

## Riemann's zeta function and Kummer function

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### Abstract

We treat Riemann's zeta function from the viewpoint of the theory of stationary processes. The point is that not only Riemann's zeta function, but also the principal integrand part of the analytic continuation for Riemann's zeta function can be regarded as the covariance functions of stationary Gaussian processes with T-positivity.

In the RIMS Project Research Conference 「Number Theory and Probability」 held on September 13-15, 2010, we reported two results (a) and (b):

(a) We gave a remark for Hilbert's conjecture. At the Second International Congress of Mathematicians held at Paris, 1900, Hilbert proved that Riemann's zeta function does not satisfy any algebraic ordinary differential equation. Contrasting with Hilbert's result, we prove that Riemann's zeta function satisfies an ordinary differential equation with time delay, by using the result of the theory of KM<sub>2</sub>O-Langevin equations with T-positivity. The point is that Riemann's zeta function can be regarded as the covariance function of a stationary process with T-positivity.

(b) We obtained a new representation theorem for an analytic continuation for Riemann's zeta function, by introducing Kummer function which is also said to be hypergeometric function of confluent type. The point is that the principal integrand part of the analytic continuation for Riemann's zeta function can be regarded as the covariance functions of stationary processes with T-positivity.

The purpose of this talk is to give a representation of the form of inner product for the Kummer function introduced in (b), by using a Hamiltonian associated with stationary process having T-positivity and investigate Riemann Hypothesis.