An analytic function in 3 complex variables related to the value-distribution of $\log L$, and the "Plancherel volume"

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The first subject of the talk is of general and elementary nature. For a continuous density measure $M(x)|dx|$ on $\mathbf{R}^d$, let μ_M denote the variance and ν_M the "Plancherel volume"

$$\nu_M = \int M(x)^2|dx| = \int |\hat{M}(y)|^2|dy|$$

($|dx|$ the self-dual Haar measure of $\mathbf{R}^d$, $\hat{}$ the Fourier transform). We first focus our attention on the natural numerical invariant $\mu_M^{d/2}\nu_M$; its meaning, basic examples with parameters, the lower bound, etc..

The second (and the main) subject is a complex analytic function in 3 variables defined explicitly as

$$\tilde{M}(s; z_1, z_2) = \prod_p F(iz_1/2, iz_2/2; 1; p^{-2s}) \qquad (\Re(s) > 1/2)$$

($i = \sqrt{-1}, F(a, b; c; t)$ Gauss' hypergeometric series). When σ is real $> 1/2$, $\tilde{M}(\sigma, z_1, z_2)$ can be interpreted as the mean value of $\{\overline{\zeta(s)}^{iz_1/2}\zeta(s)^{iz_2/2}\}$ over the vertical line $\Re(s) = \sigma$. But we consider $\tilde{M}(s; z_1, z_2)$ as an analytic function also of the complex variable s. We discuss its analytic continuation to the left of $\Re(s) > 1/2$, two other infinite product expansions, and the limit behaviours as $s \to 1/2, +\infty$. This will be applied to the determination of the corresponding limits of the above invariant $\mu_M\nu_M$ when $M = M_\sigma$ is the density measure on $\mathbf{C}$ for the distribution of $\{\log\zeta(\sigma + ti)\}_{t\in\mathbf{R}}$ or of $\{\log L(s, \chi)\}_\chi$. The connection with value-distributions is joint work with K. Matsumoto.