MAXIMAL EDGE-TRAVERSAL TIME IN FIRST PASSAGE PERCOLATION

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First passage percolation (FPP) was first introduced by Hammersley and Welsh in 1965. It can be thought of as a model for the speed to percolate some material. In this talk, we focus on the maximal edge-traversal time of optimal paths in FPP and investigate the order of the growth. We shall give precise definitions below.

Let $E(\mathbb{Z}^d)$ be the set of undirected nearest-neighbor edges. We place a non-negative random variables τ_e on each edge e as the passage time. Assume $\{\tau_e\}_{e\in E(\mathbb{Z}^d)}$ are i.i.d. random variables with distribution F. We say $\Gamma = \{x_i\}_{i=0}^k \subset \mathbb{Z}^d$ is a path from x to y (we write $\Gamma : x \to y$) if $x_0 = x$, $x_k = y$ and $|x_i - x_{i-1}|_1 = 1$ for $i = 1, \cdots, k$. Given a path $\Gamma = \{x_i\}_{i=0}^k$, the passage time of Γ is defined as $t(\Gamma) = \sum_{i=1}^k \tau_{\{x_{i-1},x_i\}}$ and we set first passage time $T(x,y)$ as $T(x,y) = \inf_{\Gamma:x\to y} t(\Gamma)$ for $x,y \in \mathbb{Z}^d$. Let Opt_n be the set of optimal paths from origin to ne_1 and $\Xi(\Gamma) = \max\{\tau_{\{x_{i-1},x_i\}} :\ 1 \le i \le k\}$ for $\Gamma = \{x_i\}_{i=0}^k \in Opt_n$.

Let $\underline{F}$ be the infimum of the support of F and $p_c(d)$, $\vec{p}_c(d)$ the critical probability of d-dim percolation, oriented precolation model, respectively. Then F is said to be useful if either holds;

$$\text{(i) } \underline{F} = 0,\ F(\{0\}) < p_c(d),\ \text{(ii) } \underline{F} > 0,\ F(\{\underline{F}\}) < \vec{p}_c(d).$$

It is easy to check that if F is useful, Opt_n is not empty almost surely. It is known from the result of van den Berg and Kesten in [1] that if F is unbounded and useful,

$$\min_{\Gamma\in Opt_n} \Xi(\Gamma) \to \infty \quad a.s.$$

Our purpose is to investigate the actual order of the growth of $\Xi(Opt_n)$.

Theorem 1. *Suppose $d \ge 2$, F is useful, and there exist $a > 1$, $c_1 - c_4$, t_1, $r > 0$ such that for any $t \ge t_1$, $c_1 e^{-c_2 t^r} \le F([t, at]) \le c_3 e^{-c_4 t^r}$. Then, there exists $K > 0$ such that,*

$$\mathbb{P}\left(K^{-1} f_{d,r}(n) \le \min_{\Gamma\in \mathrm{Opt}_n} \Xi(\Gamma) \le \max_{\Gamma\in \mathrm{Opt}_n} \Xi(\Gamma) \le K f_{d,r}(n) \right) \to 1,$$

where, we set

$$f_{d,r}(n) := \begin{cases} (\log n)^{\frac{1}{1+r}} & \textit{if } 0 < r < d-1 \\ (\log n)^{\frac{1}{d}} (\log\log n)^{\frac{d-2}{d}} & \textit{if } r = d-1 \\ (\log n)^{\frac{1}{d}} & \textit{if } d-1 < r < d \\ (\log n)^{\frac{1}{d}} (\log\log n)^{-\frac{1}{d}} & \textit{if } r = d \\ (\log n)^{\frac{1}{r}} & \textit{if } d < r. \end{cases}$$

Theorem 2. *Suppose $d \ge 2$, F is useful, $\mathbb{E}[\tau_e^4] < \infty$ and there exist $0 < \alpha$, c, t_1 and $a > 1$ such that for any $t \ge t_1$, $F([t, at]) \ge c t^{-\alpha}$. Then, there exists $K > 0$ such that,*

$$\mathbb{P}\left(K^{-1} \frac{\log n}{\log\log n} \le \min_{\Gamma\in \mathrm{Opt}_n} \Xi(\Gamma) \le \max_{\Gamma\in \mathrm{Opt}_n} \Xi(\Gamma) \le K \frac{\log n}{\log\log n} \right) \to 1.$$

References

[1] J. van den Berg and H. Kesten. Inequalities for the time constant in first-passage percolation. *Annals Applied Probability*, 56-80, 1993

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