# Scaling limits of stochastic processes associated with resistance forms

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*NB. This talk is based on the preprints [1] and [2]; the latter work is joint with B. M. Hambly (University of Oxford), and T. Kumagai (Kyoto University).*

The connections between electricity and probability are deep, and have provided many tools for understanding the behaviour of stochastic processes. In this talk, I will describe a new result in this direction, which states that if a sequence of spaces equipped with so-called 'resistance metrics' and measures converge with respect to the Gromov-Hausdorff-vague topology, and a certain non-explosion condition is satisfied, then the associated stochastic processes also converge. This result generalises previous work on trees, fractals, and various models of random graphs. Moreover, it is useful in the study of time-changed processes, including Liouville Brownian motion, the Bouchaud trap model and the random conductance model, on such spaces. I further conjecture that the result will be applicable to the random walk on the incipient infinite cluster of critical bond percolation on the high-dimensional integer lattice.

To present the main result, the objects of study will now be introduced in more detail. A resistance metric on a space $F$ is a function $R : F \times F \to \mathbb{R}$ such that, for every finite $V \subseteq F$, one can find a weighted graph with vertex set $V$ (here, 'weighted' means edges are equipped with conductances) for which $R|_{V\times V}$ is the associated effective resistance; this definition was introduced by Kigami in the study of analysis on low-dimensional fractals, see [3] for background. We write $\mathbb{F}$ for the collection of quadruples of the form $(F, R, \mu, \rho)$, where: $F$ is a non-empty set; $R$ is a resistance metric on $F$ such that closed bounded sets in $(F, R)$ are compact (note this implies $(F, R)$ is complete, separable and locally compact); $\mu$ is a locally finite Borel regular measure of full support on $(F, R)$; and $\rho$ is a marked point in $F$. Note that the resistance metric is associated with a so-called 'resistance form' $(\mathcal{E}, \mathcal{F})$ (another concept introduced by Kigami, see [3, 4]), and we will further assume that for elements of $\mathbb{F}$ this form is 'regular'. Whilst we do not give precise definitions for this terminology here, we note that it ensures the existence of a related regular Dirichlet form $(\mathcal{E}, \mathcal{D})$ on $L^2(F, \mu)$, which we suppose is recurrent, and also a Hunt process $((X_t)_{t\geq 0}, (P_x)_{x\in F})$. Writing $B_R(\rho, r)$ for the ball of radius $r$ in $(F, R)$ centred at $\rho$, and $R(\rho, B_R(\rho, r)^c)$ for the resistance from $\rho$ to the complement of $B_R(\rho, r)$, we then have the following. Note that the condition at (1) below ensures non-explosion, and is natural in the context of recurrent processes.

**Theorem 1** *Suppose that the sequence $(F_n, R_n, \mu_n, \rho_n)_{n\geq 1}$ in $\mathbb{F}$ satisfies*

$$(F_n, R_n, \mu_n, \rho_n) \to (F, R, \mu, \rho)$$

*in the Gromov-Hausdorff-vague topology for some $(F, R, \mu, \rho) \in \mathbb{F}$, and also it holds that*

$$\lim_{r\to\infty} \limsup_{n\to\infty} R_n \left(\rho_n, B_{R_n}\left(\rho_n, r\right)^c\right) = \infty. \tag{1}$$

*It is then possible to isometrically embed $(F_n, R_n)_{n\geq 1}$ and $(F, R)$ into a common metric space $(M, d_M)$ in such a way that*

$$P^n_{\rho_n}\left((X^n_t)_{t\geq 0} \in \cdot\right) \to P_\rho\left((X_t)_{t\geq 0} \in \cdot\right)$$

*weakly as probability measures on $D(\mathbb{R}_+, M)$ (that is, the space of cadlag processes on $M$, equipped with the usual Skorohod $J_1$-topology), where we have denoted by $((X^n_t)_{t\geq 0}, (P^n_x)_{x\in F_n})$ the Markov process corresponding to $(F_n, R_n, \mu_n, \rho_n)$.*

# References


[1] Croydon, D. A. (2016). *Scaling limits of stochastic processes associated with resistance forms.* Preprint available at arXiv:1609.05666.

[2] Croydon, D. A., B. M. Hambly and T. Kumagai (2016). *Time-changes of stochastic processes associated with resistance forms.* Preprint available at arXiv:1609.02120.

[3] Kigami, J. (2001). *Analysis on fractals*, Cambridge Tracts in Mathematics, vol. 143, Cambridge University Press, Cambridge.

[4] Kigami, J. (2012). *Resistance forms, quasisymmetric maps and heat kernel estimates*, Mem. Amer. Math. Soc. **216**, no. 1015, vi+132.