PROBABILISTIC DISCREPANCY THEORY

A classical topic in number theory is the study of normal numbers. A real number $x = 0.a_1a_2a_3\ldots$ (in base q) is normal if all digital blocks $b = b_1 \ldots b_k \in \{0, \ldots, q-1\}^k$ of length k asymptotically occur with frequency q^{-k}. This property is equivalent to the fact that the sequence $\{x \cdot q^n\}_{n=1}^{\infty}$ is uniformly distributed modulo 1. Gál, Erdős and Koksma studied the probabilistic behavior of the discrepancy function $D_N(xq^n)$ of such sequences, for instance they obtained a law of the iterated logarithm (LIL) for almost all x (in the sense of Lebesgue measure). Answering a question of Erdős, W. Philipp in 1975 established the following more general result:

$$\frac{1}{4} \leq \limsup_{N \to \infty} \frac{N D_N(n_k x)}{\sqrt{N \log \log N}} \leq_{Cq} \quad \text{(a.e.)},$$

where (n_k) is a sequence of positive integers satisfying the Hadamard gap condition

$$\frac{n_{k+1}}{n_k} \geq q > 1\cdot$$

This result was an initial point for *a* far ranging progress in probabilistic discrepancy theory. We focus on presenting results concerning central limit theorems for discrepancy functions including the computation of exact constants. Furthermore we construct sub-exponentially growing sequences satisfying a LIL. A main part of the presentation is devoted to permutation-invariant distribution properties. This gives final answers to a development initiated in Fourier analysis by Salem and Zygmund. The proofs depend on probabilistic and analytic tools as well as on methods from the theory of Diophantine equations, mainly on a recent version of the subspace theorem by Evertse-Schlickewei-Schmidt (2002).