

Sm1 OX Server

Edition : auto generated by oxgentexi on 27 March 2025

1 SM1

```
sm1 ox ox_sm1_forAsir . sm1.rr . sm1.rr $(OpenXM_HOME)/lib/asir-contrib . sm1
.. sm1 OpenXM/doc/kan96xx .
, sm1 server windows . cygwin , OpenXM/misc/packages/Windows sm1 windows .
, . .
 $X := \mathbf{C} \setminus \{0, 1\} = \mathbf{C} \setminus V(x(x-1))$  .  $X$  ,  $x = 0, x = 1$  1. , 1 2 . sm1 0 1 .

[283] sm1.deRham([x*(x-1), [x]]);
[1,2]
```

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Reference: [SST] Saito, M., Sturmfels, B., Takayama, N., Grobner Deformations of Hypergeometric Differential Equations, 1999, Springer. <http://www.math.kobe-u.ac.jp/KAN>

1.1 ox_sm1_forAsir

1.1.1 ox_sm1_forAsir

```
ox_sm1_forAsir
:: asir sm1 .

• ox_sm1_forAsir asir sm1.start sm1 .

,
ox_sm1_forAsir = $(OpenXM_HOME)/lib/sm1/bin/ox_sm1 + $(OpenXM_
HOME)/lib/sm1/callsm1.sm1 (macro file)
+ $(OpenXM_HOME)/lib/sm1/callsm1b.sm1 (macro file)
, , current directory, $(LOAD_SM1_PATH), $(OpenXM_HOME)/lib/sm1,
/usr/local/lib/sm1 .

• : sm1 $(OpenXM_HOME)/src/kxx/oxserver00.c, $(OpenXM_HOME)/src/kxx/sm1stackmachine.c
```

1.2

1.2.1 sm1.start

```
sm1.start()
:: localhost ox_sm1_forAsir .
```

return

- localhost ox_sm1_forAsir . ox_sm1_forAsir .
- Xm_noX = 1 ox_sm1_forAsir .
- ord . , x dx (dx $\partial/\partial x$) , sm1 , dx x . , cc sm1 .
- a z , d o , , x0, ..., x20, y0, ..., y20, z0, ..., z20 , (cf. Sm1_ord_list in sm1).

- `static Sm1_proc . sm1.get_Sm1_proc() .`
`[260] ord([da,a,db,b]);`
`[da,a,db,b,dx,dy,dz,x,y,z,dt,ds,t,s,u,v,w,`
`..... omit`
`]`
`[261] a*da;`
`a*da`
`[262] cc*dcc;`
`dcc*cc`
`[263] sm1.mul(da,a,[a]);`
`a*da+1`
`[264] sm1.mul(a,da,[a]);`
`a*da`

`ox_launch, sm1.push_int0, sm1.push_poly0, ord`

1.2.2 sm1.sm1

`sm1.sm1(p,s)`
`:: sm1 s .`

return

p

s

- `p sm1 s . (, 0)`
`[261] sm1.sm1(0," ((x-1)^2) . ");`
`0`
`[262] ox_pop_string(0);`
`x^2-2*x+1`
`[263] sm1.sm1(0," [(x*(x-1)) [(x)]] deRham ");`
`0`
`[264] ox_pop_string(0);`
`[1 , 2]`

`sm1.start, ox_push_int0, sm1.push_poly0, sm1.get_Sm1_proc().`

1.2.3 sm1.push_int0

`sm1.push_int0(p,f)`
`:: f p .`

return

p

f

- `type(f) 2 () , f (type == 7) , ox_push_cmo .`
- `type(f) 0 (zero) , , 32 bit . ox_push_cmo(P,0) CMO_NULL , , 32 bit .`
- `sm1 , 32 bit bignum . type(f) 1 () , 32 bit integer . ox_push_cmo(p,1234) bignum`
`1234 sm1 .`

- `ox_push_cmo` .

```
[219] P=sm1.start();
0
[220] sm1.push_int0(P,x*dx+1);
0
[221] A=ox_pop_cmo(P);
x*dx+1
[223] type(A);
7 (string)

[271] sm1.push_int0(0,[x*(x-1),[x]]);
0
[272] ox_execute_string(0," deRham ");
0
[273] ox_pop_cmo(0);
[1,2]
```

Reference

`ox_push_cmo`

1.2.4 `sm1.gb`

`sm1.gb([f,v,w] | proc=p,sorted=q,dehomogenize=r,needBack=n,ring_var=r)`
`:: v f .`

`sm1.gb_d([f,v,w] | proc=p)`
`:: v f . .`

return

p, q, r

f, v, w

- `v f .`
- Weight `w .` , graded reverse lexicographic order .
- `sm1.gb f (w) (w) .`
- `sm1.gb_d . . [, ,] .`
- Term order , (SST Section 1.2). `h .`
- `q , 3 , . . . r ,` dehomogenize (`h 1`).
- Reduced `in_w` , `sm1.auto_reduce(1)` .
- `needBack 1` , [groebner basis, initial, gb,1,all, [groebner basis, backward transformation]] . (`sm1.getAttribute`)
- `ring_var ring_var_for_asir . sm1 ring . reduction .`
- `diffTop=1` , . option . `diffTop` differential term order over position . [gb, init w.r.t w, init].

```
[293] sm1.gb([[x*dx+y*dy-1,x*y*dx*dy-2],[x,y]]);
[[x*dx+y*dy-1,y^2*dy^2+2],[x*dx,y^2*dy^2]]
```

, $\{x\partial_x + y\partial_y - 1, y^2\partial_y^2 + 2\}$ $1 \leq \partial_y \leq \partial_x \leq y \leq x \leq \dots$ graded reverse lexicographic order . $\{x\partial_x, y^2\partial_y\}$ leading monomial (initial monomial) .

```
[294] sm1.gb([[dx^2+dy^2-4,dx*dy-1],[x,y],[[dx,50,dy,2,x,1]]]);
      [[dx+dy^3-4*dy,-dy^4+4*dy^2-1],[dx,-dy^4]]
```

$m = x^a y^b \partial_x^c \partial_y^d$ $m' = x^{a'} y^{b'} \partial_x^{c'} \partial_y^{d'}$ weight vector $(dx, dy, x, y) = (50, 2, 1, 0)$ (m $50c+2d+a > 50c'+2d'+a'$ m') reverse lexicographic order ($50c+2d+a = 50c'+2d'+a'$ reverse lexicographic order).

```
[294] F=sm1.gb([[dx^2+dy^2-4,dx*dy-1],[x,y],[[dx,50,dy,2,x,1]]|sorted=1);
      map(print,F[2][0])$
      map(print,F[2][1])$
```

```
[595] sm1.gb(["dx*(x*dx +y*dy-2)-1","dy*(x*dx + y*dy -2)-1",
            [x,y],[[dx,1,x,-1],[dy,1]]]);
```

```
[[x*dx^2+(y*dy-h^2)*dx-h^3,x*dy*dx+y*dy^2-h^2*dy-h^3,h^3*dx-h^3*dy],
 [x*dx^2+(y*dy-h^2)*dx,x*dy*dx+y*dy^2-h^2*dy-h^3,h^3*dx]]
```

```
[596] sm1.gb_d(["dx (x dx +y dy-2)-1","dy (x dx + y dy -2)-1",
              "x,y",[dx,1,x,-1],[dy,1]]);
[[[e0,x,y,H,E,dx,dy,h],
  [[0,-1,0,0,0,1,0,0],[0,0,0,0,0,0,1,0],[1,0,0,0,0,0,0,0],
   [0,1,1,1,1,1,1,0],[0,0,0,0,0,0,-1,0],[0,0,0,0,0,-1,0,0],
   [0,0,0,0,-1,0,0,0],[0,0,0,-1,0,0,0,0],[0,0,-1,0,0,0,0,0],
   [0,0,0,0,0,0,0,1]],
  [[(1)*<<0,0,1,0,0,1,1,0>>+(1)*<<0,1,0,0,0,2,0,0>>+(-1)*<<0,0,0,0,0,1,0,2>>+(-1)*
   <<0,0,0,0,0,0,0,3>>,(1)*<<0,0,1,0,0,0,2,0>>+(1)*<<0,1,0,0,0,1,1,0>>+(-1)*<<0,0,0,
   0,0,0,1,2>>+(-1)*<<0,0,0,0,0,0,0,3>>,(1)*<<0,0,0,0,0,1,0,3>>+(-1)*<<0,0,0,0,0,
   1,3>>],
  [(1)*<<0,0,1,0,0,1,1,0>>+(1)*<<0,1,0,0,0,2,0,0>>+(-1)*<<0,0,0,0,0,1,0,2>>,(1)*<
   <0,0,1,0,0,0,2,0>>+(1)*<<0,1,0,0,0,1,1,0>>+(-1)*<<0,0,0,0,0,1,2>>+(-1)*<<0,0,0,
   0,0,0,3>>,(1)*<<0,0,0,0,0,1,0,3>>]]]
```

```
[1834] sm1.gb([[dx^2-x,dx],[x]] | needBack=1);
      [[dx,dx^2-x,1],[dx,dx^2,1],gb,1,all,[[dx,dx^2-x,1],[[0,1],[1,0],[-dx,dx^2-x]]]]
```

// computation in R²

```
[1834] F=([[dx^2-dy,0],[x*dx+2*y*dy+1/2,0]],[x,y],[[dx,1,dy,1]]];
[1835] sm1.gb([F,[x,y],[[dx,1,dy,1]]] | difftop=1);
```

```
sm1.gb sm1. . sm1 .
```

```
if ((P_sm1=sm1.get_Sm1_proc()) < 0) P_sm1=sm1.start();
//sm1.sm1(P_sm1," 13 /gb.characteristic set ");
sm1.sm1(P_sm1," [(TraceLift) 13] system_variable ");
sm1.sm1(P_sm1," [(StopDegree) 27] /gb.options set ");
```

```
sm1.auto_reduce, sm1.reduction, sm1.rat_to_p
```

1.2.5 sm1.deRham

```

sm1.deRham([f,v]|proc=p)
    ::  $C^n$  - (the zero set of  $f=0$ ) .

return

p
f
v
    •  $X = C^n \setminus V(f)$  . ,  $[\dim H^0(X,C), \dim H^1(X,C), \dim H^2(X,C), \dots, \dim H^n(X,C)]$ 
    .
    •  $v.n = \text{length}(v)$  .
    • sm1.deRham . sm1.deRham(0, [x*y*z*(x+y+z-1)*(x-y), [x,y,z]]) .
    • b-, ox_asir ox_sm1_forAsir .
      sm1(0, "[(parse) (oxasir.sm1) pushfile] extension"); , ox_asir . ox_asir_
      forAsir .
    • sm1.deRham ox_reset(sm1.get_Sm1_proc()); , sm1 , ox_shutdown(sm1.get_Sm1_
      proc()); , ox_sm1_forAsir shutdown .
      [332] sm1.deRham([x^3-y^2, [x,y]]);
      [1,1,0]
      [333] sm1.deRham([x*(x-1), [x]]);
      [1,2]

      sm1.start, deRham (sm1 command)

```

Algorithm:

Oaku, Takayama, An algorithm for de Rham cohomology groups of the complement of an affine variety via D-module computation, Journal of pure and applied algebra 139 (1999), 201–233.

1.2.6 sm1.hilbert

```

sm1.hilbert([f,v]|proc=p)
    ::  $f$  .

hilbert_polynomial(f,v)
    ::  $f$  .

return

p
f, v
    •  $f \in v \cdot h(k)$  .
    •  $h(k) = \dim_Q F_k/I \cap F_k \cdot F_k^k \cdot I \cdot f$  .
    • sm1.hilbert :  $f$  . ,  $f$  , initial monomial . ,  $f$  . , sm1 asir .

[346] load("katsura")$

```

```

[351] A=hilbert_polynomial(katsura(5),[u0,u1,u2,u3,u4,u5]);
32

[279] load("katsura")$
[280] A=gr(katsura(5),[u0,u1,u2,u3,u4,u5],0)$
[281] dp_ord();
0
[282] B=map(dp_ht,map(dp_ptod,A,[u0,u1,u2,u3,u4,u5]));
[(1)*<<0,0,0,0,0,0>>,(1)*<<0,0,0,2,0,0>>,(1)*<<0,0,1,1,0,0>>,(1)*<<0,0,2,0,0,0>>,
(1)*<<0,1,1,0,0,0>>,(1)*<<0,2,0,0,0,0>>,(1)*<<0,0,0,1,1,1>>,(1)*<<0,0,0,1,2,0>>,
(1)*<<0,0,1,0,2,0>>,(1)*<<0,1,0,0,2,0>>,(1)*<<0,1,0,1,1,0>>,(1)*<<0,0,0,0,2,2>>,
(1)*<<0,0,1,0,1,2>>,(1)*<<0,1,0,0,1,2>>,(1)*<<0,1,0,1,0,2>>,(1)*<<0,0,0,0,3,1>>,
(1)*<<0,0,0,0,4,0>>,(1)*<<0,0,0,0,1,4>>,(1)*<<0,0,1,0,4>>,(1)*<<0,0,1,0,0,4>>,
(1)*<<0,1,0,0,0,4>>,(1)*<<0,0,0,0,0,6>>]
[283] C=map(dp_dtop,B,[u0,u1,u2,u3,u4,u5]);
[u0,u3^2,u3*u2,u2^2,u2*u1,u1^2,u5*u4*u3,u4^2*u3,u4^2*u2,u4^2*u1,u4*u3*u1,
u5^2*u4^2,u5^2*u4*u2,u5^2*u4*u1,u5^2*u3*u1,u5*u4^3,u4^4,u5^4*u4,u5^4*u3,
u5^4*u2,u5^4*u1,u5^6]
[284] sm1.hilbert([C,[u0,u1,u2,u3,u4,u5]]);
32

sm1.start, sm1.gb, longname

```

1.2.7 sm1.genericAnn

```

sm1.genericAnn([f,v]|proc=p)
:: f^s . v . , s v[0] , f rest(v) .

return

p
f
v
• , f^s . v . , s v[0] , f rest(v) .
[595] sm1.genericAnn([x^3+y^3+z^3,[s,x,y,z]]);
[-x*dx-y*dy-z*dz+3*s,z^2*dy-y^2*dz,z^2*dx-x^2*dz,y^2*dx-x^2*dy]

sm1.start

```

1.2.8 sm1.wTensor0

```

sm1.wTensor0([f,g,v,w]|proc=p)
:: f g D-module 0 .

return

p
f, g, v, w
• f g D- 0 .

```

- $v \cdot w$ weight. $w[i] \ v[i]$ weight.
 - `sm1.wTensor0 ox_sm1 wRestriction0.wRestriction0, generic weight w. Weight w generic.`
 - $F \ G \ f \ g \cdot, 0 \ FG$.
 - $f, g \ D \cdot, \cdot \ D^r$.
- ```
[258] sm1.wTensor0([x*dx -1, y*dy -4], [dx+dy, dx-dy^2], [x,y], [1,2]);
 [-y*x*dx-y*x*dy+4*x+y], [5*x*dx^2+5*x*dx+2*y*dy^2+(-2*y-6)*dy+3],
 [-25*x*dx+(-5*y*x-2*y^2)*dy^2+((5*y+15)*x+2*y^2+16*y)*dy-20*x-8*y-15],
 [y^2*dy^2+(-y^2-8*y)*dy+4*y+20]]
```

### 1.2.9 sm1.reduction

```
sm1.reduction([f,g,v,w] |proc=p)
sm1.reduction([f,g,v] |proc=p)
sm1.reduction([f,g] |proc=p, ring_var=r)
sm1.reduction_verbose([f,g,v,w] |proc=p)
::

return

f

g, v, w

p (ox_sm1)
• f homogenized, g (reduce);, , f. v. w, . sm1.reduction_noH, Weyl algebra.
• : [r,c0,[c1,...,cm],g] g=[g1, ..., gm], c0 f + c1 g1 + ... + cm gm = r. r/c0 normal form
.
• , reducible.
• sm1.reduction_d(P,F,G) sm1.reduction_noH_d(P,F,G), .
• mod_reduction. ox_sm1 ring_var ring. auto_reduce(1). gb.
• reduction_verbose [r,c0,[c1,...,cm],[g1,...gm],init,order] init order r initial.

[259] sm1.reduction([x^2+y^2-4, [y^4-4*y^2+1, x+y^3-4*y], [x,y]]);
 [x^2+y^2-4, 1, [0,0], [y^4-4*y^2+1, x+y^3-4*y]]
[260] sm1.reduction([x^2+y^2-4, [y^4-4*y^2+1, x+y^3-4*y], [x,y], [[x,1]]]);
 [0,1, [-y^2+4, -x+y^3-4*y], [y^4-4*y^2+1, x+y^3-4*y]]

[1837] XM_debug=0$ S=sm1.syz([[x^2-1, x^3-1, x^4-1], [x]])$
[1838] sm1.auto_reduce(1);
1
[1839] S0=sm1.gb([S[0], [x]]);
 [[[-x^2-x-1, x+1, 0], [x^2+1, 0, -1]], [[0, x, 0], [0, 0, -1]]]
[1840] sm1.reduction([[-x^4-x^3-x^2-x, x^3+x^2+x+1, -1], S0[0]]);
 [[0, 0, 0], -1, [[x^2+1, 0, 0], [1, 0, 0]], [[-x^2-x-1, x+1, 0], [x^2+1, 0, -1]]]

XM_debug=0$
sm1.auto_reduce(1)$
F=[x*y-1, x^2+y^2-4]$
```



```

Weight_vec=[[x,10,y,1]]$
printf("\n\nsyz----\n")$
S=sm1.syz([F,[x,y],Weight_vec]); // When Weight_vec is given, the TOP order is used.
// If the Weight_vec is not given, the POT order (e.g., (1,0,0)<(0,1,0)<(0,0,1)) with
Sgb=sm1.gb([S[0],[x,y],Weight_vec]);
R0=[x+y,x^2*y+x];
P=R0[0]*F[0]+R0[1]*F[1];
R=sm1.reduction_verbose([R0,Sgb[0],[x,y],Weight_vec]);

printf("\nMinimal representation=%a\n",R[0])$
printf("The initial of minimal rep=%a\n",R[4])$
printf("Order=%a\n",R[5][1][1])$

sm1.start, d_true_nf

```

### 1.2.10 sm1.xml\_tree\_to\_prefix\_string

```

sm1.xml_tree_to_prefix_string(s|proc=p)
:: XML OpenMath s .

return String
p Number
s String
• XML OpenMath s .
• om_* .
• om_xml_to_cmo(OpenMath Tree Expression) CMO_TREE . asir CMO .
• java . (, /usr/local/jdk1.1.8/bin .)
[263] load("om");
1
[270] F=om_xml(x^4-1);
control: wait OX
Trying to connect to the server... Done.
<OMOBJ><OMA><OMS name="plus" cd="basic"/><OMA>
<OMS name="times" cd="basic"/><OMA>
<OMS name="power" cd="basic"/><OMV name="x"/><OMI>4</OMI></OMA>
<OMI>1</OMI></OMA><OMA><OMS name="times" cd="basic"/><OMA>
<OMS name="power" cd="basic"/><OMV name="x"/><OMI>0</OMI></OMA>
<OMI>-1</OMI></OMA></OMA></OMOBJ>
[271] sm1.xml_tree_to_prefix_string(F);
basic_plus(basic_times(basic_power(x,4),1),basic_times(basic_power(x,0),-1))

om_*, OpenXM/src/OpenMath, eval_str

```

### 1.2.11 sm1.syz

```

sm1.syz([f,v,w]|proc=p)
:: v f syzygy .

return

```

$p$

$f, v, w$

- :  $[s, [g, m, t]]$ .  $s$   $f$   $v$  syzygy .  $g$   $f$  weight vector  $w$  .  $m$   $f$   $g$  .  $t$   $g$  syzygy . , :  $g = m f$  ,  $s f = 0$ .
- Weight  $w$  . , graded reverse lexicographic order .
- Term order , (SST Section 1.2 ).  $h$  .

```
[293] sm1.syz([[x*dx+y*dy-1,x*y*dx*dy-2],[x,y]]);
[[[y*x*dy*dx-2,-x*dx-y*dy+1]], generators of the syzygy
[[[x*dx+y*dy-1],[y^2*dy^2+2]], grobner basis
[[[1,0],[y*dy,-1]], transformation matrix
[[[y*x*dy*dx-2,-x*dx-y*dy+1]]]]
[294] sm1.syz([[x^2*dx^2+x*dx+y^2*dy^2+y*dy-4,x*y*dx*dy-1],[x,y],[[dx,-1,x,1]]]);
[[[y*x*dy*dx-1,-x^2*dx^2-x*dx-y^2*dy^2-y*dy+4]], generators of the syzygy
[[[x^2*dx^2+h^2*x*dx+y^2*dy^2+h^2*y*dy-4*h^4],[y*x*dy*dx-h^4], GB
[h^4*x*dx+y^3*dy^3+3*h^2*y^2*dy^2-3*h^4*y*dy]],
[[[1,0],[0,1],[y*dy,-x*dx]], transformation matrix
[[[y*x*dy*dx-h^4,-x^2*dx^2-h^2*x*dx-y^2*dy^2-h^2*y*dy+4*h^4]]]]]
```

### 1.2.12 sm1.mul

```
sm1.mul(f,g,v|proc=p)
:: sm1 f g v .
```

*return*

$p$

$f, g$

$v$

- $sm1 f g v$  .
- $sm1.mul\_h$  homogenized Weyl .
- BUG:  $sm1.mul(p0*dp0,1,[p0]) dp0*p0+1 . d, .$ 

```
[277] sm1.mul(dx,x,[x]);
x*dx+1
[278] sm1.mul([x,y],[1,2],[x,y]);
x+2*y
[279] sm1.mul([[1,2],[3,4]],[[x,y],[1,2]],[x,y]);
[[x+2,y+4],[3*x+4,3*y+8]]
```

### 1.2.13 sm1.distract

```
sm1.distract([f,v,x,d,s]|proc=p)
:: sm1 f distract .
```

*return*

$p$

$f$

$v, x, d, s$

- $p$  sm1,  $f$  distraction  $v$ .
- $x, d, ,$  distract  $x, d$ . Distraction,  $s$ .
- Distraction  $x \cdot dx \ x$ . Saito, Sturmfels, Takayama : Grobner Deformations of Hypergeometric Differential Equations page 68.

```
[280] sm1.distraction([x*dx,[x],[x],[dx],[x]]);
x
[281] sm1.distraction([dx^2,[x],[x],[dx],[x]]);
x^2-x
[282] sm1.distraction([x^2,[x],[x],[dx],[x]]);
x^2+3*x+2
[283] fctr(@);
[[1,1],[x+1,1],[x+2,1]]
[284] sm1.distraction([x*dx*y+x^2*dx^2*dy,[x,y],[x],[dx],[x]]);
(x^2-x)*dy+x*y
distraction2(sm1),
```

#### 1.2.14 sm1.gkz

```
sm1.gkz([A,B]|proc=p)
:: A B GKZ (A-hypergeometric system).
```

*return*

$p$

$A, B$

- $A \ B \ GKZ$  (A-hypergeometric system).

```
[280] sm1.gkz([[[1,1,1,1],[0,1,3,4]], [0,2]]);
[[x4*dx4+x3*dx3+x2*dx2+x1*dx1,4*x4*dx4+3*x3*dx3+x2*dx2-2,
-dx1*dx4+dx2*dx3,-dx2^2*dx4+dx1*dx3^2,dx1^2*dx3-dx2^3,-dx2*dx4^2+dx3^3],
[x1,x2,x3,x4]]
```

#### 1.2.15 sm1.mgkz

```
sm1.mgkz([A,W,B]|proc=p)
:: A, weight W B modified GKZ (A-hypergeometric system).
```

*return*

$p$

$A, W, B$

- $A$ , weight vector  $W \ B$  modified GKZ (A-hypergeometric system).
- <http://arxiv.org/abs/0707.0043>

```
[280] sm1.mgkz([[[1,2,3]], [1,2,1], [a/2]]);
```

```
[[6*x3*dx3+4*x2*dx2+2*x1*dx1-a,-x4*dx4+x3*dx3+2*x2*dx2+x1*dx1,
 -dx2+dx1^2,-x4^2*dx3+dx1*dx2],[x1,x2,x3,x4]]
```

Modified A-hypergeometric system for  
 $A=(1,2,3)$ ,  $w=(1,2,1)$ ,  $\beta=a/2$ .

### 1.2.16 sm1.appell1

```
sm1.appell1(a|proc=p)
 :: F_1 F_D .
```

*return*

*p*

*a*

- Appell  $F_1$  n Lauricella  $F_D(a,b1,b2,...,bn,c;x1,...,xn)$  . ,  $a=(a,c,b1,...,bn)$ . .
- sm1 appell1 , .

```
[281] sm1.appell1([1,2,3,4]);
[[((-x1+1)*x2*dx1-3*x2)*dx2+(-x1^2+x1)*dx1^2+(-5*x1+2)*dx1-3,
 (-x2^2+x2)*dx2^2+((-x1*x2+x1)*dx1-6*x2+2)*dx2-4*x1*dx1-4,
 ((-x2+x1)*dx1+3)*dx2-4*dx1], equations
[x1,x2]] the list of variables

[282] sm1.gb(@);
[[((-x2+x1)*dx1+3)*dx2-4*dx1,((-x1+1)*x2*dx1-3*x2)*dx2+(-x1^2+x1)*dx1^2
 +(-5*x1+2)*dx1-3,(-x2^2+x2)*dx2^2+((-x2^2+x1)*dx1-3*x2+2)*dx2
 +(-4*x2-4*x1)*dx1-4,
 (x2^3+(-x1-1)*x2^2+x1*x2)*dx2^2+((-x1*x2+x1^2)*dx1+6*x2^2
 +(-3*x1-2)*x2+2*x1)*dx2-4*x1^2*dx1+4*x2-4*x1],
[x1*dx1*dx2,-x1^2*dx1^2,-x2^2*dx1*dx2,-x1*x2^2*dx2^2]]
```

```
[283] sm1.rank(sm1.appell1([1/2,3,5,-1/3]));
3
```

```
[285] Mu=2$ Beta = 1/3$
```

```
[287] sm1.rank(sm1.appell1([Mu+Beta,Mu+1,Beta,Beta,Beta]));
4
```

### 1.2.17 sm1.appell4

```
sm1.appell4(a|proc=p)
 :: F_4 F_C .
```

*return*

*p*

*a*

- Appell  $F_4$  n Lauricella  $F_C(a,b,c_1,c_2,\dots,c_n;x_1,\dots,x_n)$  . ,  $a=(a,b,c_1,\dots,c_n)$  . .
- `sm1 appell1` , .

```
[281] sm1.appell14([1,2,3,4]);
 [[-x2^2*dx2^2+(-2*x1*x2*dx1-4*x2)*dx2+(-x1^2+x1)*dx1^2+(-4*x1+3)*dx1-2,
 (-x2^2+x2)*dx2^2+(-2*x1*x2*dx1-4*x2+4)*dx2-x1^2*dx1^2-4*x1*dx1-2],
 equations
 [x1,x2]] the list of variables

[282] sm1.rank(@);
4
```

### 1.2.18 `sm1.rank`

`sm1.rank(a|proc=p)`  
 :: *a* holonomic rank .

*return*

*p*

*a*

- *a* , generic point . , holonomic rank .
- *a* .
- *a* regular holonomic `sm1.rrank` holonomic rank . `sm1.rank` .

```
[284] sm1.gkz([[[1,1,1,1],[0,1,3,4]], [0,2]]);
 [[x4*dx4+x3*dx3+x2*dx2+x1*dx1,4*x4*dx4+3*x3*dx3+x2*dx2-2,
 -dx1*dx4+dx2*dx3, -dx2^2*dx4+dx1*dx3^2,dx1^2*dx3-dx2^3,-dx2*dx4^2+dx3^3],
 [x1,x2,x3,x4]]
[285] sm1.rrank(@);
4
```

```
[286] sm1.gkz([[[1,1,1,1],[0,1,3,4]], [1,2]]);
 [[x4*dx4+x3*dx3+x2*dx2+x1*dx1-1,4*x4*dx4+3*x3*dx3+x2*dx2-2,
 -dx1*dx4+dx2*dx3, -dx2^2*dx4+dx1*dx3^2,dx1^2*dx3-dx2^3,-dx2*dx4^2+dx3^3],
 [x1,x2,x3,x4]]
[287] sm1.rrank(@);
5
```

### 1.2.19 `sm1.auto_reduce`

`sm1.auto_reduce(s|proc=p)`  
 :: "AutoReduce" *s* .

$p$

$s$

- $s = 1$ , reduced.
- $s = 0$ , reduced.

### 1.2.20 sm1.slope

`sm1.slope(ii,v,f_filtration,v_filtration|proc=p)`  
 $\therefore$  *ii* slope.

*return*

$p$

*ii*  $()$

$v$   $()$

*f\_filtration* (weight vector)

*v\_filtration*

(weight vector)

- `sm1.slope ii V filtration v_filtration` (geometric) slope.
- $v$ .
- $, 1$  slope,  $2$ , weight vector microcharacteristic variety bihomogeneous.

Algorithm: "A.Assi, F.J.Castro-Jimenez and J.M.Granger, How to calculate the slopes of a D-module, Compositio Math, 104, 1-17, 1996". Slope  $s'$ , Slope  $-s'$ .  $pF+qV$  microgap,  $-s'=q/p$ .  $s=1-1/s'$  slope.  $O(s)$ .  $s-1 \leq s$ .  $r=s-1=-1/s'$   $\kappa=1/r=-s'$ . Borel and Laplace  $1/\Gamma(1+m*r)$  factor,  $\exp(-\tau^\kappa)$ .

[284] `A= sm1.gkz([ [1,2,3], [-3] ]);`

[285] `sm1.slope(A[0],A[1],[0,0,0,1,1,1],[0,0,-1,0,0,1]);`

[286] `A2 = sm1.gkz([ [1,1,1,0],[2,-3,1,-3]], [1,0]);`  
 (\* This is an interesting example given by Laura Matusevich,  
 June 9, 2001 \*)

[287] `sm1.slope(A2[0],A2[1],[0,0,0,0,1,1,1,1],[0,0,0,-1,0,0,0,1]);`

`sm.gb`

### 1.2.21 sm1.ahg

`sm1.ahg(A)`

: It identical with `sm1.gkz(A)`.

**1.2.22 sm1.bfunction**

`sm1.bfunction(F)`  
 : It computes the global b-function of  $F$ .

Description:

It no longer calls sm1's original bfunction. Instead, it calls asir "bfct".

Algorithm:

M.Noro, Mathematical Software, icms 2002, pp.147–157.

Example:

```
sm1.bfunction(x^2-y^3);
```

**1.2.23 sm1.call\_sm1**

`sm1.call_sm1(F)`  
 : It executes  $F$  on the sm1 server. See also sm1.

**1.2.24 sm1.ecart\_homogenize01Ideal**

`sm1.ecart_homogenize01Ideal(A)`  
 : It (0,1)-homogenizes the ideal  $A[0]$ . Note that it is not an elementwise homogenization.

Example:

```
input1
F=[(1-x)*dx+1]$ FF=[F,"x,y"]$
sm1.ecart_homogenize01Ideal(FF);
input2
F=sm1.appell1([1,2,3,4]);
sm1.ecart_homogenize01Ideal(F);
```

**1.2.25 sm1.ecartd\_gb**

`sm1.ecartd_gb(A)`  
 : It returns a standard basis of  $A$  by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is used. If the option `rv="dp"` (`return_value="dp"`) is given, the answer is returned in distributed polynomials.

Note. Functions in the category `ecart` changes the global environment in the sm1 server. If you interrupted these functions, run `sm1.ecartd_gb` with a small input and terminate it normally. Then, the global environment is reset to the normal state. Reference. G. Granger, T. Oaku, N. Takayama, Tangent cone algorithm for homogeized differential operators, 2005.

Example:

```
input1
F=[2*(1-x-y)*dx+1,2*(1-x-y)*dy+1]$
```

```

FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
sm1.ecartd_gb(FF);
output1
[[(-2*x-2*y+2)*dx+h,(-2*x-2*y+2)*dy+h],[(-2*x-2*y+2)*dx,(-2*x-2*y+2)*dy]]
input2
F=[2*(1-x-y)*dx+h,2*(1-x-y)*dy+h]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]],["noAutoHomogenize",1]]$
sm1.ecartd_gb(FF);
input3
F=[x^2,y+x],[x+y,y^3],[2*x^2+x*y,y+x+x*y^3]]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]],["degreeShift",[[0,1],[-3,1]]]]$
sm1.ecartd_gb(FF);

```

### 1.2.26 sm1.ecartd\_gb\_oxRingStructure

`sm1.ecartd_gb_oxRingStructure()`

: It returns the `oxRingStructure` of the most recent `ecartd_gb` computation.

### 1.2.27 sm1.ecartd\_isSameIdeal\_h

`sm1.ecartd_isSameIdeal_h(F)`

: Here,  $F=[II, JJ, V]$ . It compares two ideals  $II$  and  $JJ$  in  $h[0,1](D)$ -alg.

Example:

```

input
II=[(1-x)^2*dx+h*(1-x)]$ JJ = [(1-x)*dx+h]$
V=[x]$
sm1.ecartd_isSameIdeal_h([II, JJ, V]);

```

### 1.2.28 sm1.ecartd\_reduction

`sm1.ecartd_reduction(F, A)`

: It returns a reduced form of  $F$  in terms of  $A$  by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is used. When the output is  $G$ ,  $G[3]$  is  $F$  and  $G[0]-(G[1]*A-\sum(k, G[2][k]*G[3][k]))=0$  holds.  $F$  must be  $(0,1)$ -homogenized (see `sm1.ecart_homogenize01Ideal`). This function does not check if the given order is admissible for the `ecart` reduction. To do this check, use `sm1.ecartd_gb`.

Example:

```

input
F=[2*(1-x-y)*dx+h,2*(1-x-y)*dy+h]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
G=sm1.ecartd_reduction(dx+dy,FF);
G[0]-(G[1]*(dx+dy)+G[2][0]*F[0]+G[2][1]*F[1]);

```



### 1.2.29 sm1.ecartd\_reduction\_noh

`sm1.ecartd_reduction_noh(F,A)`

: It returns a reduced form of  $F$  in terms of  $A$  by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is NOT used.  $A[0]$  must not contain the variable  $h$ .

Example:

```
F=[2*(1-x-y)*dx+1,2*(1-x-y)*dy+1]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
sm1.ecartd_reduction_noh(dx+dy,FF);
```

### 1.2.30 sm1.ecartd\_syz

`sm1.ecartd_syz(A)`

: It returns a syzygy of  $A$  by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is used. If the option `rv="dp"` (`return_value="dp"`) is given, the answer is returned in distributed polynomials. The return value is in the format `[s,[g,m,t]]`.  $s$  is the generator of the syzygies,  $g$  is the Grobner basis,  $m$  is the translation matrix from the generators to  $g$ .  $t$  is the syzygy of  $g$ .

Example:

```
input1
F=[2*(1-x-y)*dx+1,2*(1-x-y)*dy+1]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
sm1.ecartd_syz(FF);
input2
F=[2*(1-x-y)*dx+h,2*(1-x-y)*dy+h]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]],["noAutoHomogenize",1]]$
sm1.ecartd_syz(FF);
```

### 1.2.31 sm1.gb\_oxRingStructure

`sm1.gb_oxRingStructure()`

: It returns the `oxRingStructure` of the most recent `gb` computation.

### 1.2.32 sm1.gb\_reduction

`sm1.gb_reduction(F,A)`

: It returns a reduced form of  $F$  in terms of  $A$  by using a normal form algorithm.  $h[1,1](D)$ -homogenization is used.

Example:

```
input
F=[2*(h-x-y)*dx+h^2,2*(h-x-y)*dy+h^2]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]]]$
sm1.gb_reduction((h-x-y)^2*dx*dy,FF);
```

### 1.2.33 `sm1.gb_reduction_noh`

`sm1.gb_reduction_noh(F,A)`

: It returns a reduced form of  $F$  in terms of  $A$  by using a normal form algorithm.

Example:

```
input
F=[2*dx+1,2*dy+1]$
FF=[F,"x,y",[[dx,1,dy,1]]]$
sm1.gb_reduction_noh((1-x-y)^2*dx*dy,FF);
```

### 1.2.34 `sm1.generalized_bfunction`

`sm1.generalized_bfunction(I,V,VD,W)`

: It computes the generalized b-function (indicial equation) of  $I$  with respect to the weight  $W$ .

Description:

It no longer calls `sm1`'s original function. Instead, it calls asir "generic\_bfct".

Example:

```
sm1.generalized_bfunction([x^2*dx^2-1/2,dy^2],[x,y],[dx,dy],[-1,0,1,0]);
```

### 1.2.35 `sm1.integration`

`sm1.integration(I,V,R)`

: It computes the integration of  $I$  as a D-module to the set defined by  $R$ .  $V$  is the list of variables. When the optional variable `degree=d` is given, only the integrations from 0 to  $d$  are computed. Note that, in case of vector input, INTEGRATION VARIABLES MUST APPEAR FIRST in the list of variable  $V$ . We are using `wbfRoots` to get the roots of b-functions, so we can use only generic weight vector for now.

`sm1.integration(I,V,R | degree=key0)`

: This function allows optional variables `degree`

Algorithm:

T.Oaku and N.Takayama, [math.AG/9805006](http://math.AG/9805006), <http://www.arxiv.org>

Example:

```
sm1.integration([dt - (3*t^2-x), dx + t],[t,x],[t]);
The output [[n0,F0],[n1,F1],...] means that $H^0=D^n0/F0$, $H^{-1}=D^n1/F1$, ...
The free basis of the vector space D^n is denoted by $e0, e1, \dots$
```

### 1.2.36 `sm1.isSameIdeal_in_Dalg`

`sm1.isSameIdeal_in_Dalg(I,J,V)`

: It compares two ideals  $I$  and  $J$  in  $D\_alg$  (algebraic  $D$  with variables  $V$ , no homogenization).

Example:

```
Input1
II=[(1-x)^2*dx+(1-x)]$ JJ = [(1-x)*dx+1]$ V=[x]$
sm1.isSameIdeal_in_Dalg(II,JJ,V);
```

### 1.2.37 sm1.laplace

`sm1.laplace(L,V,VL)`  
: It returns the Laplace transformation of  $L$  for  $VL$ .  $V$  is the list of space variables. The numbers in coefficients must not be rational with a non-1 denominator. cf. `ptozp`

Example:

```
L1=sm1.laplace(dt-(3*t^2-x),[x,t],[t,dt]);
L2=sm1.laplace(dx+t,[x,t],[t,dt]);
sm1.restriction([L1,L2],[t,x],[t | degree=0]);
```

### 1.2.38 sm1.rat\_to\_p

`sm1.rat_to_p(F)`  
: It returns the denominator of  $F$  and the numerator of  $F$ . They are returned in a list. All elements of the denominator and numerator are polynomial objects with integer coefficients. Note that `dn` and `nm` do not regard rational numbers as a fractional object and this function is necessary to send data to `sm1`, which accept only integers and does not accept rational numbers.

Example:

```
sm1.rat_to_p(1/2*x+1);
[x+2,2]
sm1.rat_to_p([1/2*x,1/3*x]);
[[3*x,2*x],6]
```

### 1.2.39 sm1.restriction

`sm1.restriction(I,V,R)`  
: It computes the restriction of  $I$  as a D-module to the set defined by  $R$ .  $V$  is the list of variables. When the optional variable `degree=d` is given, only the restrictions from 0 to  $d$  are computed. Note that, in case of vector input, RESTRICTION VARIABLES MUST APPEAR FIRST in the list of variable  $V$ . We are using `wbfRoots` to get the roots of b-functions, so we can use only generic weight vector for now.

`sm1.restriction(I,V,R | degree=key0)`  
: This function allows optional variables `degree`

Algorithm:

T.Oaku and N.Takayama, [math.AG/9805006](http://math.AG/9805006), <http://xxx.lanl.gov>

Example:

```
sm1.restriction([dx^2-x,dy^2-1],[x,y],[y]);
```

The output  $[[n_0, F_0], [n_1, F_1], \dots]$  means that  $H^0 = D^n / F_0$ ,  $H^{-1} = D^{n+1} / F_1$ , ...  
The free basis of the vector space  $D^n$  is denoted by  $e_0, e_1, \dots$

### 1.2.40 sm1.saturation

`sm1.saturation(T)`  
:  $T = [I, J, V]$ . It returns saturation of  $I$  with respect to  $J^\infty$ .  $V$  is a list of variables.

Example:

```
sm1.saturation([x^2, x^2*x^4, x^2, x^4^2], [x^2, x^4], [x^2, x^4]);
```

### 1.2.41 sm1.ahg

`sm1.ahg(A)`  
: It identical with `sm1.gkz(A)`.

### 1.2.42 sm1.bfunction

`sm1.bfunction(F)`  
: It computes the global b-function of  $F$ .

Description:

It no longer calls sm1's original bfunction. Instead, it calls asir "bfct".

Algorithm:

M.Noro, Mathematical Software, icms 2002, pp.147–157.

Example:

```
sm1.bfunction(x^2-y^3);
```

### 1.2.43 sm1.call\_sm1

`sm1.call_sm1(F)`  
: It executes  $F$  on the sm1 server. See also `sm1`.

### 1.2.44 sm1.ecart\_homogenize01Ideal

`sm1.ecart_homogenize01Ideal(A)`  
: It  $(0,1)$ -homogenizes the ideal  $A[0]$ . Note that it is not an elementwise homogenization.

Example:

```
input1
F=[(1-x)*dx+1]$ FF=[F,"x,y"]$
sm1.ecart_homogenize01Ideal(FF);
input2
F=sm1.appell1([1,2,3,4]);
sm1.ecart_homogenize01Ideal(F);
```

### 1.2.45 sm1.ecartd\_gb

`sm1.ecartd_gb(A)`

: It returns a standard basis of  $A$  by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is used. If the option `rv="dp"` (`return_value="dp"`) is given, the answer is returned in distributed polynomials.

Note. Functions in the category `ecart` changes the global environment in the `sm1` server. If you interrupted these functions, run `sm1.ecartd_gb` with a small input and terminate it normally. Then, the global environment is reset to the normal state. Reference. G. Granger, T. Oaku, N. Takayama, Tangent cone algorithm for homogeized differential operators, 2005.

Example:

```
input1
F=[2*(1-x-y)*dx+1,2*(1-x-y)*dy+1]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
sm1.ecartd_gb(FF);
output1
[(-2*x-2*y+2)*dx+h,(-2*x-2*y+2)*dy+h],[(-2*x-2*y+2)*dx,(-2*x-2*y+2)*dy]]
input2
F=[2*(1-x-y)*dx+h,2*(1-x-y)*dy+h]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]],["noAutoHomogenize",1]]$
sm1.ecartd_gb(FF);
input3
F=[[x^2,y+x],[x+y,y^3],[2*x^2+x*y,y+x*x*y^3]]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]],["degreeShift",[[0,1],[-3,1]]]]$
sm1.ecartd_gb(FF);
```

### 1.2.46 sm1.ecartd\_gb\_oxRingStructure

`sm1.ecartd_gb_oxRingStructure()`

: It returns the `oxRingStructure` of the most recent `ecartd_gb` computation.

### 1.2.47 sm1.ecartd\_isSameIdeal\_h

`sm1.ecartd_isSameIdeal_h(F)`

: Here,  $F=[II,JJ,V]$ . It compares two ideals  $II$  and  $JJ$  in  $h[0,1](D)$ -alg.

Example:

```
input
II=[(1-x)^2*dx+h*(1-x)]$ JJ = [(1-x)*dx+h]$
V=[x]$
sm1.ecartd_isSameIdeal_h([II,JJ,V]);
```

### 1.2.48 `sm1.ecartd_reduction`

`sm1.ecartd_reduction(F,A)`

: It returns a reduced form of  $F$  in terms of  $A$  by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is used. When the output is  $G$ ,  $G[3]$  is  $F$  and  $G[0]-(G[1]*A-\text{sum}(k,G[2][k]*G[3][k]))=0$  holds.  $F$  must be  $(0,1)$ -homogenized (see `sm1.ecart_homogenize01Ideal`). This function does not check if the given order is admissible for the ecart reduction. To do this check, use `sm1.ecartd_gb`.

Example:

```
input
F=[2*(1-x-y)*dx+h,2*(1-x-y)*dy+h]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
G=sm1.ecartd_reduction(dx+dy,FF);
G[0]-(G[1]*(dx+dy)+G[2][0]*F[0]+G[2][1]*F[1]);
```

### 1.2.49 `sm1.ecartd_reduction_noh`

`sm1.ecartd_reduction_noh(F,A)`

: It returns a reduced form of  $F$  in terms of  $A$  by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is NOT used.  $A[0]$  must not contain the variable  $h$ .

Example:

```
F=[2*(1-x-y)*dx+1,2*(1-x-y)*dy+1]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
sm1.ecartd_reduction_noh(dx+dy,FF);
```

### 1.2.50 `sm1.ecartd_syz`

`sm1.ecartd_syz(A)`

: It returns a syzygy of  $A$  by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is used. If the option `rv="dp"` (`return_value="dp"`) is given, the answer is returned in distributed polynomials. The return value is in the format `[s,[g,m,t]]`.  $s$  is the generator of the syzygies,  $g$  is the Grobner basis,  $m$  is the translation matrix from the generators to  $g$ .  $t$  is the syzygy of  $g$ .

Example:

```
input1
F=[2*(1-x-y)*dx+1,2*(1-x-y)*dy+1]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
sm1.ecartd_syz(FF);
input2
F=[2*(1-x-y)*dx+h,2*(1-x-y)*dy+h]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]],["noAutoHomogenize",1]]$
sm1.ecartd_syz(FF);
```

### 1.2.51 `sm1.gb_oxRingStructure`

`sm1.gb_oxRingStructure()`

: It returns the `oxRingStructure` of the most recent gb computation.

### 1.2.52 `sm1.gb_reduction`

`sm1.gb_reduction(F,A)`

: It returns a reduced form of  $F$  in terms of  $A$  by using a normal form algorithm.  $h[1,1](D)$ -homogenization is used.

Example:

```
input
F=[2*(h-x-y)*dx+h^2,2*(h-x-y)*dy+h^2]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]]]$
sm1.gb_reduction((h-x-y)^2*dx*dy,FF);
```

### 1.2.53 `sm1.gb_reduction_noh`

`sm1.gb_reduction_noh(F,A)`

: It returns a reduced form of  $F$  in terms of  $A$  by using a normal form algorithm.

Example:

```
input
F=[2*dx+1,2*dy+1]$
FF=[F,"x,y",[[dx,1,dy,1]]]$
sm1.gb_reduction_noh((1-x-y)^2*dx*dy,FF);
```

### 1.2.54 `sm1.generalized_bfunction`

`sm1.generalized_bfunction(I,V,VD,W)`

: It computes the generalized b-function (indicial equation) of  $I$  with respect to the weight  $W$ .

Description:

It no longer calls `sm1`'s original function. Instead, it calls `asir "generic_bfct"`.

Example:

```
sm1.generalized_bfunction([x^2*dx^2-1/2,dy^2],[x,y],[dx,dy],[-1,0,1,0]);
```

### 1.2.55 `sm1.integration`

`sm1.integration(I,V,R)`

: It computes the integration of  $I$  as a D-module to the set defined by  $R$ .  $V$  is the list of variables. When the optional variable `degree=d` is given, only the integrations from 0 to  $d$  are computed. Note that, in case of vector input, INTEGRATION VARIABLES MUST APPEAR FIRST in the list of variable  $V$ . We are using `wbfRoots` to get the roots of b-functions, so we can use only generic weight vector for now.

`sm1.integration(I,V,R | degree=key0)`  
 : This function allows optional variables *degree*

Algorithm:

T.Oaku and N.Takayama, math.AG/9805006, <http://www.arxiv.org>

Example:

```
sm1.integration([dt - (3*t^2-x), dx + t],[t,x],[t]);
```

The output `[[n0,F0],[n1,F1],...]` means that  $H^0=D^n0/F0$ ,  $H^{-1}=D^n1/F1$ , ...  
 The free basis of the vector space  $D^n$  is denoted by  $e0, e1, \dots$

### 1.2.56 `sm1.isSameIdeal_in_Dalg`

`sm1.isSameIdeal_in_Dalg(I,J,V)`  
 : It compares two ideals  $I$  and  $J$  in  $D\text{-alg}$  (algebraic  $D$  with variables  $V$ , no homogenization).

Example:

```
Input1
II=[(1-x)^2*dx+(1-x)]$ JJ = [(1-x)*dx+1]$ V=[x]$
sm1.isSameIdeal_in_Dalg(II,JJ,V);
```

### 1.2.57 `sm1.laplace`

`sm1.laplace(L,V,VL)`  
 : It returns the Laplace transformation of  $L$  for  $VL$ .  $V$  is the list of space variables. The numbers in coefficients must not be rational with a non-1 denominator. cf. `ptozp`

Example:

```
L1=sm1.laplace(dt-(3*t^2-x),[x,t],[t,dt]);
L2=sm1.laplace(dx+t,[x,t],[t,dt]);
sm1.restriction([L1,L2],[t,x],[t] | degree=0);
```

### 1.2.58 `sm1.rat_to_p`

`sm1.rat_to_p(F)`  
 : It returns the denominator of  $F$  and the numerator of  $F$ . They are returned in a list. All elements of the denominator and numerator are polynomial objects with integer coefficients. Note that `dn` and `nm` do not regard rational numbers as a fractional object and this function is necessary to send data to `sm1`, which accept only integers and does not accept rational numbers.

Example:

```
sm1.rat_to_p(1/2*x+1);
[x+2,2]
sm1.rat_to_p([1/2*x,1/3*x]);
[[3*x,2*x],6]
```



**1.2.59 sm1.restriction**

`sm1.restriction(I,V,R)`

: It computes the restriction of  $I$  as a D-module to the set defined by  $R$ .  $V$  is the list of variables. When the optional variable *degree*= $d$  is given, only the restrictions from 0 to  $d$  are computed. Note that, in case of vector input, RESTRICTION VARIABLES MUST APPEAR FIRST in the list of variable  $V$ . We are using `wbfRoots` to get the roots of b-functions, so we can use only generic weight vector for now.

`sm1.restriction(I,V,R | degree=key0)`

: This function allows optional variables *degree*

Algorithm:

T.Oaku and N.Takayama, [math.AG/9805006](http://math.AG/9805006), <http://xxx.lanl.gov>

Example:

```
sm1.restriction([dx^2-x,dy^2-1],[x,y],[y]);
```

The output `[[n0,F0],[n1,F1],...]` means that  $H^0 = D^{n0}/F0$ ,  $H^{-1} = D^{n1}/F1$ , ...

The free basis of the vector space  $D^n$  is denoted by  $e0, e1, \dots$

**1.2.60 sm1.saturation**

`sm1.saturation(T)`

:  $T = [I,J,V]$ . It returns saturation of  $I$  with respect to  $J^\infty$ .  $V$  is a list of variables.

Example:

```
sm1.saturation([[x2^2,x2*x4, x2, x4^2], [x2,x4], [x2,x4]]);
```

# Index

(Index is nonexistent)

(Index is nonexistent)

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