Linear relations between pattern sequences in a $\langle q, r \rangle$-numeration system

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Let q and r be fixed integers with $q \geq 2$ and $0 \leq r \leq q-2$. Any positive integer n has a unique representation of the form

$$n = \sum_{i=0}^{k} a_i q^i, \quad a_i \in \Sigma_{q,r}, \quad a_k > 0,$$

where $\Sigma_{q,r} := \{-r, 1-r, \ldots, 0, 1, \ldots, q-1-r\} \supset \{0, 1\}$. The string of $\langle q, r \rangle$-*digits* $(n)_{q,r} := a_k a_{k-1} \cdots a_0$ is called the $\langle q, r \rangle$-*expansion* of n. The $\langle q, 0 \rangle$-expansion is the ordinary q-ary expansion. These numeration systems are called $\langle q, r \rangle$-numeration systems. Let w be a finite string of elements in $\Sigma_{q,r}$. We define $e_{q,r}(w; n)$ to be the number of (possibly overlapping) occurrences of w in the $\langle q, r \rangle$-expansion of n and define $e_{q,r}(w; 0) = 0$. The resulting sequence $\{e_{q,r}(w; n)\}_{n \geq 0}$ is called the *pattern sequence* for the pattern w in the $\langle q, r \rangle$-numeration system.

We investigate linear relations between pattern sequences in a $\langle q, r \rangle$-numeration system. We give a basis of the module generated by pattern sequences for words of length not exceeding l and study the expressions of pattern sequences using the basis. Similar results are obtained for the module generated by all pattern sequences.