Heap construction

```
                              11
                 9                        5
          15          7            12          8
        1    13     3    14     10    2      6    [4]
```

```
                          11
              /                        \
             9                          5
          /     \                   /        \
      [15]       14            12              8
      /  \      /  \          /   \          /   \
     1    13   3    7        10    2        6     4
```

```
                              11
                 9                        12
           15        14            10           8
         1    13    3    7        5    2      6    4
```

11

```
                              11
              ┌───────────────┴───────────────┐
             15                              12
         ┌────┴────┐                    ┌─────┴─────┐
        13        14                   10           8
       ┌─┴─┐     ┌─┴─┐                ┌─┴─┐        ┌─┴─┐
       1  [9]    3   7                5   2        6   4
```

```
                              15

            11                              12

       13         14                   10          8

     1     9    3     7             5     2      6     4
```

```
                              15
                   ┌──────────┴──────────┐
                  14                     12
             ┌─────┴─────┐          ┌─────┴─────┐
            13          ┌11┐       10           8
          ┌──┴──┐     ┌──┴──┐    ┌──┴──┐     ┌──┴──┐
          1     9     3     7    5     2      6     4
```

16

Retirement and promotion

```
            ┌─┐
            │4│
            └─┘
         ╱        ╲
       14          12
      ╱   ╲       ╱    ╲
    13     11   10      8
   ╱  ╲   ╱  ╲  ╱  ╲   ╱
  1    9 3    7 5    2 6      15
```

18

```
                            14
               4                        12
          13        11            10          8
        1     9   3     7       5     2     6       15



                            19
```

```
                               13
                  ┌────────────┴──────────────┐
                  6                            12
            ┌─────┴─────┐              ┌───────┴──────┐
            9           11            10              8
          ┌─┴─┐       ┌─┴─┐        ┌──┴──┐
          1   4       3   7        5     2      14        15
```

23

```
                              13
                   ┌──────────┴──────────┐
                  11                      12
             ┌─────┴─────┐          ┌──────┴──────┐
             9          ┌─┐        10              8
          ┌──┴──┐       │6│      ┌──┴──┐
          1     4     ┌─┴─┐      5     2      14        15
                      3   7
```

```
                          12
                 ┌────────┴────────┐
                11                 10
           ┌─────┴─────┐       ┌────┴────┐
           9           7      [2]        8
         ┌─┴─┐       ┌─┴─┐    │
         1   4       3   6    5    13    14    15
```

```
                              ┌─┐
                              │2│
                              └─┘
                     11                    10
              9            7            5         8

          1       4    3       6    12    13   14     15
```

```
                                 11
                    ┌─────────────┴─────────────┐
                    9                           10
              ┌─────┴─────┐               ┌─────┴─────┐
            ┌───┐         7               5           8
            │ 2 │       ┌─┴─┐
            └───┘       3   6
           ┌─┴─┐
           1   4                    12      13     14      15
```

```
                                    10
                    9                               6
            4               7               5               8

        1       2       3       11      12      13      14      15
```

```
                              ┌─┐
                              │2│
                              └─┘
                  7                       8

              4       3              5          6


          1       9      10     11    12    13    14      15
```

```
                              8
                  7                       6
             4         3             5         [2]

        1         9         10   11    12   13   14   15
```

```
          ┌─┐
          │1│
          └─┘
        ╱       ╲
       7          6
      ╱ ╲        ╱ ╲
     4   3      5   2

8     9    10    11    12    13    14    15
```

```
                              7
              1                              6

       4            3                  5            2


   8        9        10       11       12       13       14       15
```

```
                              6
                 4                          ┌─┐
                                            │2│
                                            └─┘
         1              3            5              7


   8        9      10       11      12      13      14      15
```

```
          ┌─┐
          │2│
          └─┘
       ╱         ╲
      4           5
    ╱   ╲
   1     3        6           7

8      9     10     11     12     13     14     15
```

```
                              5
                    4                      [2]

            1               3          6            7


      8         9       10      11     12      13      14      15
```

4

3

2

1

5

6

7

8

9

10

11

12

13

14

15

1   2   3

4   5   6   7

8   9   10   11   12   13   14   15

1   2   3

4   5   6   7

8   9   10   11   12   13   14   15

$\boxed{1}$

2   3

4   5   6   7

8   9   10   11   12   13   14   15