
2005 總 6

1 違ヤ

1.1 网荀

f_res 宴若吾, 紊紊医縹, dense 違 multipolynomial resultant 荐膊 f_res.mres, sparse 違 翫 sparse resultant 荐膊 f_res.sres, Dixon 号 resultant 荐膊 f_res.dres, 篁 違 紵荅 . 紵, resultant 縹菴翫, 羔祉や 紊縹箏ゆ翫, 違 弱阪 羔祉 莠 合翫.

号, 膩綵 紙, 後, mixed volume 膊 綽荀, や 若純 cddlib MixedVol . OpenXM 泣若 ox_sres 障 ., 純若鴻 c 鴻 ヤ若激 渦, make, 'OpenXM/src/ox_cdd' make, make install, asir や c や渦鴻若 . 箏 唇鴻 resultant 荐膊 asir 違, 'OpenXM/src/asir-contrib/packages/f_res/f_res.rr' . load, 電 篁ヲ 唇鴻眾純欲帥 ., 膩綵 紙喝後荐膊, gmp 喝荐膊茵, 羌 絨違 菴篴取膊 2 . 縹 鴻蕭, 茯ゆ黝翫. 御, f_res.gmp(), f_res.float() 若喝訝 .

1.2 Notation

ヤ 鴻 鴻, support polytope 鴻 鴻 茵 . や障, (1,1) 鴻 [1,1] ;, {(0,0), (1,0), (0,1)} polytope 鴻 鴻 [[0,0],[1,0],[0,1]] ;.

1.3 箏祉

1.3.1 f_res.mres, f_res.mresM

f_res.mres(Equations, Vars)
:: Multipolynomial resultant 縹菴

f_res.mresM(Equations, Vars)
:: 茵縹 f_res.mres 菴や 菴

return

f_res.mres
紊縹 0

f_res.mresM
茵

Equaitons 紊縹 鴻

Vars 紊違 鴻.

激

rsc 篁紙

rowidx

colidx

p 脰

sub 鴻

- *Equations* 縹 箏 絃 *Vars* 罨 縹 翫 号 *f_res.mres* resultant 縹 , *f_res.mresM* resultant 縹 茵 縹 よ 菴.
- *Equations* 縹 罨 , 罨 縹 綽 荀 .
- Rank Submatrix Construction 茵 激 *rsc* 1 絃. 翫, 違 *f_res.submatrix* 若 喝 呀 , 激 渴 鴻 篋.

```
[0] F0 = a1*x + a2*y + a3$
[1] F1 = b1*x + b2*y + b3$
[2] F2 = c1*x^2 + c2*y^2 + c3 + c4*x*y + c5*x + c6*y$
[3] f_res.mresM( [F0,F1,F2], [x,y] );
[ 0 0 0 a2 a3 a1 ]
[ 0 a2 a3 0 a1 0 ]
[ a2 a3 0 a1 0 0 ]
[ 0 b2 b3 0 b1 0 ]
[ b2 b3 0 b1 0 0 ]
[ c2 c6 c3 c4 c5 c1 ]
[4] R = f_res.mres( [F0,F1,F2], [x,y] );
(-c3*b2^2+c6*b3*b2-c2*b3^2)*a1^3+((2*c3*b2-c6*b3)*b1-c5*b3*b2+c4*b3^2)*a2+((-c6*b2+2*c2*b3)*b1+c5*b2^2-c4*b3*b2)*a3)*a1^2+((-c3*b1^2+c5*b3*b1-c1*b3^2)*a2^2+(c6*b1^2+(-c5*b2-c4*b3)*b1+2*c1*b3*b2)*a3*a2+(-c2*b1^2+c4*b2*b1-c1*b2^2)*a3^2)*a1
[5] fctr( R );
[[-1,1],[a1,1],[(c3*b2^2-c6*b3*b2+c2*b3^2)*a1^2+((-2*c3*b2+c6*b3)*b1+c5*b3*b2-c4*b3^2)*a2+((c6*b2-2*c2*b3)*b1-c5*b2^2+c4*b3*b2)*a3)*a1+(c3*b1^2-c5*b3*b1+c1*b3^2)*a2^2+(-c6*b1^2+(c5*b2+c4*b3)*b1-2*c1*b3*b2)*a3*a2+(c2*b1^2-c4*b2*b1+c1*b2^2)*a3^2,1]]
```

1.3.2 *f_res.indexof*

f_res.indexof(*Element*, *List*)
:: 鴻 箏 膾 障 箴 臀 菴

Element 縹 膾 荀 膾

List 縹 膾 荅 < 鴻

return *List* 障 *Element* や 渴 合 . *List* *Element* 障 翫 膾 -1.

- *List* 障 *Element* や 渴 合 激 菴. *List* *Element* 障 翫 -1 菴.
- *Element* c 罕 .
- *flist* 水, 違 *Asir* や c .

```
[0] f_res.indexof( 2, [1,2,3] );
1
[1] f_res.indexof( 4, [1,2,3] );
-1
[2] f_res.indexof( "nd_det", flist() );
31
[3] f_res.indexof( "nd_Det", flist() );
-1
```

1.3.3 f_res.listadd

f_res.listadd(A, B)

:: 鴻 罷

A

B 鴻

return 鴻

- 鴻 A 鴻 B 罷.
- 鴻 A 鴻 B 激 .
[0] f_res.listadd([1,2,3], [4,5,6]);
[5,7,9]
[1] f_res.listadd([a,b,c], [d,e,f]);
[a+d,b+e,c+f]

1.3.4 f_res.start

f_res.start(N)

:: ox_sres 莎桁

N 簞紙

return 賈

- <若 N 1 GMP, 簞 ュ 貴絨亥 違 OpenXM 泣若 ox_sres 莎桁, 簞 違 戎泣若 紵.
- 紵茵<や 苟や 違 若 ヤ.
- 菴賈違 篆< ュ .

1.3.5 f_res.float

f_res.float()

:: ox_sres 莎桁

return 賈

- 羌 絨亥 OpenXM 泣若 ox_sres 紵 儀, 簞 違 戎泣若 紵.
- 紵茵<や 苟や 違 若 ヤ.
- 翫 違 戎泣若 紵 違 儀 .
- 菴賈違 篆< ュ .

1.3.6 f_res.gmp

f_res.gmp()

:: ox_sres 莎桁

return 賈

- GMP OpenXM 泣若 ox_sres 紵 儀, 簞 違 戎泣若 紵.
- 紵茵<や 苟や 違 若 ヤ.
- 翫 違 戎泣若 紵 違 儀 .
- 菴賈違 篆< ュ .

1.3.7 f_res.conv

```
f_res.conv(List)
      :: polytope  橋罷
return      鴻  鴻
List      鴻茵  鴻  鴻
  • List  polytope  橋罷.
  • OpenXM 泣若 ox_sres 統      貴絨亥莎桁.
  • 鴻  婚岡  價違篁  .
      [0] f_res.conv( [ [1,1],[0,0],[0,2],[2,0],[2,2] ] );
      [[0,0],[0,2],[2,0],[2,2]]
```

1.3.8 f_res.support

```
f_res.support(Equation,Vars)
      :: 紊縵 support 菴
return      鴻  鴻
Equation  紊縵
Vars      箏紘  鴻
  • 箏紘 Vars      縵 Equation support 鴻  鴻  .
      [0] f_res.support( x^2 + x*y + y^2, [x,y] );
      [[0,2],[1,1],[2,0]]
      [1] f_res.support( x^2 + x*y + y^2, [x,y,z] );
      [[0,2,0],[1,1,0],[2,0,0]]
```

1.3.9 f_res.np

```
f_res.np(Equation,Vars)
      :: Newton polytope 菴
return      鴻  鴻
Equation  紊縵
Vars      箏紘  鴻
  • 箏紘 Vars      縵 Equation Newton polytope 鴻  鴻  .
  • OpenXM 泣若 ox_sres 統      貴絨亥莎桁.
      [0] f_res.np( x^2 + x*y + y^2, [x,y] );
      [[0,2],[2,0]]
      [1] f_res.np( x^2 + x*y + y^2, [x,y,z] );
      [[0,2,0],[2,0,0]]
```

1.3.10 f_res.msum

```
f_res.msum(Polytopes)
      :: polytope < Minkowski sum 菴
```

return 鴻 鴻
Polytopes 鴻 鴻 鴻
 激

conv 簞紙.

- *Polytopes* polytope Minkowski sum 鴻 lattice points 罷.
- *conv* 1 Minkowski sum 檣菴. OpenXM 泣若 *ox_sres* 統 貴絨亥莎桁.
 [0] Q1 = [[0,0],[1,0],[0,1]]\$
 [1] Q2 = [[0,0],[1,0],[0,1],[1,1]]\$
 [2] f_res.msum([Q1,Q1]);
 [[0,0],[0,1],[0,2],[1,0],[1,1],[2,0]]
 [3] f_res.msum([Q1,Q1] | conv=1);
 [[0,0],[0,2],[2,0]]
 [4] f_res.msum([Q1,Q1,Q1] | conv=1);
 [[0,0],[0,3],[3,0]]
 [5] f_res.msum([Q1,Q2]);
 [[0,0],[0,1],[0,2],[1,0],[1,1],[1,2],[2,0],[2,1]]
 [6] f_res.msum([Q1,Q2] | conv=1);
 [[0,0],[0,2],[1,2],[2,0],[2,1]]

1.3.11 f_res.mvol

f_res.mvol(Polytopes)
 :: polytope < mixed volume 罷

return 賈

Polytopes 鴻 鴻 鴻

- varPolytopes polytope mixed volume 罷.
- Mixed volume 臂 polytope 違 綽荀.
- OpenXM 泣若 *ox_sres* 統 貴絨亥莎桁.
 [0] Q1 = [[0,0],[1,0],[0,1]]\$
 [1] Q2 = [[0,0],[1,0],[0,1],[1,1]]\$
 [2] f_res.mvol([Q1,Q1]);
 1
 [3] f_res.mvol([Q1,Q2]);
 2
 [4] f_res.mvol([Q2,Q2]);
 2

1.3.12 f_res.sres

f_res.sres(Equations,Vars)
 :: sparse resultant 絳菴

return 紊絳

Equations 紊絳 鴻

Vars 箏紘 鴻

激

v 鴻

p 脰

sub 鴻

• *Equations* 縹 箏紘 Vars Incremental algorithm 膊 resultant 縹菴.

• 激 v v-distance 茵 鴻, 紘臂 翫 [11,12,13,...]\$ 箏帥.

• 茵 rank 膊 $GF(p)$ 箏, 茵 賢 紘 激 鴻 sub 紘 鴻 脰 撮ヤ苔箴 <. p
激 鴻 p . 脰 p 紘 翫 65521 箏帥, 鴻 sub 紘 翫 53,59,... 違箏帥.

• OpenXM 泣若 ox_sres 統 貴絨亥莎桁.

[0] F0 = a1*x + a2*y + a3\$

[1] F1 = b1*x + b2*y + b3\$

[2] F2 = c1*x^2 + c2*y^2 + c3 + c4*x*y + c5*x + c6*y\$

[3] R = f_res.sres([F0,F1,F2], [x,y]);

(c3*b2^3-c6*b3*b2^2+c2*b3^2*b2)*a1^2+((-2*c3*b2^2+c6*b3*b2)*b1+c5*b3*b2^2-c4*b3^2*b2)*a2+((c6*b2^2-2*c2*b3*b2)*b1-c5*b2^3+c4*b3*b2^2)*a3)*a1+(c3*b2*b1^2-c5*b3*b2*b1+c1*b3^2*b2)*a2^2+(-c6*b2*b1^2+(c5*b2^2+c4*b3*b2)*b1-2*c1*b3*b2^2)*a3*a2+(c2*b2*b1^2-c4*b2^2*b1+c1*b2^3)*a3^2

[4] fctr(R);

[[1,1],[b2,1],[(c3*b2^2-c6*b3*b2+c2*b3^2)*a1^2+((-2*c3*b2+c6*b3)*b1+c5*b3*b2-c4*b3^2)*a2+((c6*b2-2*c2*b3)*b1-c5*b2^2+c4*b3*b2)*a3)*a1+(c3*b1^2-c5*b3*b1+c1*b3^2)*a2^2+(-c6*b1^2+(c5*b2+c4*b3)*b1-2*c1*b3*b2)*a3*a2+(c2*b1^2-c4*b2*b1+c1*b2^2)*a3^2,1]]

1.3.13 f_res.dres, f_res.dresM

f_res.dres(*Equations*,*Vars*)

:: Dixon resultant 菴

f_res.dresM(*Equations*,*Vars*)

:: 茵縹 Dixon resultant 菴

return

f_res.dres

縹縹

f_res.dresM

茵

Equaitons 縹縹 鴻

Vars 箏紘 鴻

激

norsc 篁紙

rowidx

colidx

p 腔

sub 鴻

- *Equations* 綹 箏紘 *Vars* Dixon 号 f_res.dres resultant 綹, f_res.dresM resultant 綹茵綹 よ菴.

- Rank Submatrix Construction 茵 激 norsc 1 紘.

- 違 f_res.submatrix 若喝呀 , 激 鴻 鴻 篋.

```
[0] F0 = a1*x + a2*y + a3$
[1] F1 = b1*x + b2*y + b3$
[2] F2 = c1*x^2 + c2*y^2 + c3 + c4*x*y + c5*x + c6*y$
[3] f_res.dresM( [F0,F1,F2], [x,y] );
[ c1*b3*a2-c1*b2*a3 -c2*b3*a1+c4*b3*a2+(c2*b1-c4*b2)*a3 (c3*b2-c6*b3)*a1+(-c3*b
1+c5*b3)*a2+(c6*b1-c5*b2)*a3 ]
[ 0 -c2*b2*a1+c2*b1*a2 -c2*b3*a1+c2*b1*a3 ]
[ -c1*b2*a1+c1*b1*a2 -c4*b2*a1+c4*b1*a2 -c4*b3*a1+c1*b3*a2+(c4*b1-c1*b2)*a3 ]
[4] R = dres( [F0,F1,F2], [x,y] );
(-c3*c2*c1*b2^3+c6*c2*c1*b3*b2^2-c2^2*c1*b3^2*b2)*a1^3+(((3*c3*c2*c1*b2^2-2*c6*
c2*c1*b3*b2+c2^2*c1*b3^2)*b1-c5*c2*c1*b3*b2^2+c4*c2*c1*b3^2*b2)*a2+((-c6*c2*c1*
b2^2+2*c2^2*c1*b3*b2)*b1+c5*c2*c1*b2^3-c4*c2*c1*b3*b2^2)*a3)*a1^2+(((3*c3*c2*c
1*b2+c6*c2*c1*b3)*b1^2+(2*c5*c2*c1*b3*b2-c4*c2*c1*b3^2)*b1-c2*c1^2*b3^2*b2)*a2^
2+((2*c6*c2*c1*b2-2*c2^2*c1*b3)*b1^2-2*c5*c2*c1*b2^2*b1+2*c2*c1^2*b3*b2^2)*a3*a
2+(-c2^2*c1*b2*b1^2+c4*c2*c1*b2^2*b1-c2*c1^2*b2^3)*a3^2)*a1+(c3*c2*c1*b1^3-c5*c
2*c1*b3*b1^2+c2*c1^2*b3^2*b1)*a2^3+(-c6*c2*c1*b1^3+(c5*c2*c1*b2+c4*c2*c1*b3)*b1
^2-2*c2*c1^2*b3*b2*b1)*a3*a2^2+(c2^2*c1*b1^3-c4*c2*c1*b2*b1^2+c2*c1^2*b2^2*b1)*
a3^2*a2
[5] fctr(R);
[[-1,1],[c2,1],[c1,1],[b2*a1-b1*a2,1],[(c3*b2^2-c6*b3*b2+c2*b3^2)*a1^2+((-2*c3
*b2+c6*b3)*b1+c5*b3*b2-c4*b3^2)*a2+((c6*b2-2*c2*b3)*b1-c5*b2^2+c4*b3*b2)*a3)*a1
+(c3*b1^2-c5*b3*b1+c1*b3^2)*a2^2+(-c6*b1^2+(c5*b2+c4*b3)*b1-2*c1*b3*b2)*a3*a2+(
c2*b1^2-c4*b2*b1+c1*b2^2)*a3^2,1]]
```

1.3.14 f_res.dixonpolynomial

f_res.dixonpolynomial(*Equations*,*Vars*)

:: Dixon polynomial 菴

return 鴻

Equaitons 綹 鴻

Vars 箏紘 鴻

Equations 綹 箏紘 *Vars* Dixon polynomial 荐膊, [(Dixon polynomial), (違綹
違)] 鴻菴. 違綹違 uc c 箏紘 . 綹綹 違 違 違箏ゆ綹荀.

```
[0] F0 = a1*x + a2*y + a3$
[1] F1 = b1*x + b2*y + b3$
[2] F2 = c1*x^2 + c2*y^2 + c3 + c4*x*y + c5*x + c6*y$
[3] f_res.dixonpolynomial( [F0,F1,F2], [x,y] );
```

$$\begin{aligned} & [(-_0*c1*b2*a1+(_0*c1*b1+c1*b3)*a2-c1*b2*a3)*x+((-1*c2-_0*c4)*b2-c2*b3)*a1+((-1*c2+_0*c4)*b1+c4*b3)*a2+(c2*b1-c4*b2)*a3)*y+(c3*b2+(-1*c2-_0*c4-c6)*b3)*a1+(-c3*b1+(_0*c1+c5)*b3)*a2+((-1*c2+_0*c4+c6)*b1+(-_0*c1-c5)*b2)*a3, [_0 \ _1]] \end{aligned}$$

1.3.15 f_res.matrixdecomp

f_res.matrixdecomp(Dpoly, UC, Vars)
 :: Dixon polynomial 菌 茹 c .

return 鴻

Dpoly 素縷

UC

Vars 鴻

- dixonpolynomial Dpoly 菌 UC monomial, Vars monomial 兄続篁菌 茹 c .
- 祉や, [(UC monomial), (菌), (Vars monomial)] 綵 , $\sigma.P = V D_P W$
 V, D_P, W 菌 .

```
[0] F0 = a1*x + a2*y + a3$
[1] F1 = b1*x + b2*y + b3$
[2] F2 = c1*x^2 + c2*y^2 + c3 + c4*x*y + c5*x + c6*y$
[3] D = f_res.dixonpolynomial( [F0,F1,F2], [x,y] )$
[4] M = f_res.matrixdecomp( D[0], D[1], [x,y] );
[[ 1 _1 _0 ],[ c1*b3*a2-c1*b2*a3 -c2*b3*a1+c4*b3*a2+(c2*b1-c4*b2)*a3 (c3*b2-c6*
b3)*a1+(-c3*b1+c5*b3)*a2+(c6*b1-c5*b2)*a3 ]
[ 0 -c2*b2*a1+c2*b1*a2 -c2*b3*a1+c2*b1*a3 ]
[ -c1*b2*a1+c1*b1*a2 -c4*b2*a1+c4*b1*a2 -c4*b3*a1+c1*b3*a2+(c4*b1-c1*b2)*a3 ],[
x y 1 ]]
[5] V = M[0]*M[1]$
[6] D[0] == V[0]*M[2][0]+V[1]*M[2][1]+V[2]*M[2][2];
1
```

1.3.16 f_res.submatrix

f_res.submatrix(Matrix)
 :: 続違 菌 rank ら 菌菴.

return 菌

Matrix 菌

激

rowidx

colidx

p 脰

sub 鴻

- 菌 Matrix rank ら 菌菴.
- 菌 rank 膊 賢 紵 鴻 sub や 撮ヤ $GF(p)$ 箴 < . p 激 渦 p 箴帥.

- 箏茵罩 e 茵 障 . 絃 rowidx,colidx . 絃 Matrix (i,j)
(rowidx[i],colidx[j]) ヤ .
- 脰 p 絃 翫 65521 箏帥, 鴻 sub 絃 翫 53,59,dots 違箏帥.

```

[0] M = newmat( 3, 3, [[1,0,0],[0,a,0],[0,b,0]] );
[ 1 0 0 ]
[ 0 a 0 ]
[ 0 b 0 ]
[1] f_res.submatrix( M );
[ 1 0 ]
[ 0 a ]
[2] f_res.submatrix( M | rowidx=ltov([0,2,1]) );
[ 1 0 ]
[ 0 b ]

```

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(Index is nonexistent)

(Index is nonexistent)

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