

## Preface

This volume is the proceedings of the *Workshop on Elliptic Integrable Systems* held at Kyoto University during the period November 8 – 11, 2004. The workshop was organized by M. Noumi (Kobe University) and K. Takasaki (Kyoto University) in the framework of the RIMS Project Research 2004: *Method of Algebraic Analysis in Integrable Systems*. This volume consists of 20 articles contributed by the lecturers of the workshop and by the three visitors to RIMS who were invited in connection with this workshop.

We are grateful to the members of the Research Institute for Mathematical Sciences and the Department of Mathematics of Kyoto University for their support in the organization of the workshop. We also express our thanks to the Department of Mathematics, Kobe University for allowing us to publish this collection of articles as a volume of the Rokko Lectures in Mathematics.

We hope that this volume will be helpful for people who are interested in the new developments in the theories of elliptic special functions and elliptic integrable systems.

Masatoshi NOUMI and Kanehisa TAKASAKI  
October 2005

# Workshop on Elliptic Integrable Systems

November 8 - 11, 2004

Department of Mathematics, Kyoto University

*organized by*

Masatoshi Noumi (Kobe) and Kanehisa Takasaki (Kyoto)

*supported by*

RIMS Project Research 2004

*Method of Algebraic Analysis in Integrable Systems*

## Program

### Monday, November 8

- |   |                              |
|---|------------------------------|
| Simon Ruijsenaars (CWI, Amsterdam)  | 13:00 ~ 14:00, 14:15 ~ 15:15 |
| <i>Elliptic Calogero-Moser type systems: 1. Survey, 2. Special topics</i>                               |                              |
| Vyacheslav Spiridonov (JINR, Dubna and MPI, Bonn)   | 14:15 ~ 15:15                |
| <i>Classical elliptic hypergeometric functions and their applications 1</i>                             |                              |
| Kouichi Takemura (Yokohama CU)  | 15:50 ~ 16:00                |
| <i>Heun equation and Painlevé equation</i>  |                              |
| Yasushi Komori (Nagoya)   | 16:00 ~ 16:30                |
| <i>Elliptic Ruijsenaars operators and elliptic hypergeometric integrals</i>                             |                              |
| Satoru Odake (Sinshu)   | 16:30 ~ 17:00                |
| <i>Equilibria of 'discrete' integrable systems and deformations of classical orthogonal polynomials</i> |                              |

### Tuesday, November 9

- |  |               |
|--|---------------|
| Mikhail Olshanetsky (ITEP, Moscow)   | 9:30 ~ 10:30  |
| <i>Isomonodromic deformations and elliptic tops</i>                            |               |
| Andrei Zotov (ITEP, Moscow)  | 10:45 ~ 11:45 |
| <i>Elliptic form of <math>P_{VI}</math> and reflection equation</i>            |               |
| Jan Felipe van Diejen (Univerisity of Talca, Chile)                            | 13:00 ~ 14:00 |
| <i>Elliptic hypergeometric sums and integrals (jointly with S. Spiridonov)</i> |               |
| Vyacheslav Spiridonov (JINR, Dubna and MPI, Bonn)                              | 14:15 ~ 15:15 |
| <i>Classical elliptic hypergeometric functions and their applications 2</i>    |               |

## Tuesday, November 9

- Yuji Shimizu (ICU, Tokyo) 15:30 ~ 16:00  
*On deformation of elliptic quantum plane*
- Hitoshi Konno (Hiroshima) 16:00 ~ 16:30  
*Elliptic quantum groups, the vertex-face correspondence and elliptic 6-j symbols*
- Yas-Hiro Quano (Suzuka) 16:30 ~ 17:00  
*Form factors and vertex operators in the XYZ antiferromagnet*

## Wednesday, November 10

- Alexei Zhedanov (DIPT, Donietsk) 9:30 ~ 10:30, 10:45 ~ 11:45  
*Generalized eigenvalue problems, multipoint Padé interpolation, dynamical algebras and biorthogonal rational functions* (two lectures)
- Yasuhiro Ohta (Kobe) 13:00 ~ 14:00  
*Elliptic Painlevé equation: I. Bilinear formalism*
- Kanehisa Takasaki (Kyoto) 14:15 ~ 15:15  
*Elliptic soliton equations and Sato Grassmannian*
- Olena Korovnichenko 15:30 ~ 16:00  
*Leonard triples in classical and quantum cases*
- Yasushi Kajihara (Osaka) 16:00 ~ 16:30  
*Multiple elliptic hypergeometric series — An approach from the Cauchy determinant and thier symmetry*
- Soichi Okada (Nagoya) 16:30 ~ 17:00  
*Generalizations of Cauchy's determinant identity and Schur's Pfaffian identity*

## Thursday, November 11

- Eric Rains (UC Davis) 9:30 ~ 10:30, 10:45 ~ 11:45  
*Symmetries and recurrences of elliptic hypergeometric integrals* (two lectures)
- Yasuhiko Yamada (Kobe) 13:00 ~ 14:00  
*Elliptic Painlevé equation: II. Geometric description*
- Masahiko Itoh (Aoyama Gakuin, Tokyo) 14:15 ~ 14:45  
 *$q$ -Difference shift for BC type Jackson integrals arising from 'elementary' symmetric polynomials*
- Michael Schlosser (Vienna) 14:45 ~ 15:15  
*Multidimensional matrix inversions and elliptic hypergeometric series on root systems*

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