Probabilistic number theory on permutations

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A natural way to define additive and multiplicative functions on the symmetric group $\mathbb{S}_n$, $n \geq 1$, analogous to that under investigation in number theory, is based on the cycle decomposition of a permutation $\sigma \in \mathbb{S}_n$. Assume that σ has $k_j(\sigma) \geq 0$ independent cycles of length j, $1 \leq j \leq n$. If $h_j(k)$, $j \geq 1$, $k \geq 0$ is a double index sequence in $\mathbb{R}$ such that $h_j(0) \equiv 0$, then an additive function $h\colon \mathbb{S}_n \to \mathbb{R}$ is defined by

$$h(\sigma) = \sum_{j=1}^{n} h_j(k_j(\sigma)), \quad \sigma \in \mathbb{S}_n.$$

Let $\theta > 0$ be a constant and $w(\sigma) = k_1(\sigma) + \cdots + k_n(\sigma)$ be the number of cycles in σ. The Ewens probability $\nu_{n,\theta}(\cdot)$ on the subsets of $\mathbb{S}_n$ is defined via

$$\nu_{n,\theta}(\{\sigma\}) = \theta^{w(\sigma)}/\theta(\theta+1)\cdots(\theta+n-1), \quad \sigma \in \mathbb{S}_n.$$

In the talk, we will discuss asymptotical properties of the distributions $\nu_{n,\theta} \cdot h^{-1}$ as $n \to \infty$. The role of ideas originated in probabilistic number theory will be emphasized.