        }
    } else if ( !strcmp(kwd, "cmd") ) {
        cmd = atoi(bdy);
asir_ox_push_cmd(cmd);
    } else if ( !strcmp(kwd, "pop") ) {
        len = asir_ox_peek_cmo_size();
if ( !len )
    continue;
if ( len > result_len ) {
    result = (char *)realloc(result, len);
    result_len = len;
}

asir_ox_pop_cmo(result, len);
printf("Output>"); fflush(stdout);
printf("\n");
for ( i = 0; i < len; ) {
    printf("%02x ", result[i]);
    i++;
    if ( !(i%16) )
        printf("\n");
} printf("\n");

This program receives a line in the form of keyword body as an input and it executes the following operations according to keyword.

- **asir body**
  body is regarded as an expression written in the Asir user language. The expression is evaluated and the result is pushed onto the stack. asir_ox_execute_string() is called.

- **push body**
  body is regarded as a CMO object in the hexadecimal form. The CMO object is converted into an Asir object and is pushed onto the stack. asir_ox_push_cmo() is called.

- **pop**
  The object at the top of the stack is converted into a CMO object and it is displayed in the hexadecimal form. asir_ox.peek_cmo.size() and asir_ox.pop_cmo() are called.

- **cmd body**
  body is regarded as an SM command and the command is executed. asir_ox.push_cmo() is called.

### A.5 Appendix

#### A.5.1 Version 990831

Four years have passed since the last distribution. Though the look and feel seem unchanged, internally there are several changes such as 32-bit representation of bignums. Plotting facilities are not available on Windows.

If you have files created by bsave on the older version, you have to use bload27 to read such files.
A.5.2 Version 950831

A.5.2.1 Debugger

- One can enter the debug mode anytime.
- A command `finish` has been appended.
- One can examine any stack frame with `up`, `down` and `frame`.
- A command `trace` has been appended.

A.5.2.2 Built-in functions

- One can specify a main variable for `sdiv()` etc.
- Functions for polynomial division over finite fields such as `sdivm()` have been appended.
- `det()`, `res()` can produce results over finite fields.
- `vto1()`, conversion from a vector to a list has been appended.
- `map()` has been appended.

A.5.2.3 Groebner basis computation

- Functions for Groebner basis computation have been implemented as built-in functions.
- `grm()` and `hgrm()` have been changed to `gr()` and `hgr()` respectively.
- `gr()` and `hgr()` requires explicit specification of an ordering type.
- Extension of specification of a term ordering type.
- Groebner basis computations over finite fields.
- Lex order Groebner basis computation via a modular change of ordering algorithm.
- Several new built-in functions.

A.5.2.4 Others

- Implementation of tools for distributed computation.
- Application of modular computation for GCD computation over algebraic number fields.
- Implementation of primary decomposition of ideals.
- Porting to Windows.

A.5.3 Version 940420

The first public version.
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