

# Sm1 OX Server ㅏ

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Edition : auto generated by oxgentexi on 8 June 2017

OpenXM.org

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# 1 SM1 醇

sm1 ox 泣若 ox\_sm1\_forAsir や鴻帥 若拷 違茹h . 違 <や'sm1.rr' 臂  
 . 'sm1.rr' '\$(OpenXM\_HOME)/lib/asir-contrib' . 激鴻 sm1 小筋 違 膊 激鴻 .  
 荐膊篁 f 医江筋 糸 膊繼 筋 膊 鍵. sm1 や 吾 OpenXM/doc/kan96xx .  
 , sm1 server windows や 紹 . cygwin 医 純若鴻鴻若鴻鴻や ,  
 OpenXM/misc/packages/Windows 糸眼 sm1 泣若 windows 筋.

鴻 違, 遺違 ヌ や . 鴻 縷 違 贗違 .

腥咲  $X := \mathbf{C} \setminus \{0, 1\} = \mathbf{C} \setminus V(x(x-1))$  若鴻 悟召 荐膊 帥.  $X$  抗 や 腥  
 咲 ,  $x = 0, x = 1$  障 障篁や 若 1 電 吾主召 . c, 1 電 若鴻 悟召  
 2 . sm1 0 電 鴻 悟召 1 電 鴻 悟召 電 膈.

```
[283] sm1.deRham([x*(x-1), [x]]);
[1, 2]
```

The author of sm1 : Nobuki Takayama, takayama@math.sci.kobe-u.ac.jp

The author of sm1 packages : Toshinori Oaku, oaku@twcu.ac.jp

Reference: [SST] Saito, M., Sturmfels, B., Takayama, N., Grobner Deformations of Hypergeometric Differential Equations, 1999, Springer. <http://www.math.kobe-u.ac.jp/KAN>

## 1.1 ox\_sm1\_forAsir 泣若

### 1.1.1 ox\_sm1\_forAsir

ox\_sm1\_forAsir

:: asir sm1 泣若.

- 泣若 ox\_sm1\_forAsir asir 鴻鴻 sm1.start 儀 sm1 泣若 .  
 岡羣荐 ,  
 ox\_sm1\_forAsir = '\$(OpenXM\_HOME)/lib/sm1/bin/ox\_sm1' + '\$(OpenXM\_HOME)/lib/sm1/callsm1.s  
 (macro file)  
 + '\$(OpenXM\_HOME)/lib/sm1/callsm1b.sm1' (macro file)  
 , <や , 箒 current directory, \$(LOAD\_SM1\_PATH), \$(OpenXM\_  
 HOME)/lib/sm1, /usr/local/lib/sm1 .
- 違 若 若: sm1 茯 粹昭 泣若筋 電 < <や 荀  
 '\$(OpenXM\_HOME)/src/kxx/oxserver00.c', '\$(OpenXM\_HOME)/src/kxx/sm1stackmachine.c'

## 1.2 醇遺荀

### 1.2.1 sm1.start

sm1.start()

:: localhost ox\_sm1\_forAsir 鴻帥若.

return 贗

- localhost ox\_sm1\_forAsir 鴻帥若. 泣若 ox\_sm1\_forAsir ㊿ 激祉.
- Xm\_noX = 1 泣若 ox\_sm1\_forAsir 亥 c 鴻 儀 .

- 渦渦 ord 育綺罩 c 荐 綽荀 . , 紊 x dx 箏 小箏 (dx  $\partial/\partial x$  綽) 膊 , sm1 泣若 医激 , 紊 dx 喝眼 紊 x 窪眼 や 皿紘 . 電 < , 紊 cc sm1 膊 .
- a z , d o や , , x0, ..., x20, y0, ..., y20, z0, ..., z20 , 若 小箏 違 違 箏帥 (cf. Sm1\_ord\_list in sm1).
- 荔 ☺ 激 static Sm1\_proc 主. ☺ 激 sm1.get\_Sm1\_proc() .  

```

[260] ord([da,a,db,b]);
[da,a,db,b,dx,dy,dz,x,y,z,dt,ds,t,s,u,v,w,
..... omit .....
]
[261] a*da;
a*da
[262] cc*dcc;
dcc*cc
[263] sm1.mul(da,a,[a]);
a*da+1
[264] sm1.mul(a,da,[a]);
a*da

ox_launch, sm1.push_int0, sm1.push_poly0, ord

```

### 1.2.2 sm1.sm1

sm1.sm1(p,s)

:: 泣若 sm1 渦渦 s 紘茵 .

return

p

s 紘

- 荔 ☺ p sm1 泣若 渦渦 s 紘茵 若. (電 < , 荔 ☺ 0)

```

[261] sm1.sm1(0," ( (x-1)^2 ) . ");
0
[262] ox_pop_string(0);
x^2-2*x+1
[263] sm1.sm1(0," [(x*(x-1)) [(x)]] deRham ");
0
[264] ox_pop_string(0);
[1 , 2]

```

sm1.start, ox\_push\_int0, sm1.push\_poly0, sm1.get\_Sm1\_proc().

### 1.2.3 sm1.push\_int0

sm1.push\_int0(p,f)

:: 吾 f 荔 ☺ p 泣若檣.

return

p

$f$  吾

- `type(f)` 2 (紹医縹) ,  $f$  続 (`type == 7`) , `ox_push_cmo` 泣若檣.
- `type(f)` 0 (zero) , 泣若箏 , 32 bit 質違 . `ox_push_cmo(P,0)` 泣若 `CMO_NULL` , 泣若眼 , 32 bit 質違 .
- `sm1` 質違, 32 bit 質違 `bignum` . `type(f)` 1 () , 違 32 bit integer 泣若 . `ox_push_cmo(p,1234)` `bignum` 1234 `sm1` 泣若 絵.
- 翫 `ox_push_cmo` 若水 若喝訝.  

```
[219] P=sm1.start();
0
[220] sm1.push_int0(P,x*dx+1);
0
[221] A=ox_pop_cmo(P);
x*dx+1
[223] type(A);
7 (string)
[271] sm1.push_int0(0,[x*(x-1),[x]]);
0
[272] ox_execute_string(0," deRham ");
0
[273] ox_pop_cmo(0);
[1,2]
```

Reference

`ox_push_cmo`

#### 1.2.4 `sm1.gb`

`sm1.gb([f,v,w]|proc=p,sorted=q,dehomogenize=r)`

::  $v$  箏 小箏 違  $f$  違 阪荐膊.

`sm1.gb_d([f,v,w]|proc=p)`

::  $v$  箏 小箏 違  $f$  違 阪荐膊. 脰e縹 鴻 祉.

`return` 鴻

$p, q, r$

$f, v, w$  鴻

- $v$  箏 小箏 違  $f$  違 阪荐膊.
- Weight  $w$  ヤ . ヤ翫, graded reverse lexicographic order ヤ  $c$  阪荐膊.
- `sm1.gb` 祉ヤ  $f$  違 阪潟ヤ激  $c$  (  $w$  ) 障 ヤ激 < 縹 (  $w$  箏 ) 鴻 .
- `sm1.gb_d` e縹 鴻 祉. 紊縹 賢 障 違 阪荐膊 綺 純若 . 祉ヤ [紊医 鴻, 綺茵, 違 阪, ヤ激  $c$  障 ヤ激 < 縹] .
- Term order 綺箏翫, 電 ヤ 撮違 違 阪荐膊 (SST Section 1.2 荀). 電 紊 h 脰 .
- 激  $q$  祉 , 3 祉ヤ , 違 阪潟ヤ激 < 鴻 箏綺 純若 祉. 障 縹, 続 ; 障. 激  $r$  祉 , 祉紊縹 `dehomogenize` (  $h$  1 篁eヤ ).
- Reduced 違 弱阪障 `in-w` 荐膊 , 違 茵 `sm1.auto_reduce(1)` 紵茵 .

```

[293] sm1.gb([[x*dx+y*dy-1,x*y*dx*dy-2],[x,y]]);
[[x*dx+y*dy-1,y^2*dy^2+2],[x*dx,y^2*dy^2]]
箏      , {x∂x + y∂y - 1, y2∂y2 + 2} 1 ≤ ∂y ≤ ∂x ≤ y ≤ x ≤ ⋯ graded reverse
lexicographic order      違 阪 . {x∂x, y2∂y} 違 阪 絲障 leading monomial (initial
monomial) .

[294] sm1.gb([[dx^2+dy^2-4,dx*dy-1],[x,y],[[dx,50,dy,2,x,1]]]);
[[dx+dy^3-4*dy,-dy^4+4*dy^2-1],[dx,-dy^4]]
箏      や      m = xayb∂xc∂yd m' = xa'yb'∂xc'∂yd' weight vector (dx,dy,x,y) =
(50,2,1,0)      莠 (や障 m 50c+2d+a > 50c'+2d'+a' m' 素 ) 黠 <      莠 莢や
reverse lexicographic order      莠 (や障 50c+2d+a = 50c'+2d'+a'      reverse lexicographic
order      莠 ).

[294] F=sm1.gb([[dx^2+dy^2-4,dx*dy-1],[x,y],[[dx,50,dy,2,x,1]]|sorted=1);
map(print,F[2][0])$
map(print,F[2][1])$

[595]
sm1.gb(["dx*(x*dx +y*dy-2)-1","dy*(x*dx + y*dy -2)-1"],
[x,y],[[dx,1,x,-1],[dy,1]]);

[[x*dx^2+(y*dy-h^2)*dx-h^3,x*dy*dx+y*dy^2-h^2*dy-h^3,h^3*dx-h^3*dy],
[x*dx^2+(y*dy-h^2)*dx,x*dy*dx+y*dy^2-h^2*dy-h^3,h^3*dx]]

[596]
sm1.gb_d(["dx (x dx +y dy-2)-1","dy (x dx + y dy -2)-1"],
"x,y",[[dx,1,x,-1],[dy,1]]);
[[[e0,x,y,H,E,dx,dy,h],
[[0,-1,0,0,0,1,0,0],[0,0,0,0,0,0,1,0],[1,0,0,0,0,0,0,0],
[0,1,1,1,1,1,1,0],[0,0,0,0,0,0,-1,0],[0,0,0,0,0,-1,0,0],
[0,0,0,0,-1,0,0,0],[0,0,0,-1,0,0,0,0],[0,0,-1,0,0,0,0,0],
[0,0,0,0,0,0,0,1]]],
[[ (1)*<<0,0,1,0,0,1,1,0>>+(1)*<<0,1,0,0,0,2,0,0>>+(-1)*<<0,0,0,0,1,0,2>>+(-1)*
<<0,0,0,0,0,0,0,3>>,(1)*<<0,0,1,0,0,0,2,0>>+(1)*<<0,1,0,0,0,1,1,0>>+(-1)*<<0,0,0,
0,0,0,1,2>>+(-1)*<<0,0,0,0,0,0,0,3>>,(1)*<<0,0,0,0,0,1,0,3>>+(-1)*<<0,0,0,0,0,
1,3>>],
[(1)*<<0,0,1,0,0,1,1,0>>+(1)*<<0,1,0,0,0,2,0,0>>+(-1)*<<0,0,0,0,1,0,2>>,(1)*<
<0,0,1,0,0,0,2,0>>+(1)*<<0,1,0,0,0,1,1,0>>+(-1)*<<0,0,0,0,0,1,2>>+(-1)*<<0,0,0,
0,0,0,3>>,(1)*<<0,0,0,0,0,1,0,3>>]]]

sm1.auto_reduce, sm1.reduction, sm1.rat_to_p

```

### 1.2.5 sm1.deRham

```

sm1.deRham([f,v]|proc=p)
:: 腥咲 C^n - (the zero set of f=0)      若淵      悟召      荐膊      泣若 若.

return      鴻

p

f      紕 障 紊 縷

```

```

v      鴻
• 醇違  $X = C^n \setminus V(f)$  若鴻 悟召 荐膊. ,  $[\dim H^0(X, C), \dim H^1(X, C),$ 
 $\dim H^2(X, C), \dots, \dim H^n(X, C)]$  祉.
• v 違 鴻.  $n = \text{length}(v)$  .
• sm1.deRham 膊罌 羣紊 戎 . sm1.deRham(0, [x*y*z*(x+y+z-1)*(x-
y), [x,y,z]]) 膊 紹吾 え紊 .
• b- 違 鴻合茹 f , ox_asir ox_sm1_forAsir 餞睡 鴻 . 鴻鴻
sm1(0, "[(parse) (oxasir.sm1) pushfile] extension"); , ox_asir 篆< 吾ヤ
若 若 . 鴻鴻 ox_asir_forAsir 鴻帥若 茵 .
• sm1.deRham ox_reset(sm1.get_Sm1_proc()); 賢 , 篁コ sm1 泣若岡羣 若 ヤ篁
餞翫 , 鴻鴻 ox_shutdown(sm1.get_Sm1_proc()); , ox_sm1_forAsir 箏 shutdown
鴻帥若鴻紘 .
[332] sm1.deRham([x^3-y^2, [x,y]]);
[1,1,0]
[333] sm1.deRham([x*(x-1), [x]]);
[1,2]

sm1.start, deRham (sm1 command)

```

Algorithm:

Oaku, Takayama, An algorithm for de Rham cohomology groups of the complement of an affine variety via D-module computation, Journal of pure and applied algebra 139 (1999), 201–233.

### 1.2.6 sm1.hilbert

```

sm1.hilbert([f,v]|proc=p)
:: 紊縋 f 紊縋荐膊.

hilbert_polynomial(f,v)
:: 紊縋 f 紊縋荐膊.

return 紊縋

p
f, v      鴻
• 紊縋 f v 紊縋 h(k) 荐膊.
•  $h(k) = \dim_{\mathbb{Q}} F_k/I \cap F_k$  違 k 篁ヤ 紊縋 . I 縋 f や .
• sm1.hilbert 若: 合荐膊 f 鴻. 紘, 醇違 障 f 違 阪荐膊, initial
monomial 紊縋荐膊. c, コ f 違 阪 醇違 箏綺 違 阪 膊 . ,
sm1 紊縋違 阪荐膊 asir .

[346] load("katsura")$
[351] A=hilbert_polynomial(katsura(5), [u0,u1,u2,u3,u4,u5]);
32

[279] load("katsura")$

```

```

[280] A=gr(katsura(5),[u0,u1,u2,u3,u4,u5],0)$
[281] dp_ord();
0
[282] B=map(dp_ht,map(dp_ptod,A,[u0,u1,u2,u3,u4,u5]));
[(1)*<<1,0,0,0,0,0>>,(1)*<<0,0,0,2,0,0>>,(1)*<<0,0,1,1,0,0>>,(1)*<<0,0,2,0,0,0>>,
(1)*<<0,1,1,0,0,0>>,(1)*<<0,2,0,0,0,0>>,(1)*<<0,0,0,1,1,1>>,(1)*<<0,0,0,1,2,0>>,
(1)*<<0,0,1,0,2,0>>,(1)*<<0,1,0,0,2,0>>,(1)*<<0,1,0,1,1,0>>,(1)*<<0,0,0,0,2,2>>,
(1)*<<0,0,1,0,1,2>>,(1)*<<0,1,0,0,1,2>>,(1)*<<0,1,0,1,0,2>>,(1)*<<0,0,0,0,3,1>>,
(1)*<<0,0,0,0,4,0>>,(1)*<<0,0,0,0,1,4>>,(1)*<<0,0,0,1,0,4>>,(1)*<<0,0,1,0,0,4>>,
(1)*<<0,1,0,0,0,4>>,(1)*<<0,0,0,0,0,6>>]
[283] C=map(dp_dtop,B,[u0,u1,u2,u3,u4,u5]);
[u0,u3^2,u3*u2,u2^2,u2*u1,u1^2,u5*u4*u3,u4^2*u3,u4^2*u2,u4^2*u1,u4*u3*u1,
u5^2*u4^2,u5^2*u4*u2,u5^2*u4*u1,u5^2*u3*u1,u5*u4^3,u4^4,u5^4*u4,u5^4*u3,
u5^4*u2,u5^4*u1,u5^6]
[284] sm1.hilbert([C,[u0,u1,u2,u3,u4,u5]]);
32

sm1.start, sm1.gb, longname

```

### 1.2.7 sm1.genericAnn

```

sm1.genericAnn([f,v]|proc=p)
:: f^s 帥緇 合緇 . v 違 鴻 . , s v[0] , f rest(v) 箠 緇 .
return 鴻
p
f 紊緇
v 鴻
• 醇違, f^s 帥緇 合緇 . v 違 鴻 . , s v[0] , f rest(v) 箠 緇 .
[595] sm1.genericAnn([x^3+y^3+z^3,[s,x,y,z]]);
[-x*dx-y*dy-z*dz+3*s,z^2*dy-y^2*dz,z^2*dx-x^2*dz,y^2*dx-x^2*dy]
sm1.start

```

### 1.2.8 sm1.wTensor0

```

sm1.wTensor0([f,g,v,w]|proc=p)
:: f g D-module 0 電<渦純 荐膊.
return 鴻
p
f, g, v, w 鴻
• f g D-臂や 0 電<渦純 荐膊.
• v 違 鴻 . w weight 鴻 . 贗 w[i] v[i] weight .
• sm1.wTensor0 ox_sm1 wRestriction0 . wRestriction0, generic weight w
狗荐膊 . Weight w generic 膊 若 罩 .
• F G f g . 頑 , 0 電< 渦純 FG 帥緇 合緇膾祉 .

```

- $\cup f, g \in D$  窪や  $c$  , 箏 , 阪 怨臂  $D^r$  臂や .  
 [258] `sm1.wTensor0([x*dx -1, y*dy -4], [dx+dy, dx-dy^2], [x,y], [1,2]);`  
`[-y*x*dx-y*x*dy+4*x+y], [5*x*dx^2+5*x*dx+2*y*dy^2+(-2*y-6)*dy+3],`  
`[-25*x*dx+(-5*y*x-2*y^2)*dy^2+((5*y+15)*x+2*y^2+16*y)*dy-20*x-8*y-15],`  
`[y^2*dy^2+(-y^2-8*y)*dy+4*y+20]]`

### 1.2.9 sm1.reduction

`sm1.reduction([f,g,v,w]|proc=p)`

::

`return` 鴻

$f$  紊縷

$g, v, w$  鴻

$p$  (ox-sm1 祉合 )

- 醇違  $f$  homogenized や 撮違 , 紊縷  $g$  亜 (reduce); や障, 醇違,  $f$  牙 眼 牙 . 紊育  $v$  紘.  $w$  綺紘 や ,  $\forall$  . `sm1.reduction_noH` , Weyl algebra .
- 祉や 就 :  $[r, c0, [c1, \dots, cm], g]$   $g = [g1, \dots, gm]$  ,  $c0 f + c1 g1 + \dots + cm gm = r$  .  
 $r/c0$  normal form .
- 醇違, 箏電 reducible 臂 .
- 醇 `sm1.reduction_d(P,F,G)` `sm1.reduction_noH_d(P,F,G)` ,  $e$  縷 .

[259] `sm1.reduction([x^2+y^2-4, [y^4-4*y^2+1, x+y^3-4*y], [x,y]]);`  
`[x^2+y^2-4, 1, [0,0], [y^4-4*y^2+1, x+y^3-4*y]]`

[260] `sm1.reduction([x^2+y^2-4, [y^4-4*y^2+1, x+y^3-4*y], [x,y], [[x,1]]);`  
`[0,1, [-y^2+4, -x+y^3-4*y], [y^4-4*y^2+1, x+y^3-4*y]]`

`sm1.start, d_true_nf`

### 1.2.10 sm1.xml\_tree\_to\_prefix\_string

`sm1.xml_tree_to_prefix_string(s|proc=p)`

:: XML 吾 OpenMath ;  $s$  臀 羈 .

`return` String

$p$  Number

$s$  String

- XML 吾 OpenMath ;  $s$  臀 羈 .
- 醇違 `om_*`  $\exists$  鴻 .
- `om_xml_to_cmo(OpenMath Tree Expression)` CMO\_TREE 祉. `asir` CMO 障泣 若 .
- `java` 茵医綽荀. ( , /usr/local/jdk1.1.8/bin 瀉瀉泣若鴻  $\forall$  .)  
 [263] `load("om");`  
 1  
 [270] `F=om_xml(x^4-1);`  
`control: wait OX`



```

Trying to connect to the server... Done.
<OMOBJ><OMA><OMS name="plus" cd="basic"/><OMA>
<OMS name="times" cd="basic"/><OMA>
<OMS name="power" cd="basic"/><OMV name="x"/><OMI>4</OMI></OMA>
<OMI>1</OMI></OMA><OMA><OMS name="times" cd="basic"/><OMA>
<OMS name="power" cd="basic"/><OMV name="x"/><OMI>0</OMI></OMA>
<OMI>-1</OMI></OMA></OMA></OMOBJ>
[271] sm1.xml_tree_to_prefix_string(F);
basic_plus(basic_times(basic_power(x,4),1),basic_times(basic_power(x,0),-1))

om_*, OpenXM/src/OpenMath, eval_str

```

### 1.2.11 sm1.syz

```

sm1.syz([f,v,w]|proc=p)
:: v 箏 小箏 違 f syzygy 荐膊.

return      鴻

p
f, v, w      鴻
• 祉や      就 : [s,[g, m, t]]. s f v 紊違 緇 箏 違 syzygy . g f weight vector
w      違 阪 . m      ヌ茵 f 違 阪 g 後茵 . t      違 阪 g syzygy . 障 , 電< 縷 :
g = m f , s f = 0.
• Weight      w      や . ヤ翫, graded reverse lexicographic order や c      阪荐膊.
• Term order      綺箏翫, 電      や 撮違 違 阪荐膊 (SST      Section 1.2 荀). 電 紊 h 脰
.
[293] sm1.syz([[x*dx+y*dy-1,x*y*dx*dy-2],[x,y]]);
[[[y*x*dy*dx-2,-x*dx-y*dy+1]],      generators of the syzygy
[[[x*dx+y*dy-1],[y^2*dy^2+2]],      grobner basis
[[1,0],[y*dy,-1]],      transformation matrix
[[y*x*dy*dx-2,-x*dx-y*dy+1]]]]
[294] sm1.syz([[x^2*dx^2+x*dx+y^2*dy^2+y*dy-4,x*y*dx*dy-1],[x,y],[[dx,-1,x,1]]]);
[[[y*x*dy*dx-1,-x^2*dx^2-x*dx-y^2*dy^2-y*dy+4]], generators of the syzygy
[[[x^2*dx^2+h^2*x*dx+y^2*dy^2+h^2*y*dy-4*h^4],[y*x*dy*dx-h^4], GB
[h^4*x*dx+y^3*dy^3+3*h^2*y^2*dy^2-3*h^4*y*dy]],
[[1,0],[0,1],[y*dy,-x*dx]],      transformation matrix
[[y*x*dy*dx-h^4,-x^2*dx^2-h^2*x*dx-y^2*dy^2-h^2*y*dy+4*h^4]]]]

```

### 1.2.12 sm1.mul

```

sm1.mul(f,g,v|proc=p)
:: sm1 泣若 f g v 箏 小箏 違 c      若.

return      紊縷障      鴻

p
f, g      紊縷障      鴻
v      鴻

```

- `sm1` 泣若  $f, g, v$  箏 小箏 違  $c$  若.
- `sm1.mul_h` homogenized Weyl 篋  $f$  亥.
- BUG: `sm1.mul(p0*dp0,1,[p0])`  $dp0*p0+1$  祉.  $d$  紊違縊 育綺  $c$  , 違 膈祉 .

```
[277] sm1.mul(dx,x,[x]);
x*dx+1
[278] sm1.mul([x,y],[1,2],[x,y]);
x+2*y
[279] sm1.mul([[1,2],[3,4]],[[x,y],[1,2]],[x,y]);
[[x+2,y+4],[3*x+4,3*y+8]]
```

### 1.2.13 `sm1.distraction`

```
sm1.distraction([f,v,x,d,s]|proc=p)
:: sm1 f distraction 荐膊 .
```

`return` 鴻

$p$

$f$  紊縊

$v, x, d, s$  鴻

- 荔  $\sqcup p$  `sm1` 泣若  $f$  distraction  $v$  箏 小箏 違 膊 .
- $x, d, ,$  distract 鴻  $x$  紊,  $d$  紊違 鴻. Distraction,  $s$  紊違 茵 .
- Distraction  $x^*dx$   $x$  舟 . 荅淵 Saito, Sturmfels, Takayama : Grobner Deformations of Hypergeometric Differential Equations page 68 荀.

```
[280] sm1.distraction([x*dx,[x],[x],[dx],[x]]);
x
[281] sm1.distraction([dx^2,[x],[x],[dx],[x]]);
x^2-x
[282] sm1.distraction([x^2,[x],[x],[dx],[x]]);
x^2+3*x+2
[283] fctr(0);
[[1,1],[x+1,1],[x+2,1]]
[284] sm1.distraction([x*dx*y+x^2*dx^2*dy,[x,y],[x],[dx],[x]]);
(x^2-x)*dy+x*y
distraction2(sm1),
```

### 1.2.14 `sm1.gkz`

```
sm1.gkz([A,B]|proc=p)
:: 茵 A <若 B GKZ 膾 (A-hypergeometric system) .
```

`return` 鴻

$p$

$A, B$  鴻

- 茵  $A$  <若  $B$  GKZ 膾 (A-hypergeometric system) .

```
[280] sm1.gkz([ [[1,1,1,1],[0,1,3,4]], [0,2] ]);
[[x4*dx4+x3*dx3+x2*dx2+x1*dx1,4*x4*dx4+3*x3*dx3+x2*dx2-2,
-dx1*dx4+dx2*dx3,-dx2^2*dx4+dx1*dx3^2,dx1^2*dx3-dx2^3,-dx2*dx4^2+dx3^3],
[x1,x2,x3,x4]]
```

### 1.2.15 sm1.mgkz

```
sm1.mgkz([A,W,B]|proc=p)
:: 茵 A, weight W      <若 B      modified GKZ 膾 (A-hypergeometric system)
```

```
return      鴻
```

*p*

*A, W, B* 鴻

- 茵 *A*, weight vector *W* <若 *B* modified GKZ 膾 (A-hypergeometric system) .
- <http://arxiv.org/abs/0707.0043>

```
[280] sm1.mgkz([ [[1,2,3]], [1,2,1], [a/2]]);
[[6*x3*dx3+4*x2*dx2+2*x1*dx1-a,-x4*dx4+x3*dx3+2*x2*dx2+x1*dx1,
-dx2+dx1^2,-x4^2*dx3+dx1*dx2],[x1,x2,x3,x4]]
```

Modified A-hypergeometric system for  
 $A=(1,2,3)$ ,  $w=(1,2,1)$ ,  $\beta=(a/2)$ .

### 1.2.16 sm1.appell1

```
sm1.appell1(a|proc=p)
:: F_1 障 F_D      綽合綽膾祉祉.
```

```
return      鴻
```

*p*

*a* 鴻

- Appell F\_1 n 紊医 Lauricella F\_D( $a, b_1, b_2, \dots, b_n, c; x_1, \dots, x_n$ ) 帥緇 合綽膾祉祉. ,  $a=(a, c, b_1, \dots, b_n)$ . <若帥 違 .
- sm1 appell1 吟 , <若帥違綽綽 翫 罩  $c$  .

```
[281] sm1.appell1([1,2,3,4]);
[[((-x1+1)*x2*dx1-3*x2)*dx2+(-x1^2+x1)*dx1^2+(-5*x1+2)*dx1-3,
(-x2^2+x2)*dx2^2+((-x1*x2+x1)*dx1-6*x2+2)*dx2-4*x1*dx1-4,
((-x2+x1)*dx1+3)*dx2-4*dx1], equations
[x1,x2]] the list of variables
```

```
[282] sm1.gb(@);
[[((-x2+x1)*dx1+3)*dx2-4*dx1,((-x1+1)*x2*dx1-3*x2)*dx2+(-x1^2+x1)*dx1^2
```

```

+(-5*x1+2)*dx1-3,(-x2^2+x2)*dx2^2+((-x2^2+x1)*dx1-3*x2+2)*dx2
+(-4*x2-4*x1)*dx1-4,
(x2^3+(-x1-1)*x2^2+x1*x2)*dx2^2+((-x1*x2+x1^2)*dx1+6*x2^2
+(-3*x1-2)*x2+2*x1)*dx2-4*x1^2*dx1+4*x2-4*x1],
[x1*dx1*dx2,-x1^2*dx1^2,-x2^2*dx1*dx2,-x1*x2^2*dx2^2]]

[283] sm1.rank(sm1.appell1([1/2,3,5,-1/3]));
3

[285] Mu=2$ Beta = 1/3$
[287] sm1.rank(sm1.appell1([Mu+Beta,Mu+1,Beta,Beta,Beta]));
4

```

### 1.2.17 sm1.appell4

```

sm1.appell4(a|proc=p)
:: F_4 障 F_C 綽合綽膾祉祉.

return 鴻

P

a 鴻
• Appell F_4 n 紊医 Lauricella F_C(a,b,c1,c2,...,cn;x1,...,xn) 帥緇 合綽膾
祉祉. , a =(a,b,c1,...,cn). <若帥 違 .
• sm1 appell1 吟 , <若帥違綽緇 翫 罩 c .

[281] sm1.appell4([1,2,3,4]);
[[-x2^2*dx2^2+(-2*x1*x2*dx1-4*x2)*dx2+(-x1^2+x1)*dx1^2+(-4*x1+3)*dx1-2,
(-x2^2+x2)*dx2^2+(-2*x1*x2*dx1-4*x2+4)*dx2-x1^2*dx1^2-4*x1*dx1-2],
equations
[x1,x2]] the list of variables

[282] sm1.rank(@);
4

```

### 1.2.18 sm1.rank

```

sm1.rank(a|proc=p)
:: 緇 合綽膾 a holonomic rank 祉.

return

P

a 鴻
• 緇 合綽膾 a , generic point 茹 c 祉. , holonomic rank 若.

```

- $a$  小箴 鴻 違 鴻 .
- $a$  regular holonomic  $sm1.rrank$  holonomic rank 祉.  $c$  宴 違 鴻  $sm1.rank$  .

```
[284] sm1.gkz([ [[1,1,1,1],[0,1,3,4]], [0,2] ]);
[[x4*dx4+x3*dx3+x2*dx2+x1*dx1,4*x4*dx4+3*x3*dx3+x2*dx2-2,
  -dx1*dx4+dx2*dx3, -dx2^2*dx4+dx1*dx3^2,dx1^2*dx3-dx2^3,-dx2*dx4^2+dx3^3],
 [x1,x2,x3,x4]]
[285] sm1.rrank(@);
4

[286] sm1.gkz([ [[1,1,1,1],[0,1,3,4]], [1,2]]);
[[x4*dx4+x3*dx3+x2*dx2+x1*dx1-1,4*x4*dx4+3*x3*dx3+x2*dx2-2,
  -dx1*dx4+dx2*dx3,-dx2^2*dx4+dx1*dx3^2,dx1^2*dx3-dx2^3,-dx2*dx4^2+dx3^3],
 [x1,x2,x3,x4]]
[287] sm1.rrank(@);
5
```

### 1.2.19 sm1.auto\_reduce

$sm1.auto\_reduce(s|proc=p)$   
 :: "AutoReduce"  $s$  紘.

祉

$p$

$s$

- $s = 1$  , 箴 $\cup$ 荐膊違 阪 鴻, reduced 違 阪 .
- $s = 0$  , 荐膊違 阪 reduced 違 阪 . < 若 .

### 1.2.20 sm1.slope

$sm1.slope(ii,v,f\_filtration,v\_filtration|proc=p)$   
 :: 緇 合綬膾  $ii$  slope 祉.

return

$p$

$ii$  鴻 (合綬)

$v$  鴻 (綯)

$f\_filtration$  鴻 (weight vector)

$v\_filtration$

鴻 (weight vector)

- $sm1.slope$  緇 合綬膾  $ii$   $V$  filtration  $v\_filtration$  紘蒨認渴 何  $c$  (geometric) slope 荐膊.
- $v$  違 鴻.

- 祉や, 鴻 鴻 . 鴻 1 荀脰 slope, 脰 2 荀脰, weight vector 綽 microcharacteristic variety bihomogeneous .

Algorithm: "A.Assi, F.J.Castro-Jimenez and J.M.Granger, How to calculate the slopes of a D-module, Compositio Math, 104, 1-17, 1996" 帥. Slope  $s'$  ヤ 臂 , 脰 激莢 , 違 , Slope 偽絲上  $-s'$  祉. や障  $pF+qV$  micro 号 罕箆 gap ,  $-s'=q/p$  祉. 菴  $s=1-1/s'$  slope 若 . 茹  $c O(s)$  .  $s-1 \leq s$  羣.  $r=s-1=-1/s'$   $\kappa=1/r=-s'$  . 違 Borel and Laplace 紊  $1/\Gamma(1+m*r)$  factor,  $\exp(-\tau^\kappa)$  戎.

```
[284] A= sm1.gkz([ [[1,2,3]], [-3] ]);
```

```
[285] sm1.slope(A[0],A[1],[0,0,0,1,1,1],[0,0,-1,0,0,1]);
```

```
[286] A2 = sm1.gkz([ [[1,1,1,0],[2,-3,1,-3]], [1,0]]);
(* This is an interesting example given by Laura Matusevich,
   June 9, 2001 *)
```

```
[287] sm1.slope(A2[0],A2[1],[0,0,0,0,1,1,1,1],[0,0,0,-1,0,0,0,1]);
```

```
sm.gb
```

### 1.2.21 sm1.ahg

```
sm1.ahg(A)
```

: It is identical with `sm1.gkz(A)`.

### 1.2.22 sm1.bfunction

```
sm1.bfunction(F)
```

: It computes the global b-function of  $F$ .

Description:

It no longer calls `sm1`'s original `bfunction`. Instead, it calls `asir "bfct"`.

Algorithm:

M.Noro, Mathematical Software, icms 2002, pp.147–157.

Example:

```
sm1.bfunction(x^2-y^3);
```

### 1.2.23 sm1.call\_sm1

```
sm1.call_sm1(F)
```

: It executes  $F$  on the `sm1` server. See also `sm1`.

### 1.2.24 sm1.ecart\_homogenize01Ideal

```
sm1.ecart_homogenize01Ideal(A)
```

: It  $(0,1)$ -homogenizes the ideal  $A[0]$ . Note that it is not an elementwise homogenization.

Example:

```
input1
F=[(1-x)*dx+1]$ FF=[F,"x,y"]$
sm1.ecart_homogenize01Ideal(FF);
input2
F=sm1.appell1([1,2,3,4]);
sm1.ecart_homogenize01Ideal(F);
```

### 1.2.25 sm1.ecartd\_gb

sm1.ecartd\_gb(*A*)

: It returns a standard basis of *A* by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is used. If the option *rv*="dp" (return\_value="dp") is given, the answer is returned in distributed polynomials.

Note. Functions in the category *ecart* changes the global environment in the sm1 server. If you interrupted these functions, run sm1.ecartd\_gb with a small input and terminate it normally. Then, the global environment is reset to the normal state. Reference. G. Granger, T. Oaku, N. Takayama, Tangent cone algorithm for homogeized differential operators, 2005.

Example:

```
input1
F=[2*(1-x-y)*dx+1,2*(1-x-y)*dy+1]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
sm1.ecartd_gb(FF);
output1
[[(-2*x-2*y+2)*dx+h,(-2*x-2*y+2)*dy+h],[(-2*x-2*y+2)*dx,(-2*x-2*y+2)*dy]]
input2
F=[2*(1-x-y)*dx+h,2*(1-x-y)*dy+h]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]],["noAutoHomogenize",1]]$
sm1.ecartd_gb(FF);
input3
F=[[x^2,y+x],[x+y,y^3],[2*x^2+x*y,y+x+x*y^3]]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]],["degreeShift",[[0,1],[-3,1]]]]$
sm1.ecartd_gb(FF);
```

### 1.2.26 sm1.ecartd\_gb\_oxRingStructure

sm1.ecartd\_gb\_oxRingStructure()

: It returns the oxRingStructure of the most recent ecartd\_gb computation.

### 1.2.27 sm1.ecartd\_isSameIdeal\_h

sm1.ecartd\_isSameIdeal\_h(*F*)

: Here,  $F=[II, JJ, V]$ . It compares two ideals *II* and *JJ* in  $h[0,1](D)$ -alg.

Example:

```
input
II=[(1-x)^2*dx+h*(1-x)]$ JJ = [(1-x)*dx+h]$
V=[x]$
sm1.ecartd_isSameIdeal_h([II,JJ,V]);
```

### 1.2.28 sm1.ecartd\_reduction

`sm1.ecartd_reduction(F,A)`

: It returns a reduced form of  $F$  in terms of  $A$  by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is used. When the output is  $G$ ,  $G[3]$  is  $F$  and  $G[0]-(G[1]*A-\sum(k,G[2][k]*G[3][k]))=0$  holds.  $F$  must be  $(0,1)$ -homogenized (see `sm1.ecartd_homogenize01Ideal`). This function does not check if the given order is admissible for the ecart reduction. To do this check, use `sm1.ecartd_gb`.

Example:

```
input
F=[2*(1-x-y)*dx+h,2*(1-x-y)*dy+h]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
G=sm1.ecartd_reduction(dx+dy,FF);
G[0]-(G[1]*(dx+dy)+G[2][0]*F[0]+G[2][1]*F[1]);
```

### 1.2.29 sm1.ecartd\_reduction\_noh

`sm1.ecartd_reduction_noh(F,A)`

: It returns a reduced form of  $F$  in terms of  $A$  by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is NOT used.  $A[0]$  must not contain the variable  $h$ .

Example:

```
F=[2*(1-x-y)*dx+1,2*(1-x-y)*dy+1]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
sm1.ecartd_reduction_noh(dx+dy,FF);
```

### 1.2.30 sm1.ecartd\_syz

`sm1.ecartd_syz(A)`

: It returns a syzygy of  $A$  by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is used. If the option `rv="dp"` (`return_value="dp"`) is given, the answer is returned in distributed polynomials. The return value is in the format `[s,[g,m,t]]`.  $s$  is the generator of the syzygies,  $g$  is the Grobner basis,  $m$  is the translation matrix from the generators to  $g$ .  $t$  is the syzygy of  $g$ .

Example:

```
input1
F=[2*(1-x-y)*dx+1,2*(1-x-y)*dy+1]$
```



```

FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
sm1.ecartd_syz(FF);
input2
F=[2*(1-x-y)*dx+h,2*(1-x-y)*dy+h]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]],["noAutoHomogenize",1]]$
sm1.ecartd_syz(FF);

```

### 1.2.31 sm1.gb\_oxRingStructure

`sm1.gb_oxRingStructure()`

: It returns the `oxRingStructure` of the most recent gb computation.

### 1.2.32 sm1.gb\_reduction

`sm1.gb_reduction(F,A)`

: It returns a reduced form of  $F$  in terms of  $A$  by using a normal form algorithm.  
 $h[1,1](D)$ -homogenization is used.

Example:

```

input
F=[2*(h-x-y)*dx+h^2,2*(h-x-y)*dy+h^2]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]]]$
sm1.gb_reduction((h-x-y)^2*dx*dy,FF);

```

### 1.2.33 sm1.gb\_reduction\_noh

`sm1.gb_reduction_noh(F,A)`

: It returns a reduced form of  $F$  in terms of  $A$  by using a normal form algorithm.

Example:

```

input
F=[2*dx+1,2*dy+1]$
FF=[F,"x,y",[[dx,1,dy,1]]]$
sm1.gb_reduction_noh((1-x-y)^2*dx*dy,FF);

```

### 1.2.34 sm1.generalized\_bfunction

`sm1.generalized_bfunction(I,V,VD,W)`

: It computes the generalized b-function (indicial equation) of  $I$  with respect to the weight  $W$ .

Description:

It no longer calls sm1's original function. Instead, it calls asir "generic\_bfct".

Example:

```

sm1.generalized_bfunction([x^2*dx^2-1/2,dy^2],[x,y],[dx,dy],[-1,0,1,0]);

```

### 1.2.35 sm1.integration

`sm1.integration(I,V,R)`

: It computes the integration of  $I$  as a  $D$ -module to the set defined by  $R$ .  $V$  is the list of variables. When the optional variable `degree=d` is given, only the integrations from 0 to  $d$  are computed. Note that, in case of vector input, INTEGRATION VARIABLES MUST APPEAR FIRST in the list of variable  $V$ . We are using `wbfRoots` to get the roots of  $b$ -functions, so we can use only generic weight vector for now.

`sm1.integration(I,V,R | degree=key0)`

: This function allows optional variables `degree`

Algorithm:

T.Oaku and N.Takayama, [math.AG/9805006](http://math.AG/9805006), <http://www.arxiv.org>

Example:

```
sm1.integration([dt - (3*t^2-x), dx + t],[t,x],[t]);
```

The output `[[n0,F0],[n1,F1],...]` means that  $H^0=D^n0/F0$ ,  $H^{-1}=D^n1/F1$ , ...

The free basis of the vector space  $D^n$  is denoted by  $e_0, e_1, \dots$

### 1.2.36 sm1.isSameIdeal\_in\_Dalg

`sm1.isSameIdeal_in_Dalg(I,J,V)`

: It compares two ideals  $I$  and  $J$  in  $D_{alg}$  (algebraic  $D$  with variables  $V$ , no homogenization).

Example:

Input1

```
II=[(1-x)^2*dx+(1-x)]$ JJ = [(1-x)*dx+1]$ V=[x]$
sm1.isSameIdeal_in_Dalg(II,JJ,V);
```

### 1.2.37 sm1.laplace

`sm1.laplace(L,V,VL)`

: It returns the Laplace transformation of  $L$  for  $VL$ .  $V$  is the list of space variables. The numbers in coefficients must not be rational with a non-1 denominator. cf. `ptozp`

Example:

```
L1=sm1.laplace(dt-(3*t^2-x),[x,t],[t,dt]);
L2=sm1.laplace(dx+t,[x,t],[t,dt]);
sm1.restriction([L1,L2],[t,x],[t] | degree=0);
```

### 1.2.38 sm1.rat\_to\_p

`sm1.rat_to_p(F)`

: It returns the denominator of  $F$  and the numerator of  $F$ . They are returned in a list. All elements of the denominator and numerator are polynomial objects

with integer coefficients. Note that `dn` and `nm` do not regard rational numbers as a fractional object and this function is necessary to send data to `sm1`, which accept only integers and does not accept rational numbers.

Example:

```
sm1.rat_to_p(1/2*x+1);
    [x+2,2]
sm1.rat_to_p([1/2*x,1/3*x]);
    [[3*x,2*x],6]
```

### 1.2.39 `sm1.restriction`

`sm1.restriction(I,V,R)`

: It computes the restriction of  $I$  as a D-module to the set defined by  $R$ .  $V$  is the list of variables. When the optional variable `degree=d` is given, only the restrictions from 0 to  $d$  are computed. Note that, in case of vector input, RESTRICTION VARIABLES MUST APPEAR FIRST in the list of variable  $V$ . We are using `wbfRoots` to get the roots of b-functions, so we can use only generic weight vector for now.

`sm1.restriction(I,V,R | degree=key0)`

: This function allows optional variables `degree`

Algorithm:

T.Oaku and N.Takayama, [math.AG/9805006](http://math.AG/9805006), <http://xxx.lanl.gov>

Example:

```
sm1.restriction([dx^2-x,dy^2-1],[x,y],[y]);
The output [[n0,F0],[n1,F1],...] means that  $H^0=D^0/F_0$ ,  $H^{-1}=D^1/F_1$ , ...
The free basis of the vector space  $D^n$  is denoted by  $e_0, e_1, \dots$ 
```

### 1.2.40 `sm1.saturation`

`sm1.saturation(T)`

:  $T = [I, J, V]$ . It returns saturation of  $I$  with respect to  $J^\infty$ .  $V$  is a list of variables.

Example:

```
sm1.saturation([[x2^2,x2*x4, x2, x4^2], [x2,x4], [x2,x4]]);
```

### 1.2.41 `sm1.ahg`

`sm1.ahg(A)`

: It is identical with `sm1.gkz(A)`.

### 1.2.42 `sm1.bfunction`

`sm1.bfunction(F)`

: It computes the global b-function of  $F$ .

Description:

It no longer calls sm1's original bfunction. Instead, it calls asir "bfct".

Algorithm:

M.Noro, Mathematical Software, icms 2002, pp.147–157.

Example:

```
sm1.bfunction(x^2-y^3);
```

### 1.2.43 sm1.call\_sm1

```
sm1.call_sm1(F)
```

: It executes  $F$  on the sm1 server. See also sm1.

### 1.2.44 sm1.ecart\_homogenize01Ideal

```
sm1.ecart_homogenize01Ideal(A)
```

: It  $(0,1)$ -homogenizes the ideal  $A[0]$ . Note that it is not an elementwise homogenization.

Example:

```
input1
F=[(1-x)*dx+1]$ FF=[F,"x,y"]$
sm1.ecart_homogenize01Ideal(FF);
input2
F=sm1.appell1([1,2,3,4]);
sm1.ecart_homogenize01Ideal(F);
```

### 1.2.45 sm1.ecartd\_gb

```
sm1.ecartd_gb(A)
```

: It returns a standard basis of  $A$  by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is used. If the option `rv="dp"` (`return_value="dp"`) is given, the answer is returned in distributed polynomials.

Note. Functions in the category `ecart` changes the global environment in the sm1 server. If you interrupted these functions, run `sm1.ecartd_gb` with a small input and terminate it normally. Then, the global environment is reset to the normal state. Reference. G. Granger, T. Oaku, N. Takayama, Tangent cone algorithm for homogeized differential operators, 2005.

Example:

```
input1
F=[2*(1-x-y)*dx+1,2*(1-x-y)*dy+1]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
sm1.ecartd_gb(FF);
output1
[(-2*x-2*y+2)*dx+h,(-2*x-2*y+2)*dy+h],[(-2*x-2*y+2)*dx,(-2*x-2*y+2)*dy]]
```

```

input2
F=[2*(1-x-y)*dx+h,2*(1-x-y)*dy+h]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]],["noAutoHomogenize",1]]$
sm1.ecartd_gb(FF);
input3
F=[x^2,y+x],[x+y,y^3],[2*x^2+x*y,y+x+x*y^3]]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]],
["degreeShift",[0,1],[-3,1]]]]$
sm1.ecartd_gb(FF);

```

#### 1.2.46 sm1.ecartd\_gb\_oxRingStructure

`sm1.ecartd_gb_oxRingStructure()`

: It returns the `oxRingStructure` of the most recent `ecartd_gb` computation.

#### 1.2.47 sm1.ecartd\_isSameIdeal\_h

`sm1.ecartd_isSameIdeal_h(F)`

: Here,  $F=[II, JJ, V]$ . It compares two ideals  $II$  and  $JJ$  in  $h[0,1](D)$ -alg.

Example:

```

input
II=[(1-x)^2*dx+h*(1-x)]$ JJ = [(1-x)*dx+h]$
V=[x]$
sm1.ecartd_isSameIdeal_h([II, JJ, V]);

```

#### 1.2.48 sm1.ecartd\_reduction

`sm1.ecartd_reduction(F,A)`

: It returns a reduced form of  $F$  in terms of  $A$  by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is used. When the output is  $G$ ,  $G[3]$  is  $F$  and  $G[0]-(G[1]*A-\sum(k,G[2][k]*G[3][k]))=0$  holds.  $F$  must be  $(0,1)$ -homogenized (see `sm1.ecartd_homogenize01Ideal`). This function does not check if the given order is admissible for the `ecart` reduction. To do this check, use `sm1.ecartd_gb`.

Example:

```

input
F=[2*(1-x-y)*dx+h,2*(1-x-y)*dy+h]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
G=sm1.ecartd_reduction(dx+dy,FF);
G[0]-(G[1]*(dx+dy)+G[2][0]*F[0]+G[2][1]*F[1]);

```

#### 1.2.49 sm1.ecartd\_reduction\_noh

`sm1.ecartd_reduction_noh(F,A)`

: It returns a reduced form of  $F$  in terms of  $A$  by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is NOT used.  $A[0]$  must not contain the variable  $h$ .

Example:

```

F=[2*(1-x-y)*dx+1,2*(1-x-y)*dy+1]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
sm1.ecartd_reduction_noh(dx+dy,FF);

```

### 1.2.50 sm1.ecartd\_syz

`sm1.ecartd_syz(A)`

: It returns a syzygy of  $A$  by using a tangent cone algorithm.  $h[0,1](D)$ -homogenization is used. If the option `rv="dp"` (return\_value="dp") is given, the answer is returned in distributed polynomials. The return value is in the format `[s,[g,m,t]]`.  $s$  is the generator of the syzygies,  $g$  is the Grobner basis,  $m$  is the translation matrix from the generators to  $g$ .  $t$  is the syzygy of  $g$ .

Example:

```

input1
F=[2*(1-x-y)*dx+1,2*(1-x-y)*dy+1]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1]]]$
sm1.ecartd_syz(FF);
input2
F=[2*(1-x-y)*dx+h,2*(1-x-y)*dy+h]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]],["noAutoHomogenize",1]]$
sm1.ecartd_syz(FF);

```

### 1.2.51 sm1.gb\_oxRingStructure

`sm1.gb_oxRingStructure()`

: It returns the `oxRingStructure` of the most recent `gb` computation.

### 1.2.52 sm1.gb\_reduction

`sm1.gb_reduction(F,A)`

: It returns a reduced form of  $F$  in terms of  $A$  by using a normal form algorithm.  $h[1,1](D)$ -homogenization is used.

Example:

```

input
F=[2*(h-x-y)*dx+h^2,2*(h-x-y)*dy+h^2]$
FF=[F,"x,y",[[dx,1,dy,1],[x,-1,y,-1,dx,1,dy,1]]]$
sm1.gb_reduction((h-x-y)^2*dx*dy,FF);

```

### 1.2.53 sm1.gb\_reduction\_noh

`sm1.gb_reduction_noh(F,A)`

: It returns a reduced form of  $F$  in terms of  $A$  by using a normal form algorithm.

Example:

```

input
F=[2*dx+1,2*dy+1]$
FF=[F,"x,y",[[dx,1,dy,1]]]$
sm1.gb_reduction_noh((1-x-y)^2*dx*dy,FF);

```

### 1.2.54 sm1.generalized\_bfunction

`sm1.generalized_bfunction(I,V,VD,W)`  
: It computes the generalized b-function (indicial equation) of  $I$  with respect to the weight  $W$ .

Description:

It no longer calls sm1's original function. Instead, it calls asir "generic\_bfct".

Example:

```
sm1.generalized_bfunction([x^2*dx^2-1/2,dy^2],[x,y],[dx,dy],[-1,0,1,0]);
```

### 1.2.55 sm1.integration

`sm1.integration(I,V,R)`  
: It computes the integration of  $I$  as a D-module to the set defined by  $R$ .  $V$  is the list of variables. When the optional variable *degree*= $d$  is given, only the integrations from 0 to  $d$  are computed. Note that, in case of vector input, INTEGRATION VARIABLES MUST APPEAR FIRST in the list of variable  $V$ . We are using wbfRoots to get the roots of b-functions, so we can use only generic weight vector for now.

`sm1.integration(I,V,R | degree=key0)`  
: This function allows optional variables *degree*

Algorithm:

T.Oaku and N.Takayama, math.AG/9805006, <http://www.arxiv.org>

Example:

```
sm1.integration([dt - (3*t^2-x), dx + t],[t,x],[t]);
```

The output  $[[n_0, F_0], [n_1, F_1], \dots]$  means that  $H^0 = D^{n_0}/F_0$ ,  $H^{-1} = D^{n_1}/F_1$ , ...  
The free basis of the vector space  $D^n$  is denoted by  $e_0, e_1, \dots$

### 1.2.56 sm1.isSameIdeal\_in\_Dalg

`sm1.isSameIdeal_in_Dalg(I,J,V)`  
: It compares two ideals  $I$  and  $J$  in  $D_{\text{alg}}$  (algebraic  $D$  with variables  $V$ , no homogenization).

Example:

```

Input1
II=[(1-x)^2*dx+(1-x)]$ JJ = [(1-x)*dx+1]$ V=[x]$
sm1.isSameIdeal_in_Dalg(II,JJ,V);

```

### 1.2.57 sm1.laplace

`sm1.laplace(L,V,VL)`

: It returns the Laplace transformation of  $L$  for  $VL$ .  $V$  is the list of space variables. The numbers in coefficients must not be rational with a non-1 denominator. cf. `ptozp`

Example:

```
L1=sm1.laplace(dt-(3*t^2-x),[x,t],[t,dt]);
L2=sm1.laplace(dx+t,[x,t],[t,dt]);
sm1.restriction([L1,L2],[t,x],[t] | degree=0);
```

### 1.2.58 sm1.rat\_to\_p

`sm1.rat_to_p(F)`

: It returns the denominator of  $F$  and the numerator of  $F$ . They are returned in a list. All elements of the denominator and numerator are polynomial objects with integer coefficients. Note that `dn` and `nm` do not regard rational numbers as a fractional object and this function is necessary to send data to `sm1`, which accept only integers and does not accept rational numbers.

Example:

```
sm1.rat_to_p(1/2*x+1);
[x+2,2]
sm1.rat_to_p([1/2*x,1/3*x]);
[[3*x,2*x],6]
```

### 1.2.59 sm1.restriction

`sm1.restriction(I,V,R)`

: It computes the restriction of  $I$  as a D-module to the set defined by  $R$ .  $V$  is the list of variables. When the optional variable `degree=d` is given, only the restrictions from 0 to  $d$  are computed. Note that, in case of vector input, RESTRICTION VARIABLES MUST APPEAR FIRST in the list of variable  $V$ . We are using `wbfRoots` to get the roots of b-functions, so we can use only generic weight vector for now.

`sm1.restriction(I,V,R | degree=key0)`

: This function allows optional variables `degree`

Algorithm:

T.Oaku and N.Takayama, [math.AG/9805006](http://math.AG/9805006), <http://xxx.lanl.gov>

Example:

```
sm1.restriction([dx^2-x,dy^2-1],[x,y],[y]);
The output [[n0,F0],[n1,F1],...] means that  $H^0=D^n0/F0$ ,  $H^{-1}=D^n1/F1$ , ...
The free basis of the vector space  $D^n$  is denoted by  $e0, e1, \dots$ 
```



**1.2.60** `sm1.saturation`

`sm1.saturation( $T$ )`  
:  $T = [I, J, V]$ . It returns saturation of  $I$  with respect to  $J^{\infty}$ .  $V$  is a list of variables.

Example:

```
sm1.saturation([[x2^2, x2*x4, x2, x4^2], [x2, x4], [x2, x4]]);
```

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