

Sm1 OX Server Manual

Edition : auto generated by oxgentexi on 25 February 2020

1 SM1 Functions

This chapter describes interface functions for sm1 ox server `ox_sm1_forAsir`. These interface functions are defined in the file `sm1.rr`. The file `sm1.rr` is at `$(OpenXM_HOME)/lib/asir/contrib-packages`. The system `sm1` is a system to compute in the ring of differential operators. Many constructions of invariants in the computational algebraic geometry reduce to constructions in the ring of differential operators. Documents on `sm1` are in the directory `OpenXM/doc/kan96xx`.

The `sm1` server for windows is not distributed in the binary form. If you need to run it, compile it under the cygwin environment following the Makefile in `OpenXM/misc/packages/Windows`.

All the coefficients of input polynomials should be integers for most functions in this section. Other functions accept rational numbers as inputs and it will be explicitly noted in each explanation of these functions.

Let us evaluate the dimensions of the de Rham cohomology groups of $X := \mathbf{C} \setminus \{0, 1\} = \mathbf{C} \setminus V(x(x-1))$. The space X is a two punctured plane, so two loops that encircles the points $x = 0$ and $x = 1$ respectively spans the first homology group. Hence, the dimension of the first de Rham cohomology group is 2. `sm1` answers the dimensions of the 0th and the first cohomology groups.

```
[283] sm1.deRham([x*(x-1),[x]]);
[1,2]
```

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The author of `sm1` packages : Toshinori Oaku, oaku@twcu.ac.jp

Reference: [SST] Saito, M., Sturmfels, B., Takayama, N., Grobner Deformations of Hypergeometric Differential Equations, 1999, Springer. <http://www.math.kobe-u.ac.jp/KAN>

1.1 ox_sm1_forAsir Server

1.1.1 ox_sm1_forAsir

`ox_sm1_forAsir`

:: `sm1` server for `asir`.

- `ox_sm1_forAsir` is the `sm1` server started from `asir` by the command `sm1.start`. In the standard setting,

```
ox_sm1_forAsir = $(OpenXM_HOME)/lib/sm1/bin/ox_sm1 + $(OpenXM_
HOME)/lib/sm1/callsm1.sm1 (macro file)
+ $(OpenXM_HOME)/lib/sm1/callsm1b.sm1 (macro file)
```

The macro files `callsm1.sm1` and `callsm1b.sm1` are searched from current directory, `$(LOAD_SM1_PATH)`, `$(OpenXM_HOME)/lib/sm1`, `/usr/local/lib/sm1` in this order.

- Note for programmers: See the files `$(OpenXM_HOME)/src/kxx/oxserver00.c`, `$(OpenXM_HOME)/src/kxx/sm1stackmachine.c` to build your own server by reading `sm1` macros.

1.2 Functions

1.2.1 sm1.start

`sm1.start()`
 :: Start `ox_sm1_forAsir` on the localhost.

return Integer

- Start `ox_sm1_forAsir` on the localhost. It returns the descriptor of `ox_sm1_forAsir`.
- Set `Xm_noX = 1` to start `ox_sm1_forAsir` without a debug window.
- You might have to set suitable orders of variable by the command `ord`. For example, when you are working in the ring of differential operators on the variable `x` and `dx` (`dx` stands for $\partial/\partial x$), `sm1` server assumes that the variable `dx` is collected to the right and the variable `x` is collected to the left in the printed expression. In the example below, you must not use the variable `cc` for computation in `sm1`.
- The variables from `a` to `z` except `d` and `o` and `x0`, ..., `x20`, `y0`, ..., `y20`, `z0`, ..., `z20` can be used as variables for ring of differential operators in default. (cf. `Sm1_ord_list` in `sm1`).
- The descriptor is stored in static `Sm1_proc`. The descriptor can be obtained by the function `sm1.get_Sm1_proc()`.

```
[260] ord([da,a,db,b]);
[da,a,db,b,dx,dy,dz,x,y,z,dt,ds,t,s,u,v,w,
..... omit .....
]
[261] a*da;
a*da
[262] cc*dcc;
dcc*cc
[263] sm1.mul(da,a,[a]);
a*da+1
[264] sm1.mul(a,da,[a]);
a*da
```

Reference

`ox_launch`, `sm1.push_int0`, `sm1.push_poly0`, `ord`

1.2.2 sm1.sm1

`sm1.sm1(p,s)`
 :: ask the `sm1` server to execute the command string `s`.

return Void

`p` Number

`s` String

- It asks the `sm1` server of the descriptor number `p` to execute the command string `s`. (In the next example, the descriptor number is 0.)

```

[261] sm1.sm1(0," ( (x-1)^2 ) . ");
0
[262] ox_pop_string(0);
x^2-2*x+1
[263] sm1.sm1(0," [(x*(x-1)) [(x)]] deRham ");
0
[264] ox_pop_string(0);
[1 , 2]

```

Reference

`sm1.start`, `ox_push_int0`, `sm1.push_poly0`, `sm1.get_Sm1_proc()`.

1.2.3 `sm1.push_int0`

`sm1.push_int0(p,f)`

:: push the object f to the server with the descriptor number p .

return Void

p Number

f Object

- When `type(f)` is 2 (recursive polynomial), f is converted to a string (`type == 7`) and is sent to the server by `ox_push_cmo`.
- When `type(f)` is 0 (zero), it is translated to the 32 bit integer zero on the server. Note that `ox_push_cmo(p,0)` sends `CM0_NULL` to the server. In other words, the server does not get the 32 bit integer 0 nor the bignum 0.
- `sm1` integers are classified into the 32 bit integer and the bignum. When `type(f)` is 1 (number), it is translated to the 32 bit integer on the server. Note that `ox_push_cmo(p,1234)` send the bignum 1234 to the `sm1` server.
- In other cases, `ox_push_cmo` is called without data conversion.

```

[219] P=sm1.start();
0
[220] sm1.push_int0(P,x*dx+1);
0
[221] A=ox_pop_cmo(P);
x*dx+1
[223] type(A);
7 (string)
[271] sm1.push_int0(0,[x*(x-1),[x]]);
0
[272] ox_execute_string(0," deRham ");
0
[273] ox_pop_cmo(0);
[1,2]

```

Reference

`ox_push_cmo`

1.2.4 sm1.gb

)

:: computes the Grobner basis of f in the ring of differential operators with the variable v .

`sm1.gb_d([f,v,w] | proc=p)`

:: computes the Grobner basis of f in the ring of differential operators with the variable v .

The result will be returned as a list of distributed polynomials.

return List

p, q, r Number

f, v, w List

- It returns the Grobner basis of the set of polynomials f in the ring of differential operators with the variables v .
- The weight vectors are given by w , which can be omitted. If w is not given, the graded reverse lexicographic order will be used to compute Grobner basis.
- The return value of `sm1.gb` is the list of the Grobner basis of f and the initial terms (when w is not given) or initial ideal (when w is given).
- `sm1.gb_d` returns the results by a list of distributed polynomials. Monomials in each distributed polynomial are ordered in the given order. The return value consists of [variable names, order matrix, grobner basis in distributed polynomials, initial monomials or initial polynomials].
- When a non-term order is given, the Grobner basis is computed in the homogenized Weyl algebra (See Section 1.2 of the book of SST). The homogenization variable h is automatically added.
- When the optional variable q is set, `sm1.gb` returns, as the third return value, a list of the Grobner basis and the initial ideal with sums of monomials sorted by the given order. Each polynomial is expressed as a string temporally for now. When the optional variable r is set to one, the polynomials are dehomogenized (i.e., h is set to 1).
- If you want to have a reduced basis or compute the initial form ideal exactly, execute `sm1.auto_reduce(1)` before executing this function.
- When the `needBack` option n is 1, it returns the answer in a different format as [grobner basis, initial, gb, 1, all, [grobner basis, backward transformation]]
- The default value of `ring_var` is `ring_var_for_asir`. The server `ox_sm1` saves the ring structure used to the global `this ring_var`. See also `reduction`.

```
[293] sm1.gb([x*dx+y*dy-1,x*y*dx*dy-2],[x,y]);
      [[x*dx+y*dy-1,y^2*dy^2+2],[x*dx,y^2*dy^2]]
```

In the example above, the set $\{x\partial_x + y\partial_y - 1, y^2\partial_y^2 + 2\}$ is the Gröbner basis of the input with respect to the graded reverse lexicographic order such that

$1 \leq \partial_y \leq \partial_x \leq y \leq x \leq \dots$. The set $\{x\partial_x, y^2\partial_y\}$ is the leading monomials (the initial monomials) of the Gröbner basis.

```
[294] sm1.gb([[dx^2+dy^2-4,dx*dy-1],[x,y],[[dx,50,dy,2,x,1]]]);
      [[dx+dy^3-4*dy,-dy^4+4*dy^2-1],[dx,-dy^4]]
```

In the example above, two monomials $m = x^a y^b \partial_x^c \partial_y^d$ and $m' = x^{a'} y^{b'} \partial_x^{c'} \partial_y^{d'}$ are firstly compared by the weight vector $(dx, dy, x, y) = (50, 2, 1, 0)$ (i.e., m is larger than m' if $50c + 2d + a > 50c' + 2d' + a'$) and when the comparison is tie, then these are compared by the reverse lexicographic order (i.e., if $50c + 2d + a = 50c' + 2d' + a'$, then use the reverse lexicographic order).

```
[294] F=sm1.gb([[dx^2+dy^2-4,dx*dy-1],[x,y],[[dx,50,dy,2,x,1]]|sorted=1);
      map(print,F[2][0])$
      map(print,F[2][1])$
```

```
[595] sm1.gb(["dx*(x*dx + y*dy-2)-1","dy*(x*dx + y*dy -2)-1",
            [x,y],[[dx,1,x,-1],[dy,1]]]);
```

```
[[x*dx^2+(y*dy-h^2)*dx-h^3,x*dy*dx+y*dy^2-h^2*dy-h^3,h^3*dx-h^3*dy],
 [x*dx^2+(y*dy-h^2)*dx,x*dy*dx+y*dy^2-h^2*dy-h^3,h^3*dx]]
```

```
[596]
```

```
sm1.gb_d(["dx (x dx + y dy-2)-1","dy (x dx + y dy -2)-1",
          "x,y",[dx,1,x,-1],[dy,1]]);
```

```
[[[e0,x,y,H,E,dx,dy,h],
 [0,-1,0,0,0,1,0,0],[0,0,0,0,0,0,1,0],[1,0,0,0,0,0,0,0],
 [0,1,1,1,1,1,1,0],[0,0,0,0,0,0,-1,0],[0,0,0,0,0,-1,0,0],
 [0,0,0,0,-1,0,0,0],[0,0,0,-1,0,0,0,0],[0,0,-1,0,0,0,0,0],
 [0,0,0,0,0,0,0,1]]],
```

```
[[(1)*<<0,0,1,0,0,1,1,0>>+(1)*<<0,1,0,0,0,2,0,0>>+(-1)*<<0,0,0,0,0,1,0,2>>+
<<0,0,0,0,0,0,0,3>>,(1)*<<0,0,1,0,0,0,2,0>>+(1)*<<0,1,0,0,0,1,1,0>>+(-1)*<<0,
0,0,0,1,2>>+(-1)*<<0,0,0,0,0,0,0,3>>,(1)*<<0,0,0,0,0,1,0,3>>+(-1)*<<0,0,0,0,
1,3>>],
```

```
[(1)*<<0,0,1,0,0,1,1,0>>+(1)*<<0,1,0,0,0,2,0,0>>+(-1)*<<0,0,0,0,0,1,0,2>>,(
<<0,0,1,0,0,0,2,0>>+(1)*<<0,1,0,0,0,1,1,0>>+(-1)*<<0,0,0,0,0,1,2>>+(-1)*<<0,
0,0,0,0,3>>,(1)*<<0,0,0,0,0,1,0,3>>]]]
```

```
[1834] sm1.gb([[dx^2-x,dx],[x]] | needBack=1);
```

```
[[dx,dx^2-x,1],[dx,dx^2,1],gb,1,all,[[dx,dx^2-x,1],[[0,1],[1,0],[-dx,dx^2-x]]]
```

Reference

```
sm1.auto_reduce, sm1.reduction, sm1.rat_to_p
```

1.2.5 sm1.deRham

```
sm1.deRham([f,v]|proc=p)
```

:: ask the server to evaluate the dimensions of the de Rham cohomology groups of C^n - (the zero set of $f=0$).

return List

- p* Number
- f* String or polynomial
- v* List
- It returns the dimensions of the de Rham cohomology groups of $X = \mathbb{C}^n \setminus V(f)$. In other words, it returns $[\dim H^0(X, \mathbb{C}), \dim H^1(X, \mathbb{C}), \dim H^2(X, \mathbb{C}), \dots, \dim H^n(X, \mathbb{C})]$.
 - *v* is a list of variables. $n = \text{length}(v)$.
 - `sm1.deRham` requires huge computer resources. For example, `sm1.deRham(0, [x*y*z*(x+y+z-1)*(x-y), [x,y,z]])` is already very hard.
 - To efficiently analyze the roots of b-function, `ox_asir` should be used from `ox_sm1_forAsir`. It is recommended to load the communication module for `ox_asir` by the command
`sm1(0, "[(parse) (oxasir.sm1) pushfile] extension");` This command is automatically executed when `ox_sm1_forAsir` is started.
 - If you make an interruption to the function `sm1.deRham` by `ox_reset(sm1.get_Sm1_proc());`, the server might get out of the standard mode. So, it is strongly recommended to execute the command `ox_shutdown(sm1.get_Sm1_proc());` to interrupt and restart the server.
- ```
[332] sm1.deRham([x^3-y^2, [x,y]]);
[1,1,0]
[333] sm1.deRham([x*(x-1), [x]]);
[1,2]
```

#### Reference

`sm1.start, deRham` (sm1 command)

#### Algorithm:

Oaku, Takayama, An algorithm for de Rham cohomology groups of the complement of an affine variety via D-module computation, Journal of pure and applied algebra 139 (1999), 201–233.

### 1.2.6 sm1.hilbert

- `sm1.hilbert([f,v] | proc=p)`  
 :: ask the server to compute the Hilbert polynomial for the set of polynomials *f*.
- `hilbert_polynomial(f,v)`  
 :: ask the server to compute the Hilbert polynomial for the set of polynomials *f*.
- return*        Polynomial
- p*            Number
- f, v*        List
- It returns the Hilbert polynomial  $h(k)$  of the set of polynomials *f* with respect to the set of variables *v*.

- $h(k) = \dim_{\mathbb{Q}} F_k / I \cap F_k$  where  $F_k$  the set of polynomials of which degree is less than or equal to  $k$  and  $I$  is the ideal generated by the set of polynomials  $f$ .
- Note for `sm1.hilbert`: For an efficient computation, it is preferable that the set of polynomials  $f$  is a set of monomials. In fact, this function firstly compute a Grobner basis of  $f$ , and then compute the Hilbert polynomial of the initial monomials of the basis. If the input  $f$  is already a Grobner basis, a Grobner basis is recomputed in this function, which is a waste of time and Grobner basis computation in the ring of polynomials in `sm1` is slower than in `asir`.

```
[346] load("katsura")$
[351] A=hilbert_polynomial(katsura(5),[u0,u1,u2,u3,u4,u5]);
32

[279] load("katsura")$
[280] A=gr(katsura(5),[u0,u1,u2,u3,u4,u5],0)$
[281] dp_ord();
0
[282] B=map(dp_ht,map(dp_ptod,A,[u0,u1,u2,u3,u4,u5]));
[(1)*<<1,0,0,0,0,0>>,(1)*<<0,0,0,2,0,0>>,(1)*<<0,0,1,1,0,0>>,(1)*<<0,0,2,0,0,0>>,(1)*<<0,1,1,0,0,0>>,(1)*<<0,2,0,0,0,0>>,(1)*<<0,0,0,1,1,1>>,(1)*<<0,0,0,1,2,0>>,(1)*<<0,0,1,0,2,0>>,(1)*<<0,1,0,0,2,0>>,(1)*<<0,1,0,1,1,0>>,(1)*<<0,0,0,0,2,0>>,(1)*<<0,0,1,0,1,2>>,(1)*<<0,1,0,0,1,2>>,(1)*<<0,1,0,1,0,2>>,(1)*<<0,0,0,0,0,4>>,(1)*<<0,0,0,0,1,4>>,(1)*<<0,0,0,1,0,4>>,(1)*<<0,0,1,0,0,4>>,(1)*<<0,0,0,0,0,6>>]
[283] C=map(dp_dtop,B,[u0,u1,u2,u3,u4,u5]);
[u0,u3^2,u3*u2,u2^2,u2*u1,u1^2,u5*u4*u3,u4^2*u3,u4^2*u2,u4^2*u1,u4*u3*u1,
u5^2*u4^2,u5^2*u4*u2,u5^2*u4*u1,u5^2*u3*u1,u5*u4^3,u4^4,u5^4*u4,u5^4*u3,
u5^4*u2,u5^4*u1,u5^6]
[284] sm1.hilbert([C,[u0,u1,u2,u3,u4,u5]]);
32
```

#### Reference

`sm1.start`, `sm1.gb`, `longname`

### 1.2.7 sm1.genericAnn

`sm1.genericAnn([f,v] | proc=p)`

:: It computes the annihilating ideal for  $f^s$ .  $v$  is the list of variables. Here,  $s$  is  $v[0]$  and  $f$  is a polynomial in the variables `rest(v)`.

*return* List

$p$  Number

$f$  Polynomial

$v$  List



- This function computes the annihilating ideal for  $f^s$ .  $v$  is the list of variables. Here,  $s$  is  $v[0]$  and  $f$  is a polynomial in the variables  $\text{rest}(v)$ .

```
[595] sm1.genericAnn([x^3+y^3+z^3,[s,x,y,z]]);
 [-x*dx-y*dy-z*dz+3*s,z^2*dy-y^2*dz,z^2*dx-x^2*dz,y^2*dx-x^2*dy]
```

#### Reference

`sm1.start`

### 1.2.8 `sm1.wTensor0`

`sm1.wTensor0([f,g,v,w] | proc=p)`

:: It computes the D-module theoretic 0-th tensor product of  $f$  and  $g$ .

*return*      List

$p$             Number

$f, g, v, w$    List

- It returns the D-module theoretic 0-th tensor product of  $f$  and  $g$ .
- $v$  is a list of variables.  $w$  is a list of weights. The integer  $w[i]$  is the weight of the variable  $v[i]$ .
- `sm1.wTensor0` calls `wRestriction0` of `ox_sm1`, which requires a generic weight vector  $w$  to compute the restriction. If  $w$  is not generic, the computation fails.
- Let  $F$  and  $G$  be solutions of  $f$  and  $g$  respectively. Intuitively speaking, the 0-th tensor product is a system of differential equations which annihilates the function  $FG$ .
- The answer is a submodule of a free module  $D^r$  in general even if the inputs  $f$  and  $g$  are left ideals of  $D$ .

```
[258] sm1.wTensor0([x*dx -1, y*dy -4],[dx+dy,dx-dy^2],[x,y],[1,2]);
 [-y*x*dx-y*x*dy+4*x+y],[5*x*dx^2+5*x*dx+2*y*dy^2+(-2*y-6)*dy+3],
 [-25*x*dx+(-5*y*x-2*y^2)*dy^2+((5*y+15)*x+2*y^2+16*y)*dy-20*x-8*y-15],
 [y^2*dy^2+(-y^2-8*y)*dy+4*y+20]]
```

### 1.2.9 `sm1.reduction`

`sm1.reduction([f,g,v,w] | proc=p)`

`sm1.reduction([f,g,v] | proc=p)`

`sm1.reduction([f,g] | proc=p, ring_var=r)`

::

*return*      List

$f$             Polynomial

$g, v, w$       List

$p$             Number (the process number of `ox_sm1`)

- It reduces  $f$  by the set of polynomial  $g$  in the homogenized Weyl algebra; it applies the division algorithm to  $f$ . The set of variables is  $v$  and  $w$  is weight

vectors to determine the order, which can be omitted. `sm1.reduction_noH` is for the Weyl algebra.

- The return value is of the form  $[r, c0, [c1, \dots, cm], [g1, \dots, gm]]$  where  $g=[g1, \dots, gm]$  and  $c0 f + c1 g1 + \dots + cm gm = r$ .  $r/c0$  is the normal form.
- The function `reduction` reduces reducible terms that appear in lower order terms.
- The functions `sm1.reduction-d(P,F,G)` and `sm1.reduction_noH-d(P,F,G)` are for distributed polynomials.
- When the arguments are two, the function `mod_reduction` is called. It uses the ring structure saved in the global variable `ring_var` in the `ox-sm1` server.

```
[259] sm1.reduction([x^2+y^2-4,[y^4-4*y^2+1,x+y^3-4*y],[x,y]]);
[x^2+y^2-4,1,[0,0],[y^4-4*y^2+1,x+y^3-4*y]]
[260] sm1.reduction([x^2+y^2-4,[y^4-4*y^2+1,x+y^3-4*y],[x,y],[[x,1]]]);
[0,1,[-y^2+4,-x+y^3-4*y],[y^4-4*y^2+1,x+y^3-4*y]]

[1837] XM_debug=0$ S=sm1.syz([[x^2-1,x^3-1,x^4-1],[x]])$
[1838] sm1.auto_reduce(1);
1
[1839] S0=sm1.gb([S[0],[x]]);
[[[-x^2-x-1,x+1,0],[x^2+1,0,-1]],[[0,x,0],[0,0,-1]]]
[1840] sm1.reduction([[-x^4-x^3-x^2-x,x^3+x^2+x+1,-1], S0[0]]);
[[0,0,0],-1,[[x^2+1,0,0],[1,0,0]],[[[-x^2-x-1,x+1,0],[x^2+1,0,-1]]]
```

#### Reference

`sm1.start, d_true_nf`

#### 1.2.10 sm1.xml\_tree\_to\_prefix\_string

`sm1.xml_tree_to_prefix_string(s|proc=p)`

:: Translate OpenMath Tree Expression  $s$  in XML to a prefix notation.

*return*      String

$p$             Number

$s$             String

- It translate OpenMath Tree Expression  $s$  in XML to a prefix notation.
- This function should be moved to `om_*` in a future.
- `om_xml_to_cmo(OpenMath Tree Expression)` returns `CMO_TREE`. `asir` has not yet understood this CMO.
- `java` execution environment is required. (For example, `/usr/local/jdk1.1.8/bin` should be in the command search path.)

```
[263] load("om");
1
[270] F=om_xml(x^4-1);
```

```

control: wait OX
Trying to connect to the server... Done.
<OMOBJ><OMA><OMS name="plus" cd="basic"/><OMA>
<OMS name="times" cd="basic"/><OMA>
<OMS name="power" cd="basic"/><OMV name="x"/><OMI>4</OMI></OMA>
<OMI>1</OMI></OMA><OMA><OMS name="times" cd="basic"/><OMA>
<OMS name="power" cd="basic"/><OMV name="x"/><OMI>0</OMI></OMA>
<OMI>-1</OMI></OMA></OMA></OMOBJ>
[271] sm1.xml_tree_to_prefix_string(F);
basic_plus(basic_times(basic_power(x,4),1),basic_times(basic_power(x,0),-1))

```

## Reference

om\_\*, OpenXM/src/OpenMath, eval\_str

## 1.2.11 sm1.syz

`sm1.syz([f,v,w] | proc=p)`  
 :: computes the syzygy of  $f$  in the ring of differential operators with the variable  $v$ .

*return* List

$p$  Number

$f, v, w$  List

- The return values is of the form  $[s, [g, m, t]]$ . Here  $s$  is the syzygy of  $f$  in the ring of differential operators with the variable  $v$ .  $g$  is a Groebner basis of  $f$  with the weight vector  $w$ , and  $m$  is a matrix that translates the input matrix  $f$  to the Gr\obner basis  $g$ .  $t$  is the syzygy of the Gr\obner basis  $g$ . In summary,  $g = m f$  and  $s f = 0$  hold as matrices.
- The weight vectors are given by  $w$ , which can be omitted. If  $w$  is not given, the graded reverse lexicographic order will be used to compute Grobner basis.
- When a non-term order is given, the Grobner basis is computed in the homogenized Weyl algebra (See Section 1.2 of the book of SST). The homogenization variable  $h$  is automatically added.

```

[293] sm1.syz([[x*dx+y*dy-1,x*y*dx*dy-2],[x,y]]);
[[[y*x*dy*dx-2,-x*dx-y*dy+1]], generators of the syzygy
[[[x*dx+y*dy-1],[y^2*dy^2+2]], grobner basis
[[1,0],[y*dy,-1]], transformation matrix
[[y*x*dy*dx-2,-x*dx-y*dy+1]]]

[294] sm1.syz([[x^2*dx^2+x*dx+y^2*dy^2+y*dy-4,x*y*dx*dy-1],[x,y],[[dx,-1,x,1]
[[[y*x*dy*dx-1,-x^2*dx^2-x*dx-y^2*dy^2-y*dy+4]], generators of the syzygy
[[[x^2*dx^2+h^2*x*dx+y^2*dy^2+h^2*y*dy-4*h^4],[y*x*dy*dx-h^4], GB
[h^4*x*dx+y^3*dy^3+3*h^2*y^2*dy^2-3*h^4*y*dy]],
[[1,0],[0,1],[y*dy,-x*dx]], transformation matrix
[[y*x*dy*dx-h^4,-x^2*dx^2-h^2*x*dx-y^2*dy^2-h^2*y*dy+4*h^4]]]

```

**1.2.12 sm1.mul**

`sm1.mul(f,g,v|proc=p)`  
 :: ask the sm1 server to multiply  $f$  and  $g$  in the ring of differential operators over  $v$ .

*return*      Polynomial or List

$p$             Number

$f, g$         Polynomial or List

$v$             List

- Ask the sm1 server to multiply  $f$  and  $g$  in the ring of differential operators over  $v$ .
- `sm1.mul_h` is for homogenized Weyl algebra.
- BUG: `sm1.mul(p0*dp0,1,[p0])` returns `dp0*p0+1`. A variable order such that d-variables come after non-d-variables is necessary for the correct computation.

```
[277] sm1.mul(dx,x,[x]);
x*dx+1
[278] sm1.mul([x,y],[1,2],[x,y]);
x+2*y
[279] sm1.mul([[1,2],[3,4]],[[x,y],[1,2]],[x,y]);
[[x+2,y+4],[3*x+4,3*y+8]]
```

**1.2.13 sm1.distract**

`sm1.distract(f,v,x,d,s|proc=p)`  
 :: ask the sm1 server to compute the distraction of  $f$ .

*return*      List

$p$             Number

$f$             Polynomial

$v,x,d,s$     List

- It asks the sm1 server of the descriptor number  $p$  to compute the distraction of  $f$  in the ring of differential operators with variables  $v$ .
- $x$  is a list of x-variables and  $d$  is that of d-variables to be distracted.  $s$  is a list of variables to express the distracted  $f$ .
- Distraction is roughly speaking to replace  $x*dx$  by a single variable  $x$ . See Saito, Sturmfels, Takayama : Grobner Deformations of Hypergeometric Differential Equations at page 68 for details.

```
[280] sm1.distract([x*dx,[x],[x],[dx],[x]]);
x
[281] sm1.distract([dx^2,[x],[x],[dx],[x]]);
x^2-x
[282] sm1.distract([x^2,[x],[x],[dx],[x]]);
x^2+3*x+2
```

```
[283] fctr(0);
[[1,1],[x+1,1],[x+2,1]]
[284] sm1.distracton([x*dx*y+x^2*dx^2*dy,[x,y],[x],[dx],[x]]);
(x^2-x)*dy+x*y
```

Reference

```
distracton2(sm1),
```

### 1.2.14 sm1.gkz

`sm1.gkz([A,B] | proc=p)`  
 :: Returns the GKZ system (A-hypergeometric system) associated to the matrix  $A$  with the parameter vector  $B$ .

*return* List

$p$  Number

$A, B$  List

- Returns the GKZ hypergeometric system (A-hypergeometric system) associated to the matrix

```
[280] sm1.gkz([[[1,1,1,1],[0,1,3,4]], [0,2]]);
[[x4*dx4+x3*dx3+x2*dx2+x1*dx1,4*x4*dx4+3*x3*dx3+x2*dx2-2,
-dx1*dx4+dx2*dx3,-dx2^2*dx4+dx1*dx3^2,dx1^2*dx3-dx2^3,-dx2*dx4^2+dx3^3],
[x1,x2,x3,x4]]
```

### 1.2.15 sm1.mgkz

`sm1.mgkz([A,W,B] | proc=p)`  
 :: Returns the modified GKZ system (A-hypergeometric system) associated to the matrix  $A$  and the weight  $w$  with the parameter vector  $B$ .

*return* List

$p$  Number

$A, W, B$  List

- Returns the modified GKZ hypergeometric system (A-hypergeometric system) associated to the matrix
- <http://arxiv.org/abs/0707.0043>

```
[280] sm1.mgkz([[[1,2,3]], [1,2,1], [a/2]]);
[[6*x3*dx3+4*x2*dx2+2*x1*dx1-a,-x4*dx4+x3*dx3+2*x2*dx2+x1*dx1,
-dx2+dx1^2,-x4^2*dx3+dx1*dx2],[x1,x2,x3,x4]]
```

Modified A-hypergeometric system for  
 $A=(1,2,3)$ ,  $w=(1,2,1)$ ,  $\beta=(a/2)$ .

### 1.2.16 sm1.appell1

`sm1.appell1(a|proc=p)`

:: Returns the Appell hypergeometric system  $F_1$  or  $F_D$ .

*return* List

*p* Number

*a* List

- Returns the hypergeometric system for the Lauricella function  $F_D(a, b_1, b_2, \dots, b_n, c; x_1, \dots, x_n)$  where  $a = (a, c, b_1, \dots, b_n)$ . When  $n=2$ , the Lauricella function is called the Appell function  $F_1$ . The parameters  $a$ ,  $c$ ,  $b_1$ , ...,  $b_n$  may be rational numbers.
- It does not call `sm1` function `appell1`. As a consequence, when parameters are rational or symbolic, this function also works as well as integral parameters.

```
[281] sm1.appell1([1,2,3,4]);
[[((-x1+1)*x2*dx1-3*x2)*dx2+(-x1^2+x1)*dx1^2+(-5*x1+2)*dx1-3,
 (-x2^2+x2)*dx2^2+((-x1*x2+x1)*dx1-6*x2+2)*dx2-4*x1*dx1-4,
 ((-x2+x1)*dx1+3)*dx2-4*dx1], equations
[x1,x2]] the list of variables

[282] sm1.gb(@);
[[((-x2+x1)*dx1+3)*dx2-4*dx1,((-x1+1)*x2*dx1-3*x2)*dx2+(-x1^2+x1)*dx1^2
 +(-5*x1+2)*dx1-3,(-x2^2+x2)*dx2^2+((-x2^2+x1)*dx1-3*x2+2)*dx2
 +(-4*x2-4*x1)*dx1-4,
 (x2^3+(-x1-1)*x2^2+x1*x2)*dx2^2+((-x1*x2+x1^2)*dx1+6*x2^2
 +(-3*x1-2)*x2+2*x1)*dx2-4*x1^2*dx1+4*x2-4*x1],
[x1*dx1*dx2,-x1^2*dx1^2,-x2^2*dx1*dx2,-x1*x2^2*dx2^2]]

[283] sm1.rank(sm1.appell1([1/2,3,5,-1/3]));
3

[285] Mu=2$ Beta = 1/3$
[287] sm1.rank(sm1.appell1([Mu+Beta,Mu+1,Beta,Beta,Beta]));
4
```

### 1.2.17 sm1.appell4

`sm1.appell4(a|proc=p)`

:: Returns the Appell hypergeometric system  $F_4$  or  $F_C$ .

*return* List

*p* Number

*a* List

- Returns the hypergeometric system for the Lauricella function  $F_4(a,b,c_1,c_2,\dots,c_n;x_1,\dots,x_n)$  where  $a=(a,b,c_1,\dots,c_n)$ . When  $n=2$ , the Lauricella function is called the Appell function  $F_4$ . The parameters  $a$ ,  $b$ ,  $c_1$ , ...,  $c_n$  may be rational numbers.
- 
- It does not call `sm1` function `appell4`. As a consequence, when parameters are rational or symbolic, this function also works as well as integral parameters.

```
[281] sm1.appell4([1,2,3,4]);
 [[-x2^2*dx2^2+(-2*x1*x2*dx1-4*x2)*dx2+(-x1^2+x1)*dx1^2+(-4*x1+3)*dx1-2,
 (-x2^2+x2)*dx2^2+(-2*x1*x2*dx1-4*x2+4)*dx2-x1^2*dx1^2-4*x1*dx1-2],
 equations
 [x1,x2]] the list of variables

[282] sm1.rank(@);
4
```

### 1.2.18 sm1.rank

`sm1.rank(a|proc=p)`  
 :: Returns the holonomic rank of the system of differential equations  $a$ .  
*return*      Number  
*p*              Number  
*a*              List

- It evaluates the dimension of the space of holomorphic solutions at a generic point of the system of differential equations  $a$ . The dimension is called the holonomic rank.
- $a$  is a list consisting of a list of differential equations and a list of variables.
- `sm1.rrank` returns the holonomic rank when  $a$  is regular holonomic. It is generally faster than `sm1.rank`.

```
[284] sm1.gkz([[[1,1,1,1],[0,1,3,4]], [0,2]]);
 [[x4*dx4+x3*dx3+x2*dx2+x1*dx1,4*x4*dx4+3*x3*dx3+x2*dx2-2,
 -dx1*dx4+dx2*dx3, -dx2^2*dx4+dx1*dx3^2,dx1^2*dx3-dx2^3,-dx2*dx4^2+dx3^3],
 [x1,x2,x3,x4]]
[285] sm1.rrank(@);
4

[286] sm1.gkz([[[1,1,1,1],[0,1,3,4]], [1,2]]);
 [[x4*dx4+x3*dx3+x2*dx2+x1*dx1-1,4*x4*dx4+3*x3*dx3+x2*dx2-2,
 -dx1*dx4+dx2*dx3,-dx2^2*dx4+dx1*dx3^2,dx1^2*dx3-dx2^3,-dx2*dx4^2+dx3^3],
 [x1,x2,x3,x4]]
```

```
[287] sm1.rrank(@);
5
```

### 1.2.19 sm1.auto\_reduce

```
sm1.auto_reduce(s|proc=p)
:: Set the flag "AutoReduce" to s.
```

*return*      Number

*p*            Number

*s*            Number

- If *s* is 1, then all Grobner bases to be computed will be the reduced Grobner bases.
- If *s* is 0, then Grobner bases are not necessarily the reduced Grobner bases. This is the default.

### 1.2.20 sm1.slope

```
sm1.slope(ii,v,f_filtration,v_filtration|proc=p)
:: Returns the slopes of differential equations ii.
```

*return*      List

*p*            Number

*ii*           List (equations)

*v*            List (variables)

*f\_filtration* List (weight vector)

*v\_filtration*  
List (weight vector)

- **sm1.slope** returns the (geometric) slopes of the system of differential equations *ii* along the hyperplane specified by the V filtration *v\_filtration*.
- *v* is a list of variables.
- The return value is a list of lists. The first entry of each list is the slope and the second entry is the weight vector for which the microcharacteristic variety is not bihomogeneous.

Algorithm: see "A.Assi, F.J.Castro-Jimenez and J.M.Granger, How to calculate the slopes of a D-module, Compositio Math, 104, 1-17, 1996" Note that the signs of the slopes  $s'$  are negative, but the absolute values  $-s'$  of the slopes are returned. In other words, when  $pF+qV$  is the gap,  $-s'=q/p$  is returned. Note that  $s=1-1/s'$  is called the slope in recent literatures. Solutions belongs to  $O(s)$ . The number  $s$  satisfies  $1 \leq s$ . We have  $r=s-1=-1/s'$ , and  $\kappa=1/r=-s'$ , which is used  $1/\Gamma(1+m*r)$  factor and  $\exp(-\tau^\kappa)$  in the Borel and Laplace transformations respectively.



```

[284] A= sm1.gkz([[1,2,3]], [-3]]);

[285] sm1.slope(A[0],A[1],[0,0,0,1,1,1],[0,0,-1,0,0,1]);

[286] A2 = sm1.gkz([[1,1,1,0],[2,-3,1,-3]], [1,0]);
 (* This is an interesting example given by Laura Matusevich,
 June 9, 2001 *)

[287] sm1.slope(A2[0],A2[1],[0,0,0,0,1,1,1,1],[0,0,0,-1,0,0,0,1]);

```

## Reference

sm.gb

### 1.2.21 sm1.ahg

sm1.ahg( $A$ )  
: It is identical with sm1.gkz( $A$ ).

### 1.2.22 sm1.bfunction

sm1.bfunction( $F$ )  
: It computes the global b-function of  $F$ .

#### Description:

It no longer calls sm1's original bfunction. Instead, it calls asir "bfct".

#### Algorithm:

M.Noro, Mathematical Software, icms 2002, pp.147–157.

#### Example:

```
sm1.bfunction(x^2-y^3);
```

### 1.2.23 sm1.call\_sm1

sm1.call\_sm1( $F$ )  
: It executes  $F$  on the sm1 server. See also sm1.

### 1.2.24 sm1.ecart\_homogenize01Ideal

sm1.ecart\_homogenize01Ideal( $A$ )  
: It (0,1)-homogenizes the ideal  $A[0]$ . Note that it is not an elementwise homogenization.

#### Example:

```

input1
F=[(1-x)*dx+1]$ FF=[F,"x,y"]$
sm1.ecart_homogenize01Ideal(FF);

```

```
input2
F=sm1.appell1([1,2,3,4]);
sm1.ecart_homogenize01Ideal(F);
```

### 1.2.25 sm1.ecartd\_gb

**sm1.ecartd\_gb(A)**  
: It returns a standard basis of  $A$  by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is used. If the option `rv="dp"` (`return_value="dp"`) is given, the answer is returned in distributed polynomials.

Note. Functions in the category `ecart` changes the global environment in the `sm1` server. If you interrupted these functions, run `sm1.ecartd_gb` with a small input and terminate it normally. Then, the global environment is reset to the normal state. Reference. G. Granger, T. Oaku, N. Takayama, Tangent cone algorithm for homogeized differential operators, 2005.

Example:

```
input1
F=[2*(1-x-y)*dx+1,2*(1-x-y)*dy+1]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
sm1.ecartd_gb(FF);
output1
[[(-2*x-2*y+2)*dx+h,(-2*x-2*y+2)*dy+h], [(-2*x-2*y+2)*dx,(-2*x-2*y+2)*dy]]
input2
F=[2*(1-x-y)*dx+h,2*(1-x-y)*dy+h]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]],["noAutoHomogenize",1]]$
sm1.ecartd_gb(FF);
input3
F=[[x^2,y+x],[x+y,y^3],[2*x^2+x*y,y+x+x*y^3]]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]],["degreeShift",[[0,1],[-3,1]]]]$
sm1.ecartd_gb(FF);
```

### 1.2.26 sm1.ecartd\_gb\_oxRingStructure

**sm1.ecartd\_gb\_oxRingStructure()**  
: It returns the `oxRingStructure` of the most recent `ecartd_gb` computation.

### 1.2.27 sm1.ecartd\_isSameIdeal\_h

**sm1.ecartd\_isSameIdeal\_h(F)**  
: Here,  $F=[II, JJ, V]$ . It compares two ideals  $II$  and  $JJ$  in  $h[0,1](D)$ -alg.

Example:

```
input
II=[(1-x)^2*dx+h*(1-x)]$ JJ = [(1-x)*dx+h]$
V=[x]$
sm1.ecartd_isSameIdeal_h([II,JJ,V]);
```

### 1.2.28 sm1.ecartd\_reduction

`sm1.ecartd_reduction(F,A)`

: It returns a reduced form of  $F$  in terms of  $A$  by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is used. When the output is  $G$ ,  $G[3]$  is  $F$  and  $G[0]-(G[1]*A-\text{sum}(k,G[2][k]*G[3][k]))=0$  holds.  $F$  must be  $(0,1)$ -homogenized (see `sm1.ecart_homogenize01Ideal`). This function does not check if the given order is admissible for the ecart reduction. To do this check, use `sm1.ecartd_gb`.

Example:

```
input
F=[2*(1-x-y)*dx+h,2*(1-x-y)*dy+h]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
G=sm1.ecartd_reduction(dx+dy,FF);
G[0]-(G[1]*(dx+dy)+G[2][0]*F[0]+G[2][1]*F[1]);
```

### 1.2.29 sm1.ecartd\_reduction\_noh

`sm1.ecartd_reduction_noh(F,A)`

: It returns a reduced form of  $F$  in terms of  $A$  by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is NOT used.  $A[0]$  must not contain the variable  $h$ .

Example:

```
F=[2*(1-x-y)*dx+1,2*(1-x-y)*dy+1]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
sm1.ecartd_reduction_noh(dx+dy,FF);
```

### 1.2.30 sm1.ecartd\_syz

`sm1.ecartd_syz(A)`

: It returns a syzygy of  $A$  by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is used. If the option `rv="dp"` (`return_value="dp"`) is given, the answer is returned in distributed polynomials. The return value is in the format `[s,[g,m,t]]`.  $s$  is the generator of the syzygies,  $g$  is the Grobner basis,  $m$  is the translation matrix from the generators to  $g$ .  $t$  is the syzygy of  $g$ .

Example:

```
input1
F=[2*(1-x-y)*dx+1,2*(1-x-y)*dy+1]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
sm1.ecartd_syz(FF);
input2
F=[2*(1-x-y)*dx+h,2*(1-x-y)*dy+h]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]],["noAutoHomogenize",1]]$
sm1.ecartd_syz(FF);
```

### 1.2.31 sm1.gb\_oxRingStructure

`sm1.gb_oxRingStructure()`  
: It returns the `oxRingStructure` of the most recent gb computation.

### 1.2.32 sm1.gb\_reduction

`sm1.gb_reduction(F,A)`  
: It returns a reduced form of  $F$  in terms of  $A$  by using a normal form algorithm.  $h[1,1](D)$ -homogenization is used.

Example:

```
input
F=[2*(h-x-y)*dx+h^2,2*(h-x-y)*dy+h^2]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]]]$
sm1.gb_reduction((h-x-y)^2*dx*dy,FF);
```

### 1.2.33 sm1.gb\_reduction\_noh

`sm1.gb_reduction_noh(F,A)`  
: It returns a reduced form of  $F$  in terms of  $A$  by using a normal form algorithm.

Example:

```
input
F=[2*dx+1,2*dy+1]$
FF=[F,"x,y",[[dx,1,dy,1]]]$
sm1.gb_reduction_noh((1-x-y)^2*dx*dy,FF);
```

### 1.2.34 sm1.generalized\_bfunction

`sm1.generalized_bfunction(I,V,VD,W)`  
: It computes the generalized b-function (indicial equation) of  $I$  with respect to the weight  $W$ .

Description:

It no longer calls `sm1`'s original function. Instead, it calls `asir "generic_bfct"`.

Example:

```
sm1.generalized_bfunction([x^2*dx^2-1/2,dy^2],[x,y],[dx,dy],[-1,0,1,0]);
```

### 1.2.35 sm1.integration

`sm1.integration(I,V,R)`

: It computes the integration of  $I$  as a D-module to the set defined by  $R$ .  $V$  is the list of variables. When the optional variable *degree*= $d$  is given, only the integrations from 0 to  $d$  are computed. Note that, in case of vector input, INTEGRATION VARIABLES MUST APPEAR FIRST in the list of variable  $V$ . We are using wbf-Roots to get the roots of b-functions, so we can use only generic weight vector for now.

`sm1.integration(I,V,R | degree=key0)`

: This function allows optional variables *degree*

Algorithm:

T.Oaku and N.Takayama, math.AG/9805006, <http://www.arxiv.org>

Example:

```
sm1.integration([dt - (3*t^2-x), dx + t],[t,x],[t]);
```

The output `[[n0,F0],[n1,F1],...]` means that  $H^0=D^{n0}/F0$ ,  $H^{-1}=D^{n1}/F1$ ,

The free basis of the vector space  $D^n$  is denoted by  $e0, e1, \dots$

### 1.2.36 sm1.isSameIdeal\_in\_Dalg

`sm1.isSameIdeal_in_Dalg(I,J,V)`

: It compares two ideals  $I$  and  $J$  in  $D\_alg$  (algebraic  $D$  with variables  $V$ , no homogenization).

Example:

Input1

```
II=[(1-x)^2*dx+(1-x)]$ JJ = [(1-x)*dx+1]$ V=[x]$
```

```
sm1.isSameIdeal_in_Dalg(II,JJ,V);
```

### 1.2.37 sm1.laplace

`sm1.laplace(L,V,VL)`

: It returns the Laplace transformation of  $L$  for  $VL$ .  $V$  is the list of space variables. The numbers in coefficients must not be rational with a non-1 denominator. cf. ptozp

Example:

```
L1=sm1.laplace(dt-(3*t^2-x),[x,t],[t,dt]);
```

```
L2=sm1.laplace(dx+t,[x,t],[t,dt]);
```

```
sm1.restriction([L1,L2],[t,x],[t] | degree=0);
```

**1.2.38 sm1.rat\_to\_p****sm1.rat\_to\_p( $F$ )**

: It returns the denominator of  $F$  and the numerator of  $F$ . They are returned in a list. All elements of the denominator and numerator are polynomial objects with integer coefficients. Note that `dn` and `nm` do not regard rational numbers as a fractional object and this function is necessary to send data to `sm1`, which accept only integers and does not accept rational numbers.

Example:

```
sm1.rat_to_p(1/2*x+1);
[x+2,2]
sm1.rat_to_p([1/2*x,1/3*x]);
[[3*x,2*x],6]
```

**1.2.39 sm1.restriction****sm1.restriction( $I, V, R$ )**

: It computes the restriction of  $I$  as a D-module to the set defined by  $R$ .  $V$  is the list of variables. When the optional variable `degree=d` is given, only the restrictions from 0 to  $d$  are computed. Note that, in case of vector input, RESTRICTION VARIABLES MUST APPEAR FIRST in the list of variable  $V$ . We are using `wbfRoots` to get the roots of b-functions, so we can use only generic weight vector for now.

**sm1.restriction( $I, V, R$  | `degree=key0`)**

: This function allows optional variables `degree`

Algorithm:

T.Oaku and N.Takayama, math.AG/9805006, <http://xxx.lanl.gov>

Example:

```
sm1.restriction([dx^2-x,dy^2-1],[x,y],[y]);
The output [[n0,F0],[n1,F1],...] means that $H^0 = D^{n0}/F0$, $H^{-1} = D^{n1}/F1$,
The free basis of the vector space D^n is denoted by $e0, e1, \dots$
```

**1.2.40 sm1.saturation****sm1.saturation( $T$ )**

:  $T = [I, J, V]$ . It returns saturation of  $I$  with respect to  $J^\infty$ .  $V$  is a list of variables.

Example:

```
sm1.saturation([[x^2, x^2*x^4, x^2, x^4^2], [x^2, x^4], [x^2, x^4]]);
```

**1.2.41 sm1.ahg****sm1.ahg( $A$ )**

: It identical with `sm1.gkz( $A$ )`.

**1.2.42 sm1.bfunction**`sm1.bfunction(F)`: It computes the global b-function of  $F$ .

Description:

It no longer calls sm1's original bfunction. Instead, it calls asir "bfct".

Algorithm:

M.Noro, Mathematical Software, icms 2002, pp.147–157.

Example:

```
sm1.bfunction(x^2-y^3);
```

**1.2.43 sm1.call\_sm1**`sm1.call_sm1(F)`: It executes  $F$  on the sm1 server. See also sm1.**1.2.44 sm1.ecart\_homogenize01Ideal**`sm1.ecart_homogenize01Ideal(A)`: It (0,1)-homogenizes the ideal  $A[0]$ . Note that it is not an elementwise homogenization.

Example:

```
input1
F=[(1-x)*dx+1]$ FF=[F,"x,y"]$
sm1.ecart_homogenize01Ideal(FF);
input2
F=sm1.appell1([1,2,3,4]);
sm1.ecart_homogenize01Ideal(F);
```

**1.2.45 sm1.ecartd\_gb**`sm1.ecartd_gb(A)`

: It returns a standard basis of  $A$  by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is used. If the option `rv="dp"` (`return_value="dp"`) is given, the answer is returned in distributed polynomials.

Note. Functions in the category `ecart` changes the global environment in the sm1 server. If you interrupted these functions, run `sm1.ecartd_gb` with a small input and terminate it normally. Then, the global environment is reset to the normal state. Reference. G. Granger, T. Oaku, N. Takayama, Tangent cone algorithm for homogenized differential operators, 2005.

Example:

```

input1
F=[2*(1-x-y)*dx+1,2*(1-x-y)*dy+1]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
sm1.ecartd_gb(FF);
output1
[[(-2*x-2*y+2)*dx+h,(-2*x-2*y+2)*dy+h],[(-2*x-2*y+2)*dx,(-2*x-2*y+2)*dy]]
input2
F=[2*(1-x-y)*dx+h,2*(1-x-y)*dy+h]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]],["noAutoHomogenize",1]]$
sm1.ecartd_gb(FF);
input3
F=[x^2,y+x],[x+y,y^3],[2*x^2+x*y,y+x+x*y^3]]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]],["degreeShift",[[0,1],[-3,1]]]]$
sm1.ecartd_gb(FF);

```

#### 1.2.46 sm1.ecartd\_gb\_oxRingStructure

`sm1.ecartd_gb_oxRingStructure()`  
: It returns the `oxRingStructure` of the most recent `ecartd_gb` computation.

#### 1.2.47 sm1.ecartd\_isSameIdeal\_h

`sm1.ecartd_isSameIdeal_h(F)`  
: Here,  $F=[II, JJ, V]$ . It compares two ideals  $II$  and  $JJ$  in  $h[0,1](D)$ -alg.

Example:

```

input
II=[(1-x)^2*dx+h*(1-x)]$ JJ = [(1-x)*dx+h]$
V=[x]$
sm1.ecartd_isSameIdeal_h([II, JJ, V]);

```

#### 1.2.48 sm1.ecartd\_reduction

`sm1.ecartd_reduction(F,A)`  
: It returns a reduced form of  $F$  in terms of  $A$  by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is used. When the output is  $G$ ,  $G[3]$  is  $F$  and  $G[0]-(G[1]*A-\text{sum}(k,G[2][k]*G[3][k]))=0$  holds.  $F$  must be  $(0,1)$ -homogenized (see `sm1.ecart_homogenize01Ideal`). This function does not check if the given order is admissible for the `ecart` reduction. To do this check, use `sm1.ecartd_gb`.

Example:

```

input

```



```

F=[2*(1-x-y)*dx+h,2*(1-x-y)*dy+h]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
G=sm1.ecartd_reduction(dx+dy,FF);
G[0]-(G[1]*(dx+dy)+G[2][0]*F[0]+G[2][1]*F[1]);

```

### 1.2.49 sm1.ecartd\_reduction\_noh

`sm1.ecartd_reduction_noh(F,A)`

: It returns a reduced form of  $F$  in terms of  $A$  by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is NOT used.  $A[0]$  must not contain the variable  $h$ .

Example:

```

F=[2*(1-x-y)*dx+1,2*(1-x-y)*dy+1]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
sm1.ecartd_reduction_noh(dx+dy,FF);

```

### 1.2.50 sm1.ecartd\_syz

`sm1.ecartd_syz(A)`

: It returns a syzygy of  $A$  by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is used. If the option `rv="dp"` (`return_value="dp"`) is given, the answer is returned in distributed polynomials. The return value is in the format `[s,[g,m,t]]`.  $s$  is the generator of the syzygies,  $g$  is the Grobner basis,  $m$  is the translation matrix from the generators to  $g$ .  $t$  is the syzygy of  $g$ .

Example:

```

input1
F=[2*(1-x-y)*dx+1,2*(1-x-y)*dy+1]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
sm1.ecartd_syz(FF);
input2
F=[2*(1-x-y)*dx+h,2*(1-x-y)*dy+h]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]],["noAutoHomogenize",1]]$
sm1.ecartd_syz(FF);

```

### 1.2.51 sm1.gb\_oxRingStructure

`sm1.gb_oxRingStructure()`

: It returns the `oxRingStructure` of the most recent `gb` computation.

### 1.2.52 sm1.gb\_reduction

`sm1.gb_reduction(F,A)`

: It returns a reduced form of  $F$  in terms of  $A$  by using a normal form algorithm.  $h[1,1](D)$ -homogenization is used.

Example:

```
input
F=[2*(h-x-y)*dx+h^2,2*(h-x-y)*dy+h^2]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]]]$
sm1.gb_reduction((h-x-y)^2*dx*dy,FF);
```

### 1.2.53 sm1.gb\_reduction\_noh

`sm1.gb_reduction_noh(F,A)`

: It returns a reduced form of  $F$  in terms of  $A$  by using a normal form algorithm.

Example:

```
input
F=[2*dx+1,2*dy+1]$
FF=[F,"x,y",[[dx,1,dy,1]]]$
sm1.gb_reduction_noh((1-x-y)^2*dx*dy,FF);
```

### 1.2.54 sm1.generalized\_bfunction

`sm1.generalized_bfunction(I,V,VD,W)`

: It computes the generalized b-function (indicial equation) of  $I$  with respect to the weight  $W$ .

Description:

It no longer calls sm1's original function. Instead, it calls `asir "generic_bfct"`.

Example:

```
sm1.generalized_bfunction([x^2*dx^2-1/2,dy^2],[x,y],[dx,dy],[-1,0,1,0]);
```

### 1.2.55 sm1.integration

`sm1.integration(I,V,R)`

: It computes the integration of  $I$  as a D-module to the set defined by  $R$ .  $V$  is the list of variables. When the optional variable `degree=d` is given, only the integrations from 0 to  $d$  are computed. Note that, in case of vector input, INTEGRATION VARIABLES MUST APPEAR FIRST in the list of variable  $V$ . We are using `wbf-Roots` to get the roots of b-functions, so we can use only generic weight vector for now.

`sm1.integration(I,V,R | degree=key0)`

: This function allows optional variables `degree`

Algorithm:

T.Oaku and N.Takayama, [math.AG/9805006](http://math.AG/9805006), <http://www.arxiv.org>

Example:

```
sm1.integration([dt - (3*t^2-x), dx + t],[t,x],[t]);
```

The output  $[[n0,F0],[n1,F1],\dots]$  means that  $H^0=D^n0/F0$ ,  $H^{-1}=D^n1/F1$ ,  
The free basis of the vector space  $D^n$  is denoted by  $e0, e1, \dots$

### 1.2.56 sm1.isSameIdeal\_in\_Dalg

`sm1.isSameIdeal_in_Dalg(I,J,V)`  
: It compares two ideals  $I$  and  $J$  in  $D\_alg$  (algebraic  $D$  with variables  $V$ , no homogenization).

Example:

```
Input1
II=[(1-x)^2*dx+(1-x)]$ JJ = [(1-x)*dx+1]$ V=[x]$
sm1.isSameIdeal_in_Dalg(II,JJ,V);
```

### 1.2.57 sm1.laplace

`sm1.laplace(L,V,VL)`  
: It returns the Laplace transformation of  $L$  for  $VL$ .  $V$  is the list of space variables. The numbers in coefficients must not be rational with a non-1 denominator. cf. `ptozp`

Example:

```
L1=sm1.laplace(dt-(3*t^2-x),[x,t],[t,dt]);
L2=sm1.laplace(dx+t,[x,t],[t,dt]);
sm1.restriction([L1,L2],[t,x],[t] | degree=0);
```

### 1.2.58 sm1.rat\_to\_p

`sm1.rat_to_p(F)`  
: It returns the denominator of  $F$  and the numerator of  $F$ . They are returned in a list. All elements of the denominator and numerator are polynomial objects with integer coefficients. Note that `dn` and `nm` do not regard rational numbers as a fractional object and this function is necessary to send data to `sm1`, which accept only integers and does not accept rational numbers.

Example:

```
sm1.rat_to_p(1/2*x+1);
[x+2,2]
sm1.rat_to_p([1/2*x,1/3*x]);
[[3*x,2*x],6]
```

### 1.2.59 sm1.restriction

`sm1.restriction(I,V,R)`  
: It computes the restriction of  $I$  as a  $D$ -module to the set defined by  $R$ .  $V$  is the list of variables. When the optional variable `degree=d`

is given, only the restrictions from 0 to  $d$  are computed. Note that, in case of vector input, RESTRICTION VARIABLES MUST APPEAR FIRST in the list of variable  $V$ . We are using `wbfRoots` to get the roots of b-functions, so we can use only generic weight vector for now.

```
sm1.restriction(I,V,R | degree=key0)
: This function allows optional variables degree
```

Algorithm:

T.Oaku and N.Takayama, *math.AG/9805006*, <http://xxx.lanl.gov>

Example:

```
sm1.restriction([dx^2-x,dy^2-1],[x,y],[y]);
The output [[n0,F0],[n1,F1],...] means that $H^0 = D^{n0}/F0$, $H^{-1} = D^{n1}/F1$,
The free basis of the vector space D^n is denoted by $e0, e1, \dots$
```

### 1.2.60 sm1.saturation

```
sm1.saturation(T)
: $T = [I, J, V]$. It returns saturation of I with respect to J^∞ . V
is a list of variables.
```

Example:

```
sm1.saturation([[x2^2,x2*x4, x2, x4^2], [x2,x4], [x2,x4]]);
```

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