

Asymptotic Euler-Maclaurin formula over lattice polytopes

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Formulas for the Riemann sums over lattice polytopes defined by lattice points in the polytopes are often called Euler-Maclaurin formulas, and they appear frequently in combinatorial and geometrical theory of toric varieties. We call the asymptotic Euler-Maclaurin formula for such Riemann sums asymptotic Euler-Maclaurin formulas. An asymptotic Euler-Maclaurin formula over (simple) lattice polytopes is first obtained by Guillemin-Sternberg, which generalizes the classical Euler-Maclaurin expansion on the intervals. Thus, the problem is to find effective formulas for each term of the asymptotic expansion. In this talk, a new asymptotic Euler-Maclaurin formula over lattice polytopes will be presented. The formula is rather similar to the so-called local (exact) Euler-Maclaurin formula due to Berline-Vergne. Indeed, it can be shown that the differential operators appearing in each term of the asymptotic expansion which will be given in this talk coincide with the homogeneous parts of Berline-Vergne operators. But our approach is quite different from Berline-Vergne, and it gives an independent proof for some class of polytopes (called Delzant polytopes). In the talk, the proof is sketched in the one-dimensional case along with our strategy, which gives a formula for each coefficients of the expansion of the twisted Riemann sums over intervals, will be given.