Stochastic differential equations for infinite particle systems of jump types with long range interactions

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In this talk we study infinite particle systems with interactions in which each particle is undergoing the jump type process on $\mathbb{R}^d$ with rate function $p_x(y) = p(|x-y|)$ from x to y satisfying conditions (p.1)—(p.2):

(p.1) $p(r) = O(r^{-(d+\alpha)})$ as $r \to \infty$ for some $\alpha > 0$.

(p.2) $p(r) = O(r^{-(d+\beta)})$ as $r \to +0$ for some $0 < \beta < 2$.

Our theorems can be applied to the systems with Dyson, Ginibre, Airy and Bessel interactions. In particular, we can give the SDE representations for the interacting α-stable particle systems for any $\alpha \in (\kappa, 2)$, where κ is the growth order of the density (the 1-correlation function) of μ, that is, $\rho^1(x) = O(|x|^\kappa)$, $|x| \to \infty$.

Suppose that a state space is d-dimensional Euclidian space $\mathbb{R}^d$. Then the configuration space is represented as $\mathfrak{M} = \left\{\xi = \sum_i \delta_{x_i}; \xi(K) < \infty \text{ for all compact sets } K \subset \mathbb{R}^d\right\}$, where δ_a stands for the delta measure at a. We endow $\mathfrak{M}$ with the vague topology. Then $\mathfrak{M}$ is a Polish space. For $x, y \in \mathbb{R}^d$ and $\xi \in \mathfrak{M}$, we write $\xi^{xy} = \xi - \delta_x + \delta_y$ and $\xi \setminus x = \xi - \delta_x$ if $\xi(\{x\}) \geq 1$.

Let μ be a probability measure on $\mathfrak{M}$, which describes an equilibrium measure for the system. We consider a Dirichlet form $\mathfrak{E}$ defined by

$$\mathfrak{E}(f,f) = \frac{1}{2}\int_{\mathfrak{M}} \mu(d\xi) \int_{\mathbb{R}^d} \xi(dx) \int_{\mathbb{R}^d} p(x,y)\{f(\xi^{xy}) - f(\xi)\}^2 dy,$$

with some positive measurable function p on $\mathbb{R}^d \times \mathbb{R}^d$, which is a jump rate satisfying the above condition (p.1)–(p.2). Under suitable assumptions we can construct the associated unlabeled particle system by using the Dirichlet form technique [1].

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In this talk, we give the ISDE representations for the unlabeled particle systems by generalizing the method in [2]. We introduce the rate function given by $c(\xi, x; y) = 0$ if $\xi(\{x\}) = 0$, and

$$c(\xi, x; y) = p(|x - y|) \left(1 + \frac{d\mu_y}{d\mu_x}(\xi \setminus x)\frac{\rho^1(y)}{\rho^1(x)}\right), \quad \text{if } \xi(\{x\}) \geq 1.$$

Here, μ_x is the reduced Palm measure defined by $\mu_x = \mu\left(\cdot - \delta_x \middle| \xi(\{x\}) \geq 1\right)$ for $x \in \mathbb{R}^d$, $\rho^1(x)$ is the 1-correlation function of μ and $d\mu_y/d\mu_x$ is the Radon-Nikodym derivative of μ_y with respect to μ_x. Then the labeled process $(X_j(t))_{j\in\mathbb{N}}$ solves the following ISDE:

$$X_j(t) = X_j(0) + \int_0^t \int_{\mathbb{R}^d} \int_0^\infty u\ a\left(u, r, X_j(s-), \sum_{i\neq j} \delta_{X_i(s-)}\right) N_j(dsdudr), \tag{1}$$

where $a(u, r, x, \xi) = \mathbf{1}\left(0 \leq r \leq c(\xi, x; x + u)\right)$, and $N_j,\ j \in \mathbb{N}$ are independent Poisson random point fields on $[0, \infty) \times \mathbb{R}^d \times [0, \infty)$ whose intensity measure is the Lebesgue measure $dsdudr$.

We also discuss the uniqueness of solutions of ISDE (1) by applying the argument in [3], where systems of interacting Brownian motions are studied.

References

[1] Esaki, S., Infinite particle systems of long range jumps with long range interactions. to appear in Tohoku Mathematical Journal arXiv:1508.06795 [math.PR].

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