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Risa/Asir

1.0
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by Shuhei Todo

1

1.1

- 2
-
- neighborhood graph tree
-
- (adjoint curves)
-
-

x, y, z

1.2 Notation

- $[x, y, z] \quad (x:y:z)_{z=0} \quad z=1$
- $Q \quad \overline{Q}$

1.3

1.3.1 intersect

$\text{intersect}(F, G)$
 $\quad \quad \quad \therefore 2F=0, G=0 .$

return

$F \quad G \quad \quad x, y, z$

- $2F=0, G=0 \quad [x, y, z]$
- F, G

```
[1] intersect(y^2-x*z, (x^2+y^2)^3-4*x^2*y^2*z^2);
[[0,0,1],[(#4),(#5),1]]
[2] defpoly(alg(4));
t#4^3+3*t#4^2+3*t#4-3
[3] defpoly(alg(5));
t#5^2-t#4
[4] intersect(x^2-y^2, x^3+y*x^2+(y^2-z^2)*x+y^3-z^2*y);
***two curve have common components***
```

1.3.2 sing

$\text{sing}(F) \quad \quad \therefore F=0 .$

return

F x,y,z

- $F=0$ $[x,y,z]$ $F_x(x,y,z) = F_y(x,y,z) = F_z(x,y,z) = 0$
- F

```
[1] sing(16*x^6-24*z^2*x^4+9*z^4*x^2+4*z^2*y^4-4*z^4*y^2);
[[0,0,1],[(#4),0,1],[1/2,(#3),1],[-1/2,(#3),1],[0,1,0]]
[2] defpoly(alg(3));
2*t#3^2-1
[3] defpoly(alg(4));
4*t#4^2-3
[4] sing((x-y)*(y^2-x*z));
[[1,1,1],[0,0,1]]
[5] sing((x-y)^2*(y^2-x*z));
***Argument has multiple divisor***
```

Section 1.3.3 [nbh], page 2, Section 1.4.4 [multia], page 5,

1.3.3 nbh

$\text{nbh}(F)$ $:: F=0$ neighborhood graph

return

F x,y,z

- $F=0$ neighborhood graph neighborhood graph tree


```
[ , , [, (=1)(=-1)], [("terminal")] ]
[ , [, (=1)(=-1)], [("terminal")] ]
```
- F

```
=1
[1] F=x^6+3*y^2*x^4+(3*y^4-4*z^2*y^2)*x^2+y^6;
x^6+3*y^2*x^4+(3*y^4-4*z^2*y^2)*x^2+y^6
[2] sing(F);
[[0,0,1],[(#0),1,0]]
[3] nbh(F);
[ 1 [0,0,1] [4,-1] [[ 1 [2,1] [terminal] ],[ 1 [2,1] [terminal] ]] ]
[ 2 [(#0),1,0] [2,-1] [[ 1 [1,1] [terminal] ]] ]
[0,0,1] 4 22 [(#0),1,0]
```

Section 1.3.2 [sing], page 1,

1.3.4 genus

$\text{genus}(F)$ $:: F=0$.

return 0

F x,y,z

- $F=0$

- $F \overline{Q}[x, y, z] \mathbb{Q}[x, y, z] \overline{Q}[x, y, z]$

```

[1] genus(x^6+3*y^2*x^4+(3*y^4-4*z^2*y^2)*x^2+y^6);
0
[2] genus(y^2*z-x^3-z^3);
1
[3] genus(x^2+y^2+z^2-x*y-y*z-z*x);
-1
[4] fctr(x^2+y^2+z^2-x*y-y*z-z*x);
[[1,1],[x^2+(-y-z)*x+y^2-z*y+z^2,1]]
[5] irr_conic(x^2+y^2+z^2-x*y-y*z-z*x);
reducible

```

Section 1.4.5 [irr_conic], page 6,

1.3.5 adjoint1, adjoint2

adjoint1(F)

adjoint2(F)

:: $F=0$ n-1, n-2 (adjoint curve) (n=deg(F))

return x,y,z

F x,y,z

- $n-2 \ G=0 \ F=0 \ r-1 \ G=0 \ F=0 \ n-2$ (adjoint curve) $n-1 \ G_0 = 0, G_1 = 0, \dots, G_{n-2} = 0 \ n-2$
 $c_0 G_0 + c_1 G_1 + \dots + c_{n-2} G_{n-2} \ (c_i) \ \text{adjoint2}(F) \ n-1 \ n-1 \ n-1 \ 2n-1 \ n-1 \ \text{adjoint1}(F)$
-
- F

```

[1] adjoint2(x^6+3*y^2*x^4+(3*y^4-4*z^2*y^2)*x^2+y^6);
[c2,c3,c4,c6,c7] 5
(c2-c4)*x^4+c3*y*x^3+(c2*y^2+c6*z*y)*x^2+(c3*y^3+c7*z*y^2)*x+c4*y^4
[2] adjoint1(F);
[c1,c7,c11,c12,c13,c15,c16,c17,c18,c19,c20] 11
(c1*y+(c11-c15+c18-c20)*z)*x^4+(c13*y^2+c7*z*y+c11*z^2)*x^3+(c17*z*y^2+c12*z^2*y
+c15*z^3)*x^2+(c13*z^2*y^2+c16*z^3*y+c18*z^4)*x+c17*z^3*y^2+c19*z^4*y+c20*z^5

```

Section 1.4.7 [restriction], page 6,

1.3.6 intpt

intpt(F) :: $F=0$ [x,y,z] no integer solution

return no integer solution.

F x,y,z

- $F=0$ (affine) [x,y,z] x,y, z no integer solution
- LegendreF

```

[1] intpt(22*x^2-10*y^2+z^2+5*x*y+13*y*x-z*x);
[71,-121,473]
[2] intpt(22*x^2-10*y^2+z^2+5*x*y+12*y*x-z*x);
no integer solution

```

1.3.7 parametrize

```

parametrize(F)
    :: F=0

return
F      x,y,z
• F=00t      P(t),Q(t),R(t)      x,y,zS(x,y,z),T(x,y,z)(x:y:z)=(P(t):Q(t):R(t)),
  t=T(x,y,z)/S(x,y,z)      parametrize(F)      [P(t),Q(t),R(t),T(x,y,z)/S(x,y,z)]
  GCD(P(t),Q(t),R(t))=1 P(t),Q(t),R(t)
• F Q[x,y,z] 0
  [1] parametrize(x^4+(2*y^2-z^2)*x^2+y^4+z^2*y^2);
  [-t^3-t,t^3-t,t^4+1,(-x^2-y^2)/(z*x+z*y)]
  [2] parametrize((x^2+y^2)^3-4*x^2*y^2*z^2);
  heuristic2 failed...
  heuristic3 succeed
  [32256*t^6-133120*t^5-129024*t^4+1064960*t^3-516096*t^2
  -2129920*t+2064384,-127008*t^6+1048320*t^5-2671232*t^4
  +10684928*t^2-16773120*t+8128512,274625*t^6-3194100*t^5
  +15678780*t^4-41555808*t^3+62715120*t^2-51105600*t+17576000,
  (-126*x^4+1040*y*x^3-382*y^2*x^2+1040*y^3*x-256*y^4)
  /(-65*x^4+520*y*x^3+(-65*y^2-32*z*y)*x^2+(520*y^3+256*z*y^2)*x)]
  [3] parametrize(22*x^2-10*y^2+z^2+5*x*y+12*y*x-z*x);
  [(220*#6-10)*t^2+(-22*#6+1),(374*#6-17)*t^2+(-22*#6-43)*t,
  (220*#6+210)*t^2+(-374*#6+17)*t+22,(-y)/((22*#6-1)*x+z)]

```

Section 1.3.4 [genus], page 2,

1.4

1.4.1 tdeg

```

tdeg(Poly)
    :: Poly

return      0

Poly
• Poly
  [1] tdeg(u^3+v^3-x*y*z*w);
  4
  [956] tdeg((x^3+y^2+z)*(a^2+b+1));
  5

```

1.4.2 homzation

```

homzation(AF)
    :: x,yx,y,z

return      x,y,z

```

F *x,y*

- *x,yx,y,zx,y*

```
[1] homzation((x^2+4*x^3+6*x^4)-4*x^4*y
+(-2*x-4*x^2-2*x^3)*y^2+y^4);
(-4*y+6*z)*x^4+(-2*y^2+4*z^2)*x^3
+(-4*z*y^2+z^3)*x^2-2*z^2*y^2*x+z*y^4
[958] homzation(u*v+1);
Input must be polynomial of variable x,y
```

1.4.3 random_line

random_line(*Pt*,*B*[,*Seed*])
 :: *Pt*(=[*x,y,z*])

return *x,y,z*

Pt

B

Seed

- *Pt*(=[*x,y,z*]) -BB
- *Seedrandom*([*Seed*])

```
[1] random_line([0,0,1],1);
x-8*y
```

1.4.4 multia

multia(*F*,*Pt*)
 :: *F=0 Pt*(=[*x,y,z*])

return 0

F *x,y,z*

Pt

- *F=0 Pt*(=[*x,y,z*]) FN *Pt* 0NF=*0Pt*

```
[1] multia((4*y^2+4*z^2)*x^4+8*z^3*x^3+8*z^2*y^2*x^2-8*z^5*x+
4*z^4*y^2-4*z^6,[0,0,1]);
0
[2] multia((4*y^2+4*z^2)*x^4+8*z^3*x^3+8*z^2*y^2*x^2-8*z^5*x+
4*z^4*y^2-4*z^6,[0,1,0]);
4
[3] multia((4*y^2+4*z^2)*x^4+8*z^3*x^3+8*z^2*y^2*x^2-8*z^5*x+
4*z^4*y^2-4*z^6,[1,0,0]);
2
```

Section 1.3.2 [sing], page 1, Section 1.3.3 [nbh], page 2,

1.4.5 irr_conic

```
irr_conic(F)
    ::  $F \overline{Q}[x, y, z]$ 

return

F      x,y,z
•  $F \overline{Q}[x, y, z]$  irreducible
  [1] irr_conic( $x^2+y^2+z^2-x*y-y*z-z*x$ );
  reducible
  [2] fctr( $x^2+y^2+z^2-x*y-y*z-z*x$ );
  [[1,1],[ $x^2+(-y-z)*x+y^2-z*y+z^2$ ,1]]
```

1.4.6 lissajou

```
lissajou(M,N)
    ::  $x = \sin(M\theta), y = \cos(N\theta)$ 

return      x,y,z

M N
•  $x = \sin(M\theta), y = \cos(N\theta)$  x,y,z
  [984] lissajou(3,4);
   $64*x^8-128*z^2*x^6+80*z^4*x^4-16*z^6*x^2+16*z^2*y^6$ 
   $-24*z^4*y^4+9*z^6*y^2$ 
  [985] lissajou(2,7);
   $4096*x^{14}-14336*z^2*x^{12}+19712*z^4*x^{10}-13440*z^6*x^8$ 
   $+4704*z^8*x^6-784*z^{10}*x^4+49*z^{12}*x^2+4*z^{10}*y^4-4*z^{12}*y^2$ 
```

1.4.7 restriction

```
restriction(A,List)
    ::

return      x,y,z

A      adjoint1,adjoint2 x,y,z

List    [x,y,z]
• adjoint1,adjoint2 List Q
• List intersectsing
  ⟨undefined⟩ [adjoint1], page ⟨undefined⟩,
```


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